Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Wednesday 23 October 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. Two particles, P and Q, have masses $3m$ and $2m$ respectively. The particles are connected by a light inextensible string. Initially P and Q are at rest on a smooth horizontal plane with the string slack.

Particle P is then projected along the plane directly away from Q with speed $4u$. At the same instant, particle Q is projected along the plane in the opposite direction with speed $3u$.

Find

(a) the common speed of the particles immediately after the string becomes taut,

(3)

(b) the magnitude of the impulse exerted on Q at the instant when the string becomes taut.

(3)

(a) Take the direction of motion of P as positive.

Let the common speed after the string becomes taut be v.

CLM: $3m \times 4u - 2m \times 3u = 3mv + 2mv$

$12mu - 6mu = 5mv$

$6mu = 5mv$

$v = \frac{6u}{5}$

Common speed $= \frac{6u}{5}$

(b) Impulse on Q = change in momentum of Q

$I = 2m(\frac{6u}{5} - (-3u))$

$I = 2m \times \frac{21u}{5}$

Magnitude of impulse $= \frac{42mu}{5}$

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Question 1 continued

Q1

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2. A small ball is released from rest from a point that is 40 m above horizontal ground. The ball bounces on the ground and rebounds vertically. Each time the ball bounces on the ground, the speed of the ball is instantaneously reduced by 50%. The ball is modelled as a particle moving freely under gravity, from the instant when it is released until it first hits the ground, and between each successive bounce.

(a) Find the time from the instant when the ball is released from rest to the instant when it hits the ground for the second time.

(5)

(b) Find the maximum height reached by the ball above the ground after the ball's third bounce.

(4)

(a) Downwards positive. First drop: $u = 0$, $s = 40$, $a = 9.8$

$s = ut + \frac{1}{2}at^2$: $\quad 40 = \frac{1}{2} \times 9.8 \times t_1^2$

$t_1 = \sqrt{(80 \div 9.8)} = 2.857\ldots$ s

Speed on hitting ground: $v_1 = \sqrt{(2 \times 9.8 \times 40)} = 28$ m s^{-1}

Speed after 1st bounce = 50% of 28 = 14 m s^{-1}

Time from 1st bounce to 2nd bounce (up and back down):

$t_2 = \frac{2 \times 14}{9.8} = \frac{20}{7} = 2.857\ldots$ s

Total time = $2.857\ldots + 2.857\ldots = 5.714\ldots$

Total time = 5.71 s

(b) Speeds after successive bounces:

$v_2 = 14$, $\quad v_3 = 7$, $\quad v_4 = 3.5$ m s^{-1} (after the 3rd bounce)

At maximum height $v = 0$: $\quad v^2 = u^2 + 2as$

$0 = 3.5^2 - 2 \times 9.8 \times h$

$h = \frac{12.25}{19.6} = 0.625$

Maximum height = 0.625 m

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Question 2 continued

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Question 2 continued

Q2

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3. A car of mass 800 kg is towing a trailer of mass 400 kg up a straight road using a towbar. The towbar is parallel to the road and parallel to the direction of motion of the car. The road is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{7}$. The engine of the car produces a constant driving force of magnitude D newtons. The resistance to the motion of the car from non-gravitational forces is modelled as a single force of magnitude 420 N. The resistance to the motion of the trailer from non-gravitational forces is modelled as a single force of magnitude 300 N. The car and trailer are modelled as particles and the towbar is modelled as a light rod.

Given that the tension in the towbar is 2060 N, find the value of D.

(7)

$\sin\alpha = \frac{1}{7}$. Let the acceleration up the road be a m s^{-2}.

Trailer, N2L up the slope:

$2060 - 300 - 400g \times \frac{1}{7} = 400a$

$2060 - 300 - 560 = 400a$

$1200 = 400a \quad \Rightarrow \quad a = 3$ m s^{-2}

Car, N2L up the slope:

$D - 420 - 800g \times \frac{1}{7} - 2060 = 800a$

$D - 420 - 1120 - 2060 = 800 \times 3$

$D - 3600 = 2400$

$D = 6000$

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Question 3 continued

Q3

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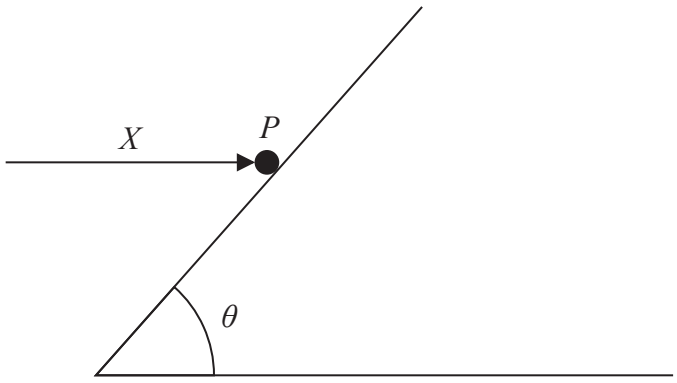


Figure 1

A particle, P, of mass km lies on a fixed rough plane. The plane is inclined to the horizontal at an acute angle θ. A horizontal force of magnitude X acts on P, as shown in Figure 1. The line of action of the force lies in the vertical plane which contains the line of greatest slope of the inclined plane that passes through P. The coefficient of friction between P and the inclined plane is μ.

When $X = mg$, the particle P is in equilibrium and on the point of sliding down the plane.

(a) Show that $\mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}$

(10)

(b) Deduce that, when $k = 1$, θ must be greater than 45°

(2)

(a) P is on the point of sliding down, so friction F acts up the plane.

Resolving perpendicular to the plane (X = mg):

$R = kmg\cos\theta + mg\sin\theta$

Resolving parallel to the plane:

$F + mg\cos\theta = kmg\sin\theta$

$F = kmg\sin\theta - mg\cos\theta$

Limiting friction: $F = \mu R$

$kmg\sin\theta - mg\cos\theta = \mu(kmg\cos\theta + mg\sin\theta)$

Cancel mg: $k\sin\theta - \cos\theta = \mu(k\cos\theta + \sin\theta)$

$\mu = \dfrac{k\sin\theta - \cos\theta}{k\cos\theta + \sin\theta}$

Divide top and bottom by cos θ:

$\mu = \dfrac{k\sin\theta \div \cos\theta - \cos\theta \div \cos\theta}{k\cos\theta \div \cos\theta + \sin\theta \div \cos\theta}$

$\mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}$ as required

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Question 4 continued

(b) When $k = 1$:

$$\mu = \frac{\tan\theta - 1}{1 + \tan\theta}$$

The plane is rough, so $\mu > 0$.

θ is acute, so $\tan\theta > 0$ and $1 + \tan\theta > 0$.

Hence $\tan\theta - 1 > 0$

$\tan\theta > 1 \quad \Rightarrow \quad \theta > 45°$

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Question 4 continued

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Question 4 continued

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Q4

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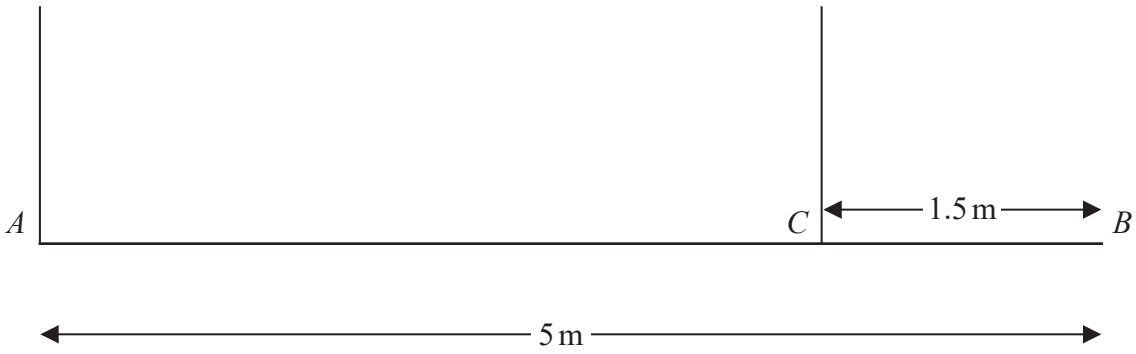


Figure 2

A non-uniform beam, AB, has length 5 m and mass 12 kg. The beam is suspended in a horizontal position by two vertical ropes. One rope is attached to the beam at A. The other rope is attached to the beam at C, where $CB = 1.5$ m, as shown in Figure 2. The distance of the centre of mass of the beam from A is 1.75 m. The beam is modelled as a non-uniform rod and the ropes are modelled as light inextensible strings.

A particle of mass M kg is now placed on the beam at B and the beam remains in equilibrium in a horizontal position.

(a) Find the largest possible value of M.

(3)

The particle at B is now removed and a particle of mass 15 kg is now placed on the beam at the point D, where $AD = x$ metres. The beam remains in equilibrium in a horizontal position.

Given that the tension in the rope attached to the beam at C is now twice the tension in the rope attached to the beam at A,

(b) find the value of x.

(5)

(a) M is largest when the beam is about to tip about C, so $T_A = 0$.

$AC = 5 - 1.5 = 3.5$ m, so G is $3.5 - 1.75 = 1.75$ m from C.

$M(C)$: $Mg \times 1.5 = 12g \times 1.75$

$1.5M = 21$

$M = 14$

(b) Let $T_A = T$, so $T_C = 2T$.

Resolving vertically: $T + 2T = 12g + 15g$

$3T = 27g \Rightarrow T = 9g$

$M(A)$: $2T \times 3.5 = 12g \times 1.75 + 15g \times x$

$18g \times 3.5 = 21g + 15gx$

$63 = 21 + 15x$

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Question 5 continued

$15x = 42$

$x = 2.8$

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Question 5 continued

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Question 5 continued

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6. An athlete runs a 200 m race along a straight horizontal track.

In a model of the motion of the athlete, air resistance is ignored, the athlete starts from rest at time $t = 0$ seconds and moves with uniform acceleration $0.8\,\text{m s}^{-2}$ for T seconds, reaching a speed of $V\,\text{m s}^{-1}$. She then maintains this speed until she crosses the finishing line.

The total time from when the athlete starts to when she crosses the finishing line is 30 s.

(a) Sketch a speed-time graph for the model of the motion of the athlete from the instant when she starts to the instant when she crosses the finishing line.

(2)

(b) Write down an expression for V in terms of T.

(1)

(c) Show that $T^2 - kT + 500 = 0$, where k is a constant to be found.

(4)

(d) Hence find the value of T, justifying your answer carefully.

(3)

(e) Considering your speed-time graph or otherwise, state two ways, apart from including air resistance, in which the model could be made to be more realistic.

(2)

(a)



(b) $v = u + at$: $V = 0 + 0.8T$

$V = 0.8T$

(c) Distance = area under the graph (trapezium) = 200

$$\frac{(30 + (30 - T))V}{2} = 200$$

$$\frac{(60 - T) \times 0.8T}{2} = 200$$

$$(60 - T) \times 0.8T = 400$$

$$48T - 0.8T^2 = 400$$

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Question 6 continued

$0.8T^2 - 48T + 400 = 0$

$T^2 - 60T + 500 = 0$, so $k = 60$

(d) $(T - 10)(T - 50) = 0$

$T = 10$ or $T = 50$

The total time is 30 s, so $T < 30$ and $T = 50$ is impossible.

$T = 10$

(e) 1. The acceleration would not be constant (uniform).

2. The change from accelerating to constant speed would not be instantaneous (and she would not keep exactly constant speed).

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Question 6 continued

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Question 6 continued

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7. Two forces, **F** and **G**, act on a particle. The force **F** has magnitude 4 N and acts in a direction with a bearing of 120° and the force **G** has magnitude 6 N and acts due north.

Given that $\mathbf{P} = 2\mathbf{F} + \mathbf{G}$, find

(i) the magnitude of **P**

(ii) the direction of **P**, giving your answer as a bearing to the nearest degree.

(7)

(i) Take i east and j north. 2F has magnitude 8 N on a bearing of 120°.

$2F = 8 \sin 120° \, i + 8 \cos 120° \, j = 4\sqrt{3} \, i - 4j$

$G = 6j$

$P = 2F + G = 4\sqrt{3} \, i + 2j$

$|P|^2 = (8 \cos 30°)^2 + (6 - 8 \sin 30°)^2 = 48 + 4 = 52$

$|P| = \sqrt{52} = 7.2111...$

Magnitude of P = 7.21 N

(ii) Let β be the angle between P and north (clockwise).

$$\tan \beta = \frac{8 \cos 30°}{6 - 8 \sin 30°} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$$

$\beta = 73.897...°$

Bearing of P = 074°

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Question 7 continued

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8. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity. The velocity of A is $20\,\text{km h}^{-1}$ due west and the velocity of B is $40\,\text{km h}^{-1}$ on a bearing of 150°. The boats are modelled as particles.

At noon, the position vector of A is $60\mathbf{i}$ km and B is at the origin O. At time t hours after noon, the position vector of A is $\mathbf{r}$ km and the position vector of B is $\mathbf{s}$ km.

(a) Find the velocity of B in the form $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(3)

(b) Find expressions for $\mathbf{r}$ and $\mathbf{s}$ in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(3)

(c) Find the time, to the nearest minute, at which the distance between the boats is the same as it was at noon.

(8)

(a) Bearing 150° is 60° below the east direction.

$v_B = (40\cos 60°)i + (-40\sin 60°)j$

$v_B = (20i - 20\sqrt{3}\,j)$ km h^{-1}

(b) $v_A = -20i$

$r = 60i + t(-20i) = (60 - 20t)i$

$s = t(20i - 20\sqrt{3}\,j) = 20t\,i - 20\sqrt{3}\,t\,j$

(c) Distance at noon = 60 km

$s - r = t(20i - 20\sqrt{3}\,j) - [60i + t(-20i)]$

$s - r = (40t - 60)i - 20\sqrt{3}\,t\,j$

$|s - r| = 60$:

$(40t - 60)^2 + (-20\sqrt{3}\,t)^2 = 60^2$

$1600t^2 - 4800t + 3600 + 1200t^2 = 3600$

$2800t^2 - 4800t = 0$

$400t(7t - 12) = 0$

$t = 0$ (noon) or $t = \frac{12}{7} = 1.714\ldots$ hours

$\frac{12}{7}$ h = 1 h 42.857... min = 1 h 43 min to the nearest minute

Time = 1:43 pm (13:43)

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Q8

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