Please check the examination details below before entering your candidate information

Candidate surname: StemUp Model Solution | Other names:

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 9 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA11/01R**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. Find

$$\int \left(\frac{1}{2}x^3 + \frac{3}{\sqrt{x}} - 4 \right) \mathrm{d}x$$

writing your answer in simplest form.

(4)

$\int (\frac{1}{2}x^3 + \frac{3}{\sqrt{x}} - 4)\,dx = \int (\frac{1}{2}x^3 + 3x^{-1/2} - 4)\,dx$

$= \frac{1}{2} \times \frac{x^4}{4} + 3 \times \frac{x^{1/2}}{1/2} - 4x + c$

$= \frac{1}{8}x^4 + 6x^{1/2} - 4x + c$

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Question 1 continued

(Total for Question 1 is 4 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Solve

$$5(x + 3) > 4(2x - 5)$$

(2)

(b) (i) Write

$$x^2 - 6x + 1$$

in the form $(x + a)^2 + b$ where a and b are constants.

(ii) Hence solve

$$x^2 - 6x + 1 \geqslant 0$$

(4)

(c) Hence find the values of x that satisfy both

$$5(x + 3) > 4(2x - 5) \quad \textbf{and} \quad x^2 - 6x + 1 \geqslant 0$$

(1)

(a) $5x + 15 > 8x - 20$

$35 > 3x$

$x < \frac{35}{3}$

(b)(i) $x^2 - 6x + 1 = (x - 3)^2 - 9 + 1$

$= (x - 3)^2 - 8$

(ii) $(x - 3)^2 - 8 = 0$

$(x - 3)^2 = 8 \Rightarrow x - 3 = \pm\sqrt{8}$

critical values $x = 3 - \sqrt{8}$, $x = 3 + \sqrt{8}$

$x^2 - 6x + 1 \geq 0 \Rightarrow x \leq 3 - \sqrt{8}$ or $x \geq 3 + \sqrt{8}$

(i.e. $x \leq 3 - 2\sqrt{2}$ or $x \geq 3 + 2\sqrt{2}$)

(c) $x \leq 3 - \sqrt{8}$ or $3 + \sqrt{8} \leq x < \frac{35}{3}$

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Question 2 continued

(Total for Question 2 is 7 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$y = x^3 + 96\sqrt{x} + 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in simplest form.

(3)

(b) Find the solution of the equation

$$\frac{d^2y}{dx^2} = 0$$

writing the answer in the form 2^k where k is a constant.

(3)

$y = x^3 + 96x^{1/2} + 5$

(a) $\frac{dy}{dx} = 3x^2 + 96 \times \frac{1}{2} x^{-1/2}$

$\frac{dy}{dx} = 3x^2 + \frac{48}{\sqrt{x}}$

(b) $\frac{d^2y}{dx^2} = 6x + 48 \times (-\frac{1}{2}) x^{-3/2} = 6x - 24x^{-3/2}$

$6x - \frac{24}{x^{3/2}} = 0 \quad \Rightarrow \quad 6x^{5/2} = 24$

$x^{5/2} = 4$

$x = 4^{2/5} = (2^2)^{2/5}$

$x = 2^{4/5}$

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Question 3 continued

(Total for Question 3 is 6 marks)

4. The curve C has equation

$$y = \frac{2}{x} + 3x - 4 \qquad x \neq 0$$

The straight line l has equation

$$y = kx + 2$$

where k is a constant.

(a) Show that l meets C when

$$(k-3)x^2 + 6x - 2 = 0$$

(2)

(b) Hence find the value of k for which l is a tangent to C

(3)

(a) $kx + 2 = \frac{2}{x} + 3x - 4$

$\times x$: $kx^2 + 2x = 2 + 3x^2 - 4x$

$kx^2 - 3x^2 + 2x + 4x - 2 = 0$

$(k-3)x^2 + 6x - 2 = 0$ as required

(b) l is a tangent $\Rightarrow b^2 - 4ac = 0$

$6^2 - 4(k-3)(-2) = 0$

$36 + 8(k-3) = 0$

$36 + 8k - 24 = 0 \Rightarrow 8k = -12$

$k = -\frac{3}{2}$

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Question 4 continued

(Total for Question 4 is 5 marks)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Fully factorise

$$9x^3 - 10x^2 + x$$

(2)

(b) Hence solve

$$9 \times 27^y - 10 \times 9^y + 3^y = 0$$

(3)

(a) $9x^3 - 10x^2 + x = x(9x^2 - 10x + 1)$

$= x(9x - 1)(x - 1)$

(b) $27^y = (3^y)^3$, $9^y = (3^y)^2$ Let $x = 3^y$

$9(3^y)^3 - 10(3^y)^2 + 3^y = 0 \Rightarrow 9x^3 - 10x^2 + x = 0$

$x(9x - 1)(x - 1) = 0$

$3^y = 0$ (no solutions), $3^y = \frac{1}{9}$, $3^y = 1$

$3^y = \frac{1}{9} = 3^{-2} \Rightarrow y = -2$; $3^y = 1 = 3^0 \Rightarrow y = 0$

$y = 0$, $y = -2$

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Question 5 continued

(Total for Question 5 is 5 marks)

6.

Diagram not drawn to scale

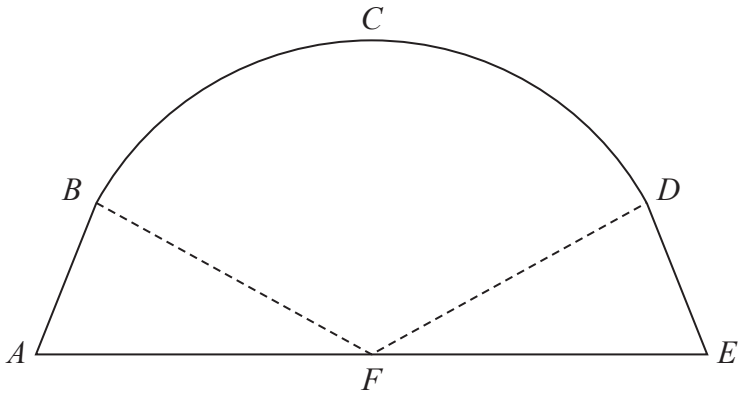


Figure 1

Figure 1 shows a sketch of the entrance to a tunnel.

The shape of the entrance consists of a sector $BCDF$, of a circle centre F, joined to two congruent (identical) triangles ABF and EDF.

Given that

AFE is a straight line

$AF = FE = 6.4\,\text{m}$

$FB = FD = 6.2\,\text{m}$

angle $BFD = 2.275$ radians

(a) Show that angle $AFB = 0.433$ radians to 3 decimal places.

(1)

(b) Find the perimeter of the entrance to the tunnel, $ABCDEFA$, in metres, to one decimal place.

(4)

(c) Find the cross-sectional area of the entrance to the tunnel, $ABCDEFA$, in m^2, to one decimal place.

(4)

(a) Angle $AFB = \frac{\pi - 2.275}{2} = \frac{0.866592...}{2} = 0.433296...$

$= 0.433$ radians (3 d.p.) as required

(b) Arc $BCD = r\theta = 6.2 \times 2.275 = 14.105$ m

$AB^2 = 6.4^2 + 6.2^2 - 2 \times 6.4 \times 6.2 \times \cos 0.4333$

$AB^2 = 7.3739... \Rightarrow AB = 2.7154...$ m $(= DE)$

Perimeter $= AB +$ arc $BCD + DE + EA$

$= 2 \times 2.7154... + 14.105 + 12.8$

$= 32.335...$

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Question 6 continued

Perimeter = 32.3 m (1 d.p.)

(c) Area of sector BCDF $= \frac{1}{2} r^2\theta = \frac{1}{2} \times 6.2^2 \times 2.275 = 43.7255$

Area of triangle ABF $= \frac{1}{2} ab \sin C = \frac{1}{2} \times 6.4 \times 6.2 \times \sin 0.4333$

$= 8.3301...$

Total area $= 2 \times 8.3301... + 43.7255$

$= 60.385...$

Area $= 60.4\ m^2$ (1 d.p.)

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7.

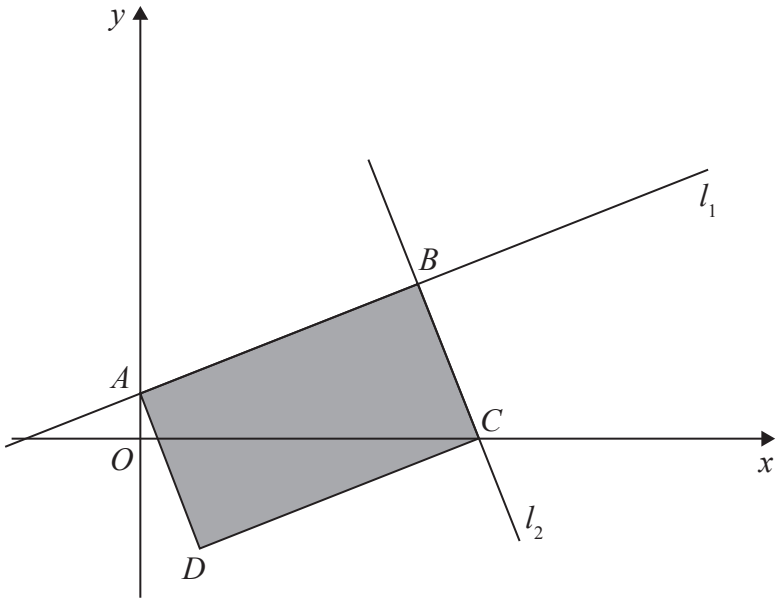


Figure 2

The straight line l_1 shown in Figure 2 has equation $5y = 2x + 10$

The points A and B lie on l_1 such that

- point A lies on the y-axis
- point B has x coordinate 10

(a) Find the distance AB writing your answer as a fully simplified surd.

(3)

The straight line l_2 also shown in Figure 2

- passes through B
- is perpendicular to l_1

(b) Find an equation for l_2 writing your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Line l_2 crosses the x-axis at the point C.

Point D is such that the points A, B, C and D form the vertices of a rectangle, shown shaded in Figure 2.

(c) Find the area of rectangle $ABCD$.

(3)

(a) A: $x = 0 \Rightarrow 5y = 10 \Rightarrow y = 2$, $A(0, 2)$

B: $x = 10 \Rightarrow 5y = 30 \Rightarrow y = 6$, $B(10, 6)$

$AB^2 = (10 - 0)^2 + (6 - 2)^2 = 100 + 16 = 116$

$AB = \sqrt{116} = 2\sqrt{29}$

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Question 7 continued

(b) l_1: $y = \frac{2}{5}x + 2 \Rightarrow$ gradient of $l_1 = \frac{2}{5}$

l_2 perpendicular $\Rightarrow$ gradient of $l_2 = -\frac{5}{2}$

$y - 6 = -\frac{5}{2}(x - 10)$

$2y - 12 = -5x + 50$

$5x + 2y - 62 = 0$

(c) C: $y = 0 \Rightarrow 5x = 62 \Rightarrow x = \frac{62}{5}$, $C\left(\frac{62}{5}, 0\right)$

$BC^2 = \left(\frac{62}{5} - 10\right)^2 + (0 - 6)^2 = \frac{144}{25} + 36 = \frac{1044}{25}$

$BC = \frac{\sqrt{1044}}{5} = \frac{6\sqrt{29}}{5}$ $(= 6.462...)$

Area $ABCD = AB \times BC = 2\sqrt{29} \times \frac{6\sqrt{29}}{5} = \frac{348}{5}$

Area = 69.6

Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

8. A curve C with equation $y = f(x)$ passes through the point $R(4, 13)$.

Given that

$$f'(x) = 2(x - 3)(3x + 2)$$

(a) use integration to find $f(x)$, giving your answer in simplest form.

(5)

(b) Given that $f(x)$ can be written in the form

$$(x - 3)^2(px + q)$$

find the value of the constant p and the value of the constant q.

(2)

(c) Sketch the graph of $y = f(2x)$, showing the coordinates of any points where the curve touches or crosses the coordinate axes.

(4)

(a) $f'(x) = 2(3x^2 + 2x - 9x - 6) = 6x^2 - 14x - 12$

$f(x) = \frac{6x^3}{3} - \frac{14x^2}{2} - 12x + c$

$f(x) = 2x^3 - 7x^2 - 12x + c$

$R(4, 13)$: $13 = 2 \times 64 - 7 \times 16 - 12 \times 4 + c$

$13 = 128 - 112 - 48 + c = -32 + c \Rightarrow c = 45$

$f(x) = 2x^3 - 7x^2 - 12x + 45$

(b) $(x - 3)^2(px + q) = (x^2 - 6x + 9)(px + q)$

Compare x^3: $p = 2$; constants: $9q = 45 \Rightarrow q = 5$

Check: $(x^2 - 6x + 9)(2x + 5) = 2x^3 - 7x^2 - 12x + 45$ ✓

$p = 2$, $q = 5$

(c) $f(2x) = (2x - 3)^2(4x + 5)$

touches x-axis at $x = \frac{3}{2}$, crosses at $x = -\frac{5}{4}$, y-intercept 45

Sketch shown on the next page.

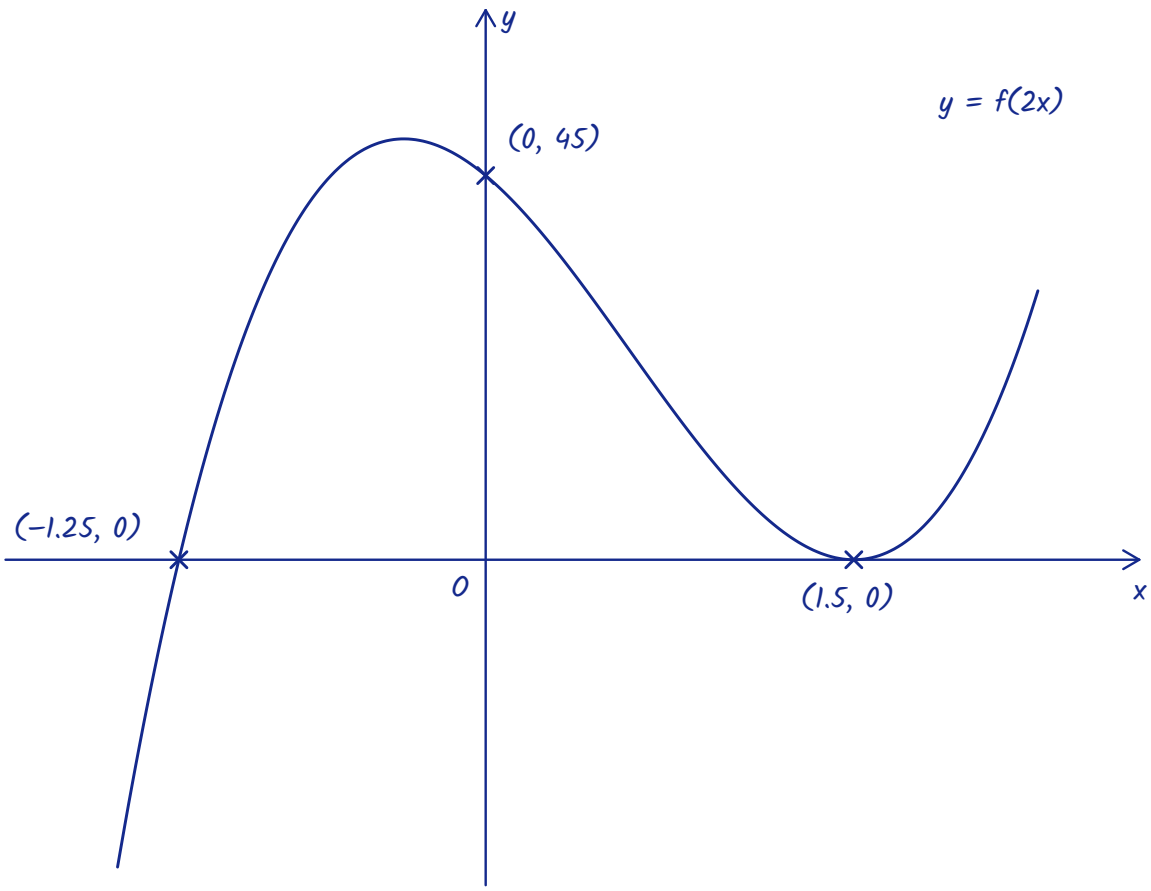
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Question 8 continued

Question 8 continued

(c)



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Question 8 continued

(Total for Question 8 is 11 marks)

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9.

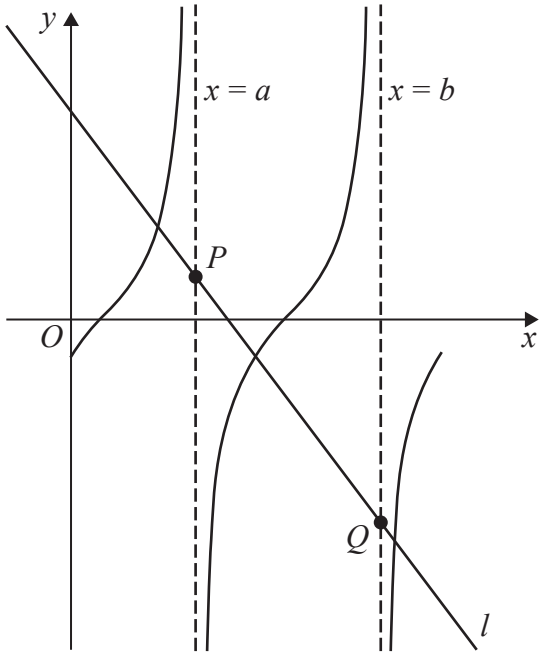


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan\left(x - \frac{\pi}{6}\right)$ for $0 \leqslant x \leqslant 2\pi$
- part of the straight line l with equation $y = \pi - x$

(a) State the number of solutions of the equation

(i) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leqslant x \leqslant 2\pi$

(ii) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leqslant x \leqslant 100\pi$

(iii) $\tan\left(x - \frac{\pi}{6}\right) = \pi + x$ in the interval $0 \leqslant x \leqslant 2\pi$

(3)

The line with equation $x = a$, shown in Figure 3, is the asymptote to the curve with the smallest positive x coordinate.

(b) State the value of a

(1)

The line with equation $x = b$, also shown in Figure 3, is the asymptote to the curve with the second smallest positive x coordinate.

The line l meets $x = a$ at point P and meets $x = b$ at point Q as shown in Figure 3.

(c) Find the midpoint of the line segment PQ.

(4)

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Question 9 continued

(a)(i) 3

(ii) 101

(iii) 2

(b) Asymptote where $x - \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow a = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$

(c) Period of tan is π, so $b = a + \pi = \frac{2\pi}{3} + \pi = \frac{5\pi}{3}$

P: $y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$, $P\left(\frac{2\pi}{3}, \frac{\pi}{3}\right)$

Q: $y = \pi - \frac{5\pi}{3} = -\frac{2\pi}{3}$, $Q\left(\frac{5\pi}{3}, -\frac{2\pi}{3}\right)$

Midpoint of PQ $= \left(\frac{1}{2}\left(\frac{2\pi}{3} + \frac{5\pi}{3}\right), \frac{1}{2}\left(\frac{\pi}{3} - \frac{2\pi}{3}\right)\right)$

$= \left(\frac{7\pi}{6}, -\frac{\pi}{6}\right)$

(Total for Question 9 is 8 marks)

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10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{3}x^3 - 25x - \frac{56}{x} + \frac{194}{3} \qquad x > 0$$

The point P, which lies on C, has coordinates $(2, -8)$

(a) Show that an equation of the tangent to C at P is

$$y = -3x - 2$$

(5)

The point Q also lies on C.

Given that the tangent to C at Q is parallel to the tangent to C at P,

(b) find, using algebra and showing your working, the exact x coordinate of Q.

(5)

(a) $y = \frac{2}{3}x^3 - 25x - 56x^{-1} + \frac{194}{3}$

$\frac{dy}{dx} = 2x^2 - 25 + 56x^{-2} = 2x^2 - 25 + \frac{56}{x^2}$

At $x = 2$: $\frac{dy}{dx} = 2 \times 4 - 25 + \frac{56}{4} = 8 - 25 + 14 = -3$

Tangent: $y - (-8) = -3(x - 2)$

$y + 8 = -3x + 6$

$y = -3x - 2$ as required

(b) Parallel $\Rightarrow$ same gradient: $2x^2 - 25 + \frac{56}{x^2} = -3$

$\times x^2$: $2x^4 - 25x^2 + 56 = -3x^2$

$2x^4 - 22x^2 + 56 = 0$

$x^4 - 11x^2 + 28 = 0$

$(x^2 - 4)(x^2 - 7) = 0$

$x^2 = 4$ or $x^2 = 7$

$x = 2$ is the point P (and $x > 0$), so for Q: $x^2 = 7$

$x = \sqrt{7}$

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Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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