Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Wednesday 9 October 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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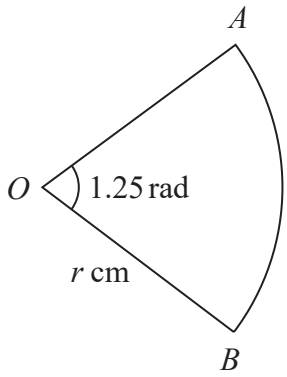


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is 1.25 radians.

Given that the area of the sector AOB is 15 cm^2

(a) find the exact value of r,

(2)

(b) find the exact length of the perimeter of the sector. Write your answer in simplest form.

(3)

(a) Area $= \frac{1}{2} r^2\theta \Rightarrow \frac{1}{2} r^2 \times 1.25 = 15$

$r^2 = \frac{15 \times 2}{1.25} = 24$

$r = \sqrt{24} = 2\sqrt{6}$

(b) Arc $AB = r\theta = 2\sqrt{6} \times 1.25 = 2.5\sqrt{6}$

Perimeter $= 2r + r\theta = 2 \times 2\sqrt{6} + 2\sqrt{6} \times 1.25$

$= 4\sqrt{6} + 2.5\sqrt{6}$

$= \frac{13\sqrt{6}}{2}$ cm

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Question 1 continued

Q1

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2. A tree was planted in the ground.

Exactly 2 years after it was planted, the height of the tree was 1.85 m.

Exactly 7 years after it was planted, the height of the tree was 3.45 m.

Given that the height, H metres, of the tree, t years after it was planted in the ground, can be modelled by the equation

$$H = at + b$$

where a and b are constants,

(a) find the value of a and the value of b.

(4)

(b) State, according to the model, the height of the tree when it was planted.

(1)

(a) $t = 2$, $H = 1.85$: $1.85 = 2a + b$

$t = 7$, $H = 3.45$: $3.45 = 7a + b$

Subtract: $1.6 = 5a \Rightarrow a = 0.32$

$b = 1.85 - 2 \times 0.32 = 1.85 - 0.64 = 1.21$

$a = 0.32$, $b = 1.21$

(b) When planted, $t = 0$: $H = b = 1.21$

Height when planted = 1.21 m

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Question 2 continued

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3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

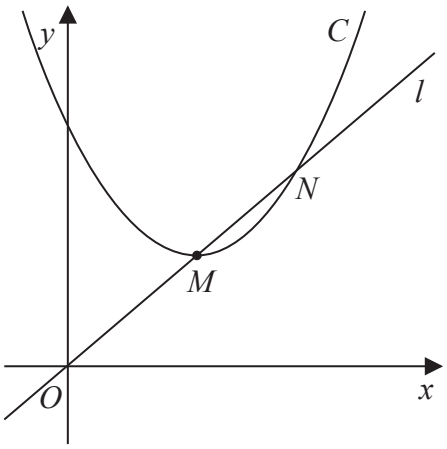


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = x^2 - 5x + 13$

The point M is the minimum point of C.

The straight line l passes through the origin O and intersects C at the points M and N as shown.

Find, showing your working,

(a) the coordinates of M,

(3)

(b) the coordinates of N.

(5)

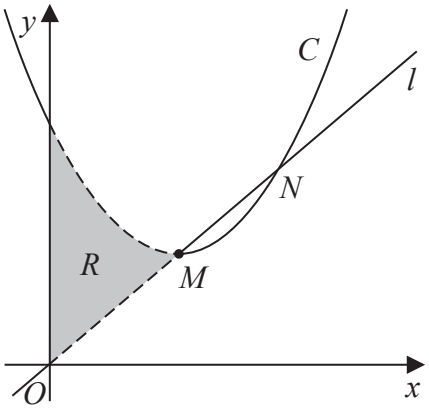


Figure 3

Figure 3 shows the curve C and the line l. The finite region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Use inequalities to define the region R.

(2)

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Question 3 continued

(a) $y = x^2 - 5x + 13 = (x - 2.5)^2 - 2.5^2 + 13$

$= (x - 2.5)^2 + 6.75$

Minimum when $x - 2.5 = 0$: $M = (2.5, 6.75)$

(b) l passes through O and M: gradient $= \frac{6.75}{2.5} = 2.7$

l: $y = 2.7x$

$2.7x = x^2 - 5x + 13$

$x^2 - 7.7x + 13 = 0$

$(x - 2.5)(x - 5.2) = 0$

$x = 2.5$ (at M) or $x = 5.2$

$y = 2.7 \times 5.2 = 14.04$

$N = (5.2, 14.04)$

(c) R: $y < x^2 - 5x + 13$, $y > 2.7x$, $0 < x < 2.5$

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Question 3 continued

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Question 3 continued

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4. A parallelogram $ABCD$ has area $40\,\text{cm}^2$

Given that AB has length $10\,\text{cm}$, BC has length $6\,\text{cm}$ and angle DAB is obtuse, find

(a) the size of angle DAB, in degrees, to 2 decimal places,

(3)

(b) the length of diagonal BD, in cm, to one decimal place.

(2)

(a) Area $= AB \times AD \times \sin\theta \quad \Rightarrow \quad 40 = 10 \times 6 \times \sin\theta$

$\sin\theta = \frac{40}{60} = \frac{2}{3}$

$\theta = \sin^{-1}\frac{2}{3} = 41.8103...^\circ \quad$ or $\quad \theta = 180^\circ - 41.8103...^\circ$

Angle DAB is obtuse: $\quad DAB = 138.1896...^\circ$

$DAB = 138.19^\circ$

(b) $BD^2 = AB^2 + AD^2 - 2 \times AB \times AD \times \cos DAB$

$BD^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \times \cos 138.1896...^\circ$

$BD^2 = 136 + 89.4427... = 225.4427...$

$BD = 15.0147... = 15.0\ \text{cm}$

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Question 4 continued

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5. A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving the answer in simplest form.

(3)

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

(b) Find the equation of the normal to the curve at P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

(a) $y = \frac{x^3}{6} + 4x^{1/2} - 15$

$\frac{dy}{dx} = \frac{3x^2}{6} + 4 \times \frac{1}{2} x^{-1/2}$

$\frac{dy}{dx} = \frac{1}{2} x^2 + 2x^{-1/2}$

(b) At $x = 4$: $\frac{dy}{dx} = \frac{1}{2} \times 4^2 + \frac{2}{\sqrt{4}} = 8 + 1 = 9$

Gradient of normal $= -\frac{1}{9}$

$y - \frac{11}{3} = -\frac{1}{9}(x - 4)$

$9y - 33 = -x + 4$

$x + 9y - 37 = 0$

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Question 5 continued

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6. The curve C has equation $y = \frac{4}{x} + k$, where k is a positive constant.

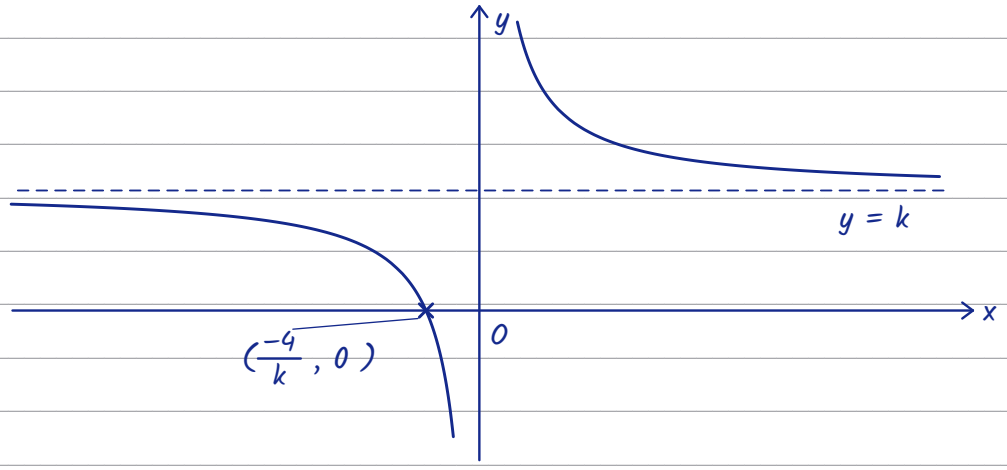
(a) Sketch a graph of C, stating the equation of the horizontal asymptote and the coordinates of the point of intersection with the x-axis.

(3)

The line with equation $y = 10 - 2x$ is a tangent to C.

(b) Find the possible values for k.

(5)



(a) Horizontal asymptote: $y = k$

Meets x-axis where $\frac{4}{x} + k = 0 \Rightarrow x = -\frac{4}{k}$: $(-\frac{4}{k}, 0)$

(b) $10 - 2x = \frac{4}{x} + k$

$10x - 2x^2 = 4 + kx$

$2x^2 + (k - 10)x + 4 = 0$

Tangent ⇒ equal roots: $b^2 - 4ac = 0$

$(k - 10)^2 - 4 \times 2 \times 4 = 0$

$(k - 10)^2 = 32$

$k - 10 = \pm\sqrt{32} = \pm 4\sqrt{2}$

$k = 10 + 4\sqrt{2}$ or $k = 10 - 4\sqrt{2}$

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Question 6 continued

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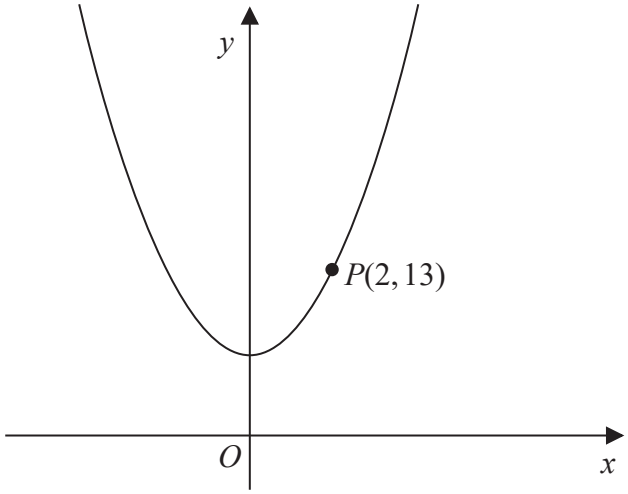


Figure 4

Figure 4 shows part of the curve with equation $y = 2x^2 + 5$

The point $P(2,13)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Give your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

(a) $\frac{dy}{dx} = 4x$. At $x = 2$: gradient of tangent $= 4 \times 2 = 8$

(b) $y_Q = 2(2 + h)^2 + 5 = 2(4 + 4h + h^2) + 5 = 13 + 8h + 2h^2$

Gradient $PQ = \frac{(13 + 8h + 2h^2) - 13}{(2 + h) - 2} = \frac{8h + 2h^2}{h}$

Gradient $PQ = 8 + 2h$

(c) As $h \to 0$, gradient $PQ = 8 + 2h \to 8$, which is the gradient of the tangent at P found in (a) (the limit of the chord gradient).

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Question 7 continued

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8. Solve, using algebra, the equation

$$x - 6x^{\frac{1}{2}} + 4 = 0$$

Fully simplify your answers, writing them in the form $a + b\sqrt{c}$, where a, b and c are integers to be found.

(5)

Let $y = x^{1/2}$, so $x = y^2$:

$y^2 - 6y + 4 = 0$

$(y - 3)^2 - 9 + 4 = 0 \quad \Rightarrow \quad (y - 3)^2 = 5$

$x^{1/2} = y = 3 \pm \sqrt{5}$ (both positive, so both valid)

$x = (3 \pm \sqrt{5})^2 = 9 \pm 6\sqrt{5} + 5$

$x = 14 + 6\sqrt{5}$ or $x = 14 - 6\sqrt{5}$

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Question 8 continued

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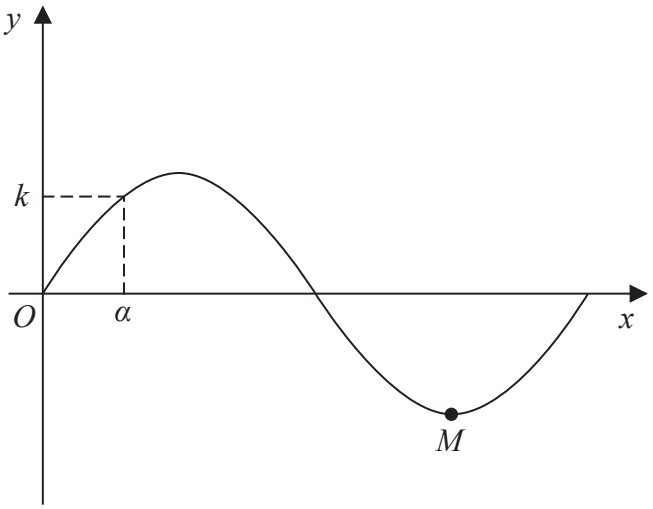


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = \sin\left(\frac{x}{12}\right)$, where x is measured in radians. The point M shown in Figure 5 is a minimum point on C.

(a) State the period of C.

(1)

(b) State the coordinates of M.

(1)

The smallest positive solution of the equation $\sin\left(\frac{x}{12}\right) = k$, where k is a constant, is α.

Find, in terms of α,

(c) (i) the negative solution of the equation $\sin\left(\frac{x}{12}\right) = k$ that is closest to zero,

(ii) the smallest positive solution of the equation $\cos\left(\frac{x}{12}\right) = k$.

(2)

(a) Period $= 2\pi \div \frac{1}{12} = 24\pi$

(b) Minimum when $\frac{x}{12} = \frac{3\pi}{2} \Rightarrow x = 18\pi$; $M = (18\pi, -1)$

(c)(i) $\frac{x}{12} = -\pi - \frac{\alpha}{12} \Rightarrow x = -12\pi - \alpha$

(ii) $\cos\frac{x}{12} = k = \sin\frac{\alpha}{12} = \cos\left(\frac{\pi}{2} - \frac{\alpha}{12}\right) \Rightarrow x = 6\pi - \alpha$

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Question 9 continued

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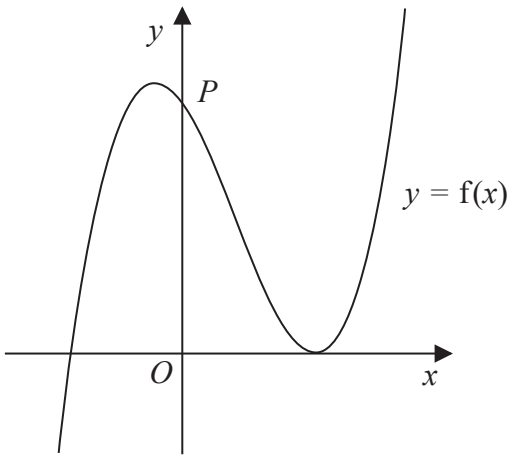


Figure 6

Figure 6 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x + 5)(x - 3)^2$$

(a) Deduce the values of x for which $f(x) \leqslant 0$

(2)

The curve crosses the y-axis at the point P, as shown.

(b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

(c) Hence, or otherwise, find

(i) the coordinates of P,

(ii) the gradient of the curve at P.

(2)

The curve with equation $y = f(x)$ is translated two units in the positive x direction to a curve with equation $y = g(x)$.

(d) (i) Find $g(x)$, giving your answer in a simplified factorised form.

(ii) Hence state the y intercept of the curve with equation $y = g(x)$.

(3)

(a) $f(x) = 0$ at $x = -\frac{5}{2}$ (crosses) and $x = 3$ (touches)

$f(x) \leqslant 0$: $x \leqslant -\frac{5}{2}$ or $x = 3$

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Question 10 continued

(b) $f(x) = (2x + 5)(x - 3)^2 = (2x + 5)(x^2 - 6x + 9)$

$= 2x^3 - 12x^2 + 18x + 5x^2 - 30x + 45$

$f(x) = 2x^3 - 7x^2 - 12x + 45$

(c) (i) $x = 0$: $f(0) = 45$ $P = (0, 45)$

(ii) $f'(x) = 6x^2 - 14x - 12$

At P: $f'(0) = -12$ gradient $= -12$

(d) (i) $g(x) = f(x - 2) = (2(x - 2) + 5)(x - 2 - 3)^2$

$g(x) = (2x + 1)(x - 5)^2$

(ii) $g(0) = (1)(-5)^2 = 25$: y intercept is 25

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Question 10 continued

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Question 10 continued

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11. A curve has equation $y = f(x)$.

The point $P\left(4, \frac{32}{3}\right)$ lies on the curve.

Given that

- $f''(x) = \frac{4}{\sqrt{x}} - 3$
- $f'(x) = 5$ at P

find

(a) the equation of the tangent to the curve at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found,

(2)

(b) $f(x)$.

(8)

(a) Tangent at P: $y - \frac{32}{3} = 5(x - 4)$

$y = 5x - 20 + \frac{32}{3}$

$y = 5x - \frac{28}{3}$

(b) $f'(x) = \frac{4}{\sqrt{x}} - 3 = 4x^{-1/2} - 3$

$f(x) = \frac{4x^{1/2}}{1/2} - 3x + k = 8x^{1/2} - 3x + k$

$f(4) = 5$: $8 \times 2 - 12 + k = 5 \Rightarrow k = 1$

$f(x) = 8x^{1/2} - 3x + 1$

$f(x) = \frac{8x^{3/2}}{3/2} - \frac{3x^2}{2} + x + d = \frac{16}{3}x^{3/2} - \frac{3}{2}x^2 + x + d$

$f(4) = \frac{32}{3}$: $\frac{16}{3} \times 8 - \frac{3}{2} \times 16 + 4 + d = \frac{32}{3}$

$\frac{128}{3} - 24 + 4 + d = \frac{32}{3} \Rightarrow d = \frac{32}{3} - \frac{128}{3} + 20 = -12$

$f(x) = \frac{16}{3}x^{3/2} - \frac{3}{2}x^2 + x - 12$

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Question 11 continued

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Question 11 continued

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