Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 14 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01R**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. The sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+2} = 3u_{n+1} - 2u_n$$

Given that

- $u_1 = 7$
- $u_3 = 4$

(a) find the value of u_2

(2)

(b) find $\sum_{r=1}^{4} (u_r + 2r)$

(3)

(a) $u_3 = 3u_2 - 2u_1$

$4 = 3u_2 - 2 \times 7 \Rightarrow 3u_2 = 18$

$u_2 = 6$

(b) $u_4 = 3u_3 - 2u_2 = 3 \times 4 - 2 \times 6 = 0$

$\Sigma_{r=1}^{4} (u_r + 2r) = (7 + 2) + (6 + 4) + (4 + 6) + (0 + 8)$

$= 9 + 10 + 10 + 8$

$= 37$

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Question 1 continued

(Total for Question 1 is 5 marks)

2. The table shows corresponding values of x and y for a continuous curve with equation $y = f(x)$ between $x = -4$ and $x = 5$, where a is a constant.

x	-4	-2.5	-1	0.5	2	3.5	5
y	4.16	2.91	a	1.73	1.37	1.43	2.28

The trapezium rule is used with all the y values in the table to find an approximation for

$$\int_{-4}^{5} f(x)\,dx$$

Given that the value of this approximation is 19.3

(a) find the value of the constant a to 3 significant figures.

(3)

(b) Use the given answer of 19.3 to find an approximate value for

$$\int_{-4}^{5} (2f(x) - 3)\,dx$$

(2)

(a) $h = (5 - (-4)) \div 6 = 1.5$

$(1.5/2)\{4.16 + 2.28 + 2(2.91 + a + 1.73 + 1.37 + 1.43)\} = 19.3$

$0.75\{6.44 + 2(7.44 + a)\} = 19.3$

$21.32 + 2a = 19.3 \div 0.75 = 25.7333...$

$2a = 4.41333... \Rightarrow a = 2.20666...$

$a = 2.21$ (3 s.f.)

(b) $\int_{-4}^{5} (2f(x) - 3)\,dx = 2\int_{-4}^{5} f(x)\,dx - [3x]_{-4}^{5}$

$= 2 \times 19.3 - (15 - (-12))$

$= 38.6 - 27$

$= 11.6$

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Question 2 continued

(Total for Question 2 is 5 marks)

3. A circle has equation

$$x^2 + y^2 + 8x - 14y - 79 = 0$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is nearest the origin O,

(b) find the exact length of OP

(2)

(a) $x^2 + 8x + y^2 - 14y = 79$

$(x + 4)^2 - 16 + (y - 7)^2 - 49 = 79$

$(x + 4)^2 + (y - 7)^2 = 144$

(i) Centre $(-4, 7)$

(ii) Radius $= \sqrt{144} = 12$

(b) Distance from O to centre:

$OC = \sqrt{((-4)^2 + 7^2)} = \sqrt{(16 + 49)} = \sqrt{65}$

O lies inside the circle ($\sqrt{65} < 12$), and P lies on the line OC extended,

so OP = radius $- OC$

$OP = 12 - \sqrt{65}$

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Question 3 continued

(Total for Question 3 is 5 marks)

4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(3 + 2x)^6$$

giving each coefficient in simplest form.

(4)

(b) Hence find the coefficient of x^2 in the expansion of

$$\left(2x^2 - \frac{1}{6x}\right)(3 + 2x)^6$$

(3)

(a) $(3 + 2x)^6 = 3^6 + {}^6C_1 \times 3^5 \times (2x) + {}^6C_2 \times 3^4 \times (2x)^2$

$+ {}^6C_3 \times 3^3 \times (2x)^3 + \ldots$

$= 729 + 6 \times 243 \times 2x + 15 \times 81 \times 4x^2 + 20 \times 27 \times 8x^3 + \ldots$

$= 729 + 2916x + 4860x^2 + 4320x^3 + \ldots$

(b) $(2x^2 - 1/(6x))(729 + 2916x + 4860x^2 + 4320x^3 + \ldots)$

x^2 terms: $2x^2 \times 729 = 1458x^2$

$-1/(6x) \times 4320x^3 = -720x^2$

Coefficient of $x^2 = 1458 - 720$

$= 738$

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Question 4 continued

(Total for Question 4 is 7 marks)

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The depth of water, D metres, in a harbour on a particular day is given by the equation

$$D = 8 + 5\sin\left(\frac{\pi t}{6} + 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after **midnight**.

(a) Show that the depth of water in the harbour at 2 am is just over 4 metres.

(1)

(b) Find, to the nearest minute, the first time after **midday** when the depth of water in the harbour is exactly 6 metres.

(5)

(a) $t = 2$: $D = 8 + 5\sin(2\pi/6 + 3) = 8 + 5\sin(4.0472...)$

$= 8 + 5 \times (-0.78680...) = 4.066... = 4.07$ m, just over 4 metres

(b) $6 = 8 + 5\sin(\pi t/6 + 3) \Rightarrow \sin(\pi t/6 + 3) = -2/5$

$\arcsin(-0.4) = -0.41152...$

So $\pi t/6 + 3 = \pi + 0.41152... = 3.5531...$, $2\pi - 0.41152... = 5.8717...$,

or $+ 2\pi$: $9.8363...$, $12.1549...$

After midday $t \geqslant 12$, so $\pi t/6 + 3 \geqslant 2\pi + 3 = 9.283...$

First solution: $\pi t/6 + 3 = 9.8363...$

$t = 6(9.8363... - 3)/\pi = 6 \times 6.8363.../\pi = 13.0564...$ hours

$0.0564... \times 60 = 3.38$ minutes

Time = 13:03 (1:03 pm)

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Question 5 continued

(Total for Question 5 is 6 marks)

6. $f(x) = 4x^3 + px^2 + 8x + q$

where p and q are constants.

Given that

- $(2x + 3)$ is a factor of $f(x)$
- $f(x)$ has a remainder of -5 when divided by $(x + 2)$

(a) (i) show that $p = 10$

(ii) find the value of q.

(5)

(b) Hence find the range of values of x for which $f(x)$ is decreasing.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

(a)(i) $(2x + 3)$ is a factor $\Rightarrow f(-3/2) = 0$

$4(-27/8) + p(9/4) + 8(-3/2) + q = 0$

$-27/2 + 9p/4 - 12 + q = 0 \Rightarrow 9p + 4q = 102$ (1)

Remainder -5 when divided by $(x + 2) \Rightarrow f(-2) = -5$

$4(-8) + 4p - 16 + q = -5 \Rightarrow 4p + q = 43$ (2)

(2) $\times 4$: $16p + 4q = 172$

Subtract (1): $7p = 70 \Rightarrow p = 10$ as required

(ii) $q = 43 - 4 \times 10$

$q = 3$

(b) $f(x) = 4x^3 + 10x^2 + 8x + 3$

$f'(x) = 12x^2 + 20x + 8$

$f'(x) = 0$: $4(3x^2 + 5x + 2) = 0$

$4(3x + 2)(x + 1) = 0 \Rightarrow x = -2/3, \ x = -1$

$f(x)$ is decreasing when $f'(x) < 0$, i.e. between the roots

$-1 < x < -2/3$

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 2\pi$, the equation

$$3\sin x \tan x = 11 + \cos x$$

giving the answers in radians to 3 decimal places.

(5)

(ii) Given that

- $0 < \theta < 90°$
- $\cos\theta = \frac{1}{3}$

find, in simplest form, the exact value of $\tan\theta$

(2)

(i) $3 \sin x \tan x = 11 + \cos x$, $\tan x = \sin x/\cos x$

$3 \sin^2 x/\cos x = 11 + \cos x$

$3 \sin^2 x = 11 \cos x + \cos^2 x$

$3(1 - \cos^2 x) = 11 \cos x + \cos^2 x$

$4\cos^2 x + 11 \cos x - 3 = 0$

$(4 \cos x - 1)(\cos x + 3) = 0$

$\cos x = 1/4$ ($\cos x = -3$ has no solutions)

$x = \arccos(1/4) = 1.31811...$, $x = 2\pi - 1.31811... = 4.96506...$

$x = 1.318,\ 4.965$

(ii) $\sin^2\theta = 1 - \cos^2\theta = 1 - 1/9 = 8/9$

$0 < \theta < 90° \Rightarrow \sin\theta = \sqrt{8}/3 = 2\sqrt{2}/3$

$\tan\theta = \sin\theta/\cos\theta = (2\sqrt{2}/3) \div (1/3)$

$\tan\theta = 2\sqrt{2}$

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Question 7 continued

(Total for Question 7 is 7 marks)

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8. (i) (a) In an **arithmetic** series the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

(3)

(b) Hence find

$$900 + 892 + 884 + \ldots + 500$$

(3)

(ii) Given that the first three terms of a **geometric** series are

$$k+4 \qquad k-2 \qquad 11-k$$

where k is a constant,

(a) show that

$$2k^2 - 11k - 40 = 0$$

(3)

Given also that this series is convergent,

(b) find the value of S_∞

(4)

(i)(a) $S_n = a + (a + d) + \ldots + (a + (n-2)d) + (a + (n-1)d)$

$S_n = (a + (n-1)d) + (a + (n-2)d) + \ldots + (a + d) + a$

Adding: $2S_n = (2a + (n-1)d) + (2a + (n-1)d) + \ldots + (2a + (n-1)d)$

$2S_n = n(2a + (n-1)d) \Rightarrow S_n = (n/2)(2a + (n-1)d)$ as required

(b) $a = 900$, $d = -8$, last term 500

$900 + (n-1)(-8) = 500 \Rightarrow n - 1 = 400/8 = 50 \Rightarrow n = 51$

$S_{51} = (51/2)(2 \times 900 + 50 \times (-8)) = (51/2)(1800 - 400)$

$= (51/2) \times 1400 = 35700$

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Question 8 continued

(ii)(a) Common ratio: $(k - 2)/(k + 4) = (11 - k)/(k - 2)$

$(k - 2)^2 = (k + 4)(11 - k)$

$k^2 - 4k + 4 = 11k - k^2 + 44 - 4k$

$k^2 - 4k + 4 = 44 + 7k - k^2$

$2k^2 - 11k - 40 = 0$ as required

(b) $(2k + 5)(k - 8) = 0 \Rightarrow k = -5/2$ or $k = 8$

$k = -5/2$: terms $3/2, -9/2, 27/2$, $r = -3$, $|r| > 1$ so not convergent

$k = 8$: terms $12, 6, 3$, $a = 12$, $r = 6/12 = 1/2$, $|r| < 1$ so convergent

$S_\infty = a/(1 - r) = 12/(1 - 1/2)$

$S_\infty = 24$

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 13 marks)

9. Given that

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

(a) $t = 4$: $3\log_2(4+4) - 2\log_2(4-2) = 3\log_2 8 - 2\log_2 2$

$= 3 \times 3 - 2 \times 1 = 9 - 2 = 7$ ✓ so $t = 4$ is a solution

(b) $\log_2(t+4)^3 - \log_2(t-2)^2 = 7$

$\log_2[(t+4)^3/(t-2)^2] = 7$

$(t+4)^3/(t-2)^2 = 2^7 = 128$

$(t+4)^3 = 128(t-2)^2$

$t^3 + 12t^2 + 48t + 64 = 128(t^2 - 4t + 4) = 128t^2 - 512t + 512$

$t^3 - 116t^2 + 560t - 448 = 0$ as required

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Question 9 continued

(c) $t = 4$ is a root, so $(t - 4)$ is a factor

$t^3 - 116t^2 + 560t - 448 = (t - 4)(t^2 - 112t + 112)$

$t^2 - 112t + 112 = 0$

$t = (112 \pm \sqrt{(112^2 - 4 \times 1 \times 112)})/2 = (112 \pm \sqrt{12096})/2$

$\sqrt{12096} = \sqrt{(576 \times 21)} = 24\sqrt{21}$

$t = (112 \pm 24\sqrt{21})/2 = 56 \pm 12\sqrt{21}$

$56 - 12\sqrt{21} = 1.009... < 2$, so $\log_2(t - 2)$ is undefined: reject

$56 + 12\sqrt{21} > 2$ is valid

$t = 4, \quad t = 56 + 12\sqrt{21}$

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 9 marks)

10.

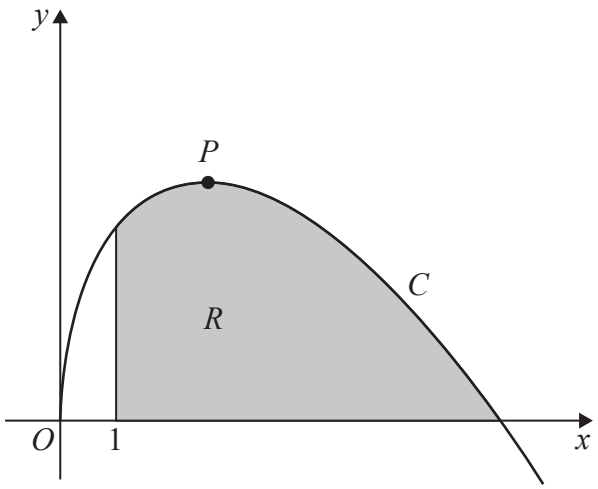


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \qquad x > 0$$

The point P is the stationary point on C.

(a) Find, using calculus, the x coordinate of P.

(4)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 1$

(b) Using calculus, calculate the exact area of R.

(5)

(a) $y = (9x - x^2)/(2\sqrt{x}) = (9/2)x^{1/2} - (1/2)x^{3/2}$

$dy/dx = (9/4)x^{-1/2} - (3/4)x^{1/2}$

At P, $dy/dx = 0$: $(9/4)x^{-1/2} = (3/4)x^{1/2}$

Multiply by $x^{1/2}$: $9/4 = (3/4)x \Rightarrow x = (9/4) \times (4/3)$

$x = 3$

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Question 10 continued

(b) Curve meets x-axis: $9x - x^2 = 0, x > 0 \Rightarrow x = 9$ (upper limit)

$\int ((9/2)x^{1/2} - (1/2)x^{3/2})\,dx = (9/2)(2/3)x^{3/2} - (1/2)(2/5)x^{5/2}$

$= 3x^{3/2} - (1/5)x^{5/2}$

Area $R = [3x^{3/2} - (1/5)x^{5/2}]_1^9$

$= (3 \times 27 - (1/5) \times 243) - (3 \times 1 - (1/5) \times 1)$

$= (81 - 243/5) - (3 - 1/5)$

$= 162/5 - 14/5$

$= 148/5$

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

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