Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Q:1/1/1/1/

P 7 2 4 5 8 A

Pearson

Write your answers in the D1 answer book for this paper.

1. 175 135 210 105 100 150 60 20 70 125

The numbers in the list above represent the weights, in kilograms, of ten crates. The crates are to be transported in trucks that can each hold a maximum total crate weight of 300 kg.

(a) Calculate a lower bound for the number of trucks that will be needed to transport the crates.

(2)

(b) Using the list provided, carry out a bubble sort to produce a list of the weights in descending order. You need only give the state of the list after each pass.

(4)

(c) Use the first-fit decreasing bin packing algorithm to allocate the crates to the trucks.

(3)

(Total for Question 1 is 9 marks)

2.

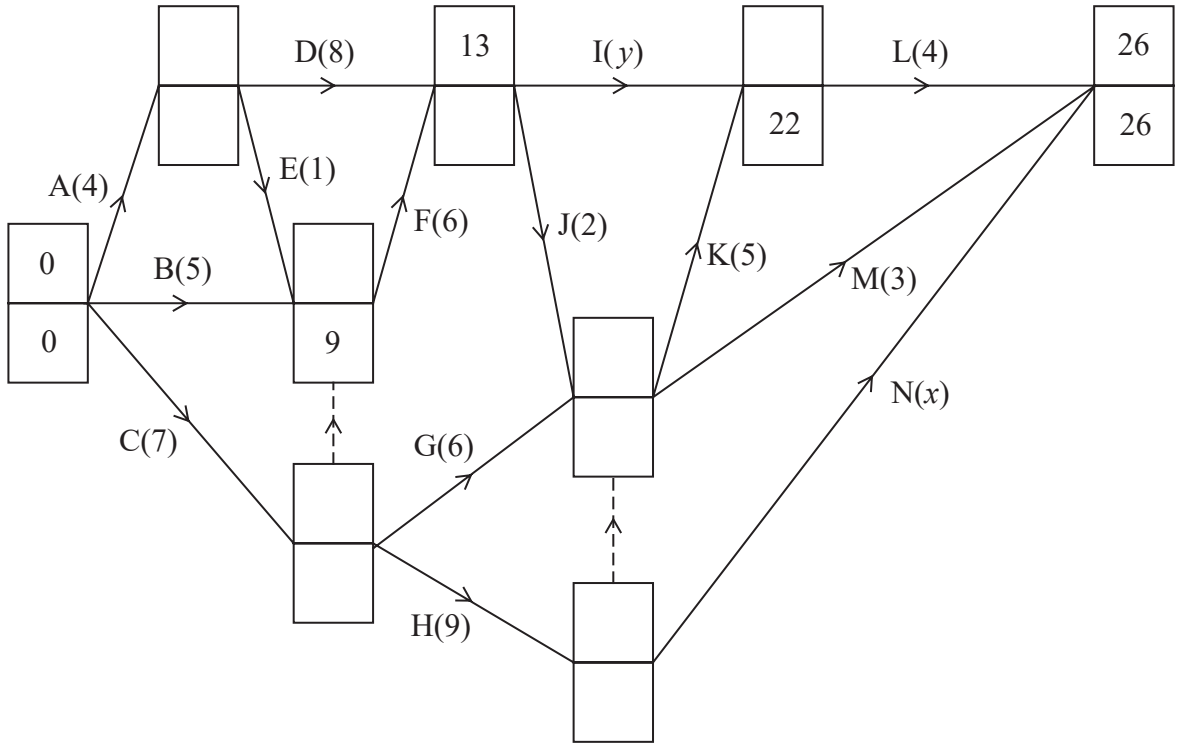


Figure 1

The network in Figure 1 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times and late event times are to be shown at each vertex and some have been completed.

Given that

- CHN is the critical path for the project
- the total float on activity B is twice the duration of the total float on activity I

(a) find the value of x and show that the value of y is 7

(2)

(b) Calculate the missing early event times and late event times and hence complete Diagram 1 in your answer book.

(3)

Each activity requires one worker, and the project must be completed in the shortest possible time.

(c) Draw a cascade chart for this project on Grid 1 in your answer book, and use it to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities.

(6)

(Total for Question 2 is 11 marks)

3.

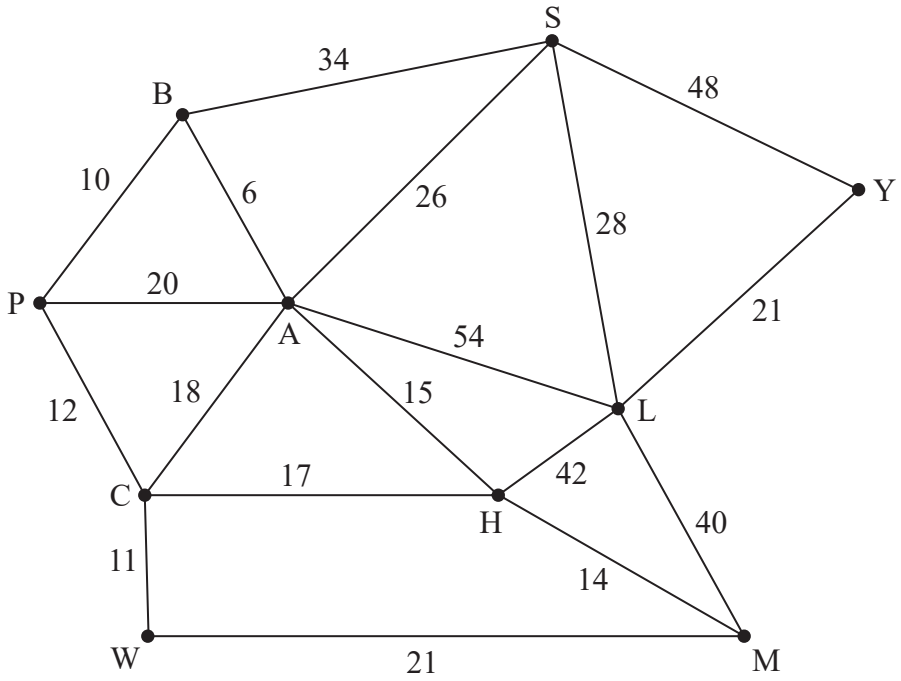


Figure 2

The network in Figure 2 shows the distances, in miles, between ten towns, A, B, C, H, L, M, P, S, W and Y.

(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

The table shows the shortest distances, in miles, between the ten towns.

(b) Use Prim's algorithm on the table, starting at A, to find the minimum spanning tree for this network. You must clearly state the order in which you select the arcs of your tree.

(3)

(c) State the weight of the minimum spanning tree found in (b).

(1)

Sharon needs to visit all of the towns, starting and finishing in the same town, and wishes to minimise the total distance she travels.

(d) Use your answer to (c) to calculate an initial upper bound for the length of Sharon's route.

(1)

(e) Use the nearest neighbour algorithm on the table, starting at W, to find an upper bound for the length of Sharon's route. Write down the route which gives this upper bound.

(2)

Using the nearest neighbour algorithm, starting at Y, an upper bound of length 212 miles was found.

(f) State the best upper bound that can be obtained by using this information and your answers from (d) and (e). Give the reason for your answer.

(1)

(g) By deleting W and all of its arcs, find a lower bound for the length of Sharon's route.

(2)

Sharon decides to take the route found in (e).

(h) Interpret this route in terms of the actual towns visited.

(1)

(Total for Question 3 is 14 marks)



4. A linear programming problem in x, y and z is described as follows.

Maximise $P = -x + y$

subject to

$$x + 2y + z \leqslant 15$$

$$3x - 4y + 2z \geqslant 1$$

$$2x + y + z = 14$$

$$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$$

(a) (i) Eliminate z from the first two inequality constraints, simplifying your answers.

(ii) Hence state the maximum possible value of P

(4)

Given that P takes the maximum possible value found in (a)(ii),

(b) (i) determine the maximum possible value of x

(ii) Hence find a solution to the linear programming problem.

(3)

(Total for Question 4 is 7 marks)

5. The precedence table shows the eleven activities required to complete a project.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A, B
E	A, B
F	B, C
G	B, C
H	D
I	D, E, F, G
J	H, I
K	D, E, F

(a) Draw the activity network for the project, using activity on arc and the minimum number of dummies.

(5)

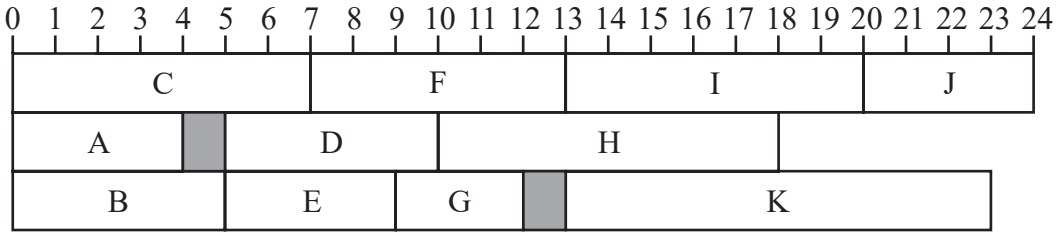


Figure 3

Figure 3 shows a schedule for the project. Each of the activities shown in the precedence table requires one worker. The time taken to complete each activity is in hours and the project is to be completed in the minimum possible time.

(b) (i) State the minimum completion time for the project.

(ii) State the critical activities.

(iii) State the total float on activity G and the total float on activity K.

(4)

(Total for Question 5 is 9 marks)

6.

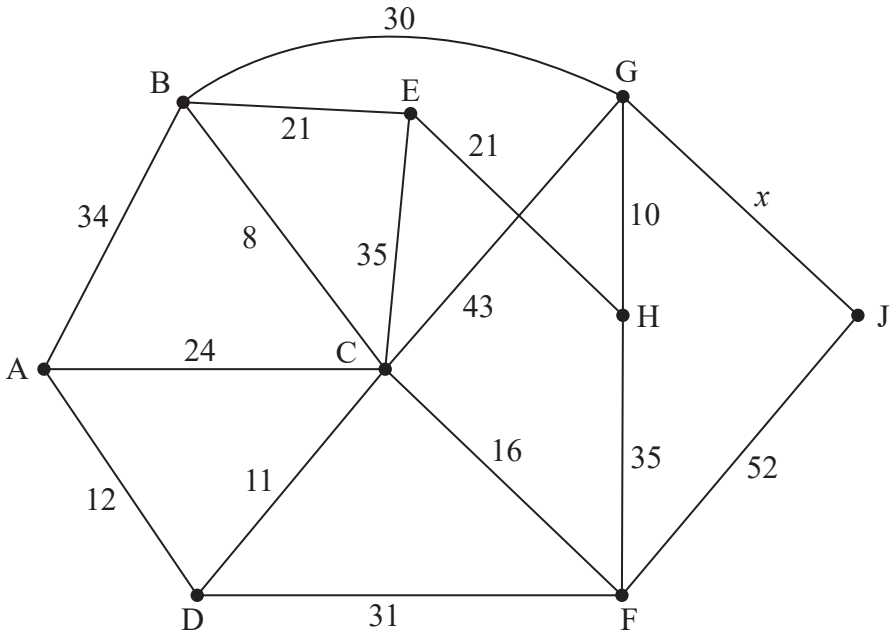


Figure 4

[*The total weight of the network is* $383 + x$]

Figure 4 models a network of roads. The number on each edge gives the time, in minutes, to travel along the corresponding road. The vertices, A, B, C, D, E, F, G, H and J represent nine towns. Ezra wishes to travel from A to H as fast as possible.

The time taken to travel between towns G and J is unknown and is denoted by x minutes.

Dijkstra's algorithm is to be used to find the fastest time to travel from A to H. On Diagram 1 in the answer book the "Order of labelling" and "Final value" at A and J, and the "Working values" at J, have already been completed.

(a) Use Dijkstra's algorithm to find the fastest time to travel from A to H. State the quickest route.

(6)

Ezra needs to travel along each road to check it is in good repair. He wishes to minimise the total time required to traverse the network. Ezra plans to start and finish his inspection route at A. It is given that his route will take at least 440 minutes.

(b) Use the route inspection algorithm and the completed Diagram 1 to find the range of possible values of x.

(6)

(c) Write down a possible route for Ezra.

(1)

A new direct road from D to H is under construction and will take 25 minutes to travel along. Ezra will include this new road in a minimum length inspection route starting and finishing at A. It is given that this inspection route takes exactly 488 minutes.

(d) Determine the value of x. You must give reasons for your answer.

(2)

(Total for Question 6 is 15 marks)

7.

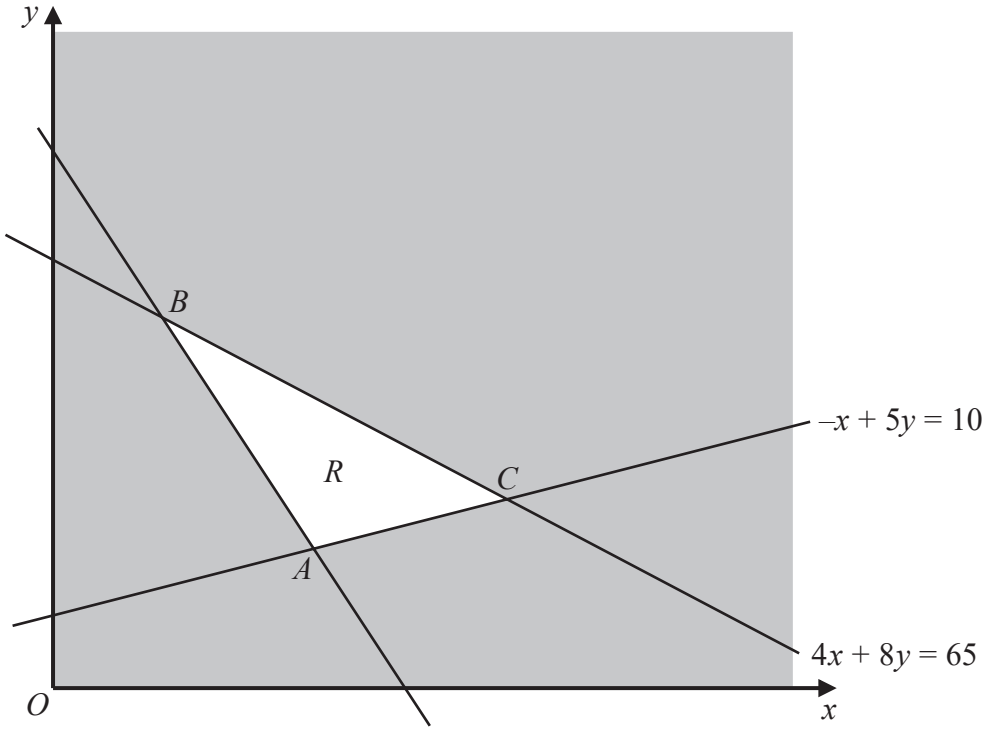


Figure 5

Figure 5 shows the constraints of a linear programming problem in x and y, where R is the feasible region. The equations of two of the lines and the three intersection points, A, B and C, are shown. The coordinates of C are $\left(\frac{35}{4}, \frac{15}{4}\right)$

The objective function is $P = x + 3y$

When the objective is to maximise $x + 3y$, the value of P is 24

When the objective is to minimise $x + 3y$, the value of P is 10

(a) (i) Find the coordinates of A and B.

(ii) Determine the inequalities that define R.

(8)

An additional constraint, $y \geqslant kx$, where k is a positive constant, is added to the linear programming problem.

(b) Determine the greatest value of k for which this additional constraint does not affect the feasible region.

(2)

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER: 75 MARKS

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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference WDM11/01

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

Answer Book
Do not return the question paper with the answer book.

Total Marks

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Q:1/1/1/1/

P 7 2 4 5 8 A 0 1 2 0

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1.

175 135 210 105 100 150 60 20 70 125

(a) Total = 175 + 135 + 210 + 105 + 100 + 150 + 60 + 20 + 70 + 125

= 1150

1150 ÷ 300 = 3.833...

Lower bound = 4 trucks

(b) Bubble sort into descending order:

Start	175	135	210	105	100	150	60	20	70	125
Pass 1	175	210	135	105	150	100	60	70	125	20
Pass 2	210	175	135	150	105	100	70	125	60	20
Pass 3	210	175	150	135	105	100	125	70	60	20
Pass 4	210	175	150	135	105	125	100	70	60	20
Pass 5	210	175	150	135	125	105	100	70	60	20
Pass 6	210	175	150	135	125	105	100	70	60	20

No swaps in pass 6, so the list is now sorted:

210 175 150 135 125 105 100 70 60 20

(c) First-fit decreasing (capacity 300 kg):

Truck 1: 210 70 20

Truck 2: 175 125

Truck 3: 150 135

Truck 4: 105 100 60

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Question 1 continued

(Total for Question 1 is 9 marks)

2.

(a) CHN is critical: $7 + 9 + x = 26 \Rightarrow x = 10$

Total float on B $= 9 - 0 - 5 = 4$

Total float on I $= 22 - 13 - y = 9 - y$

$4 = 2(9 - y) \Rightarrow 4 = 18 - 2y \Rightarrow 2y = 14 \Rightarrow y = 7$

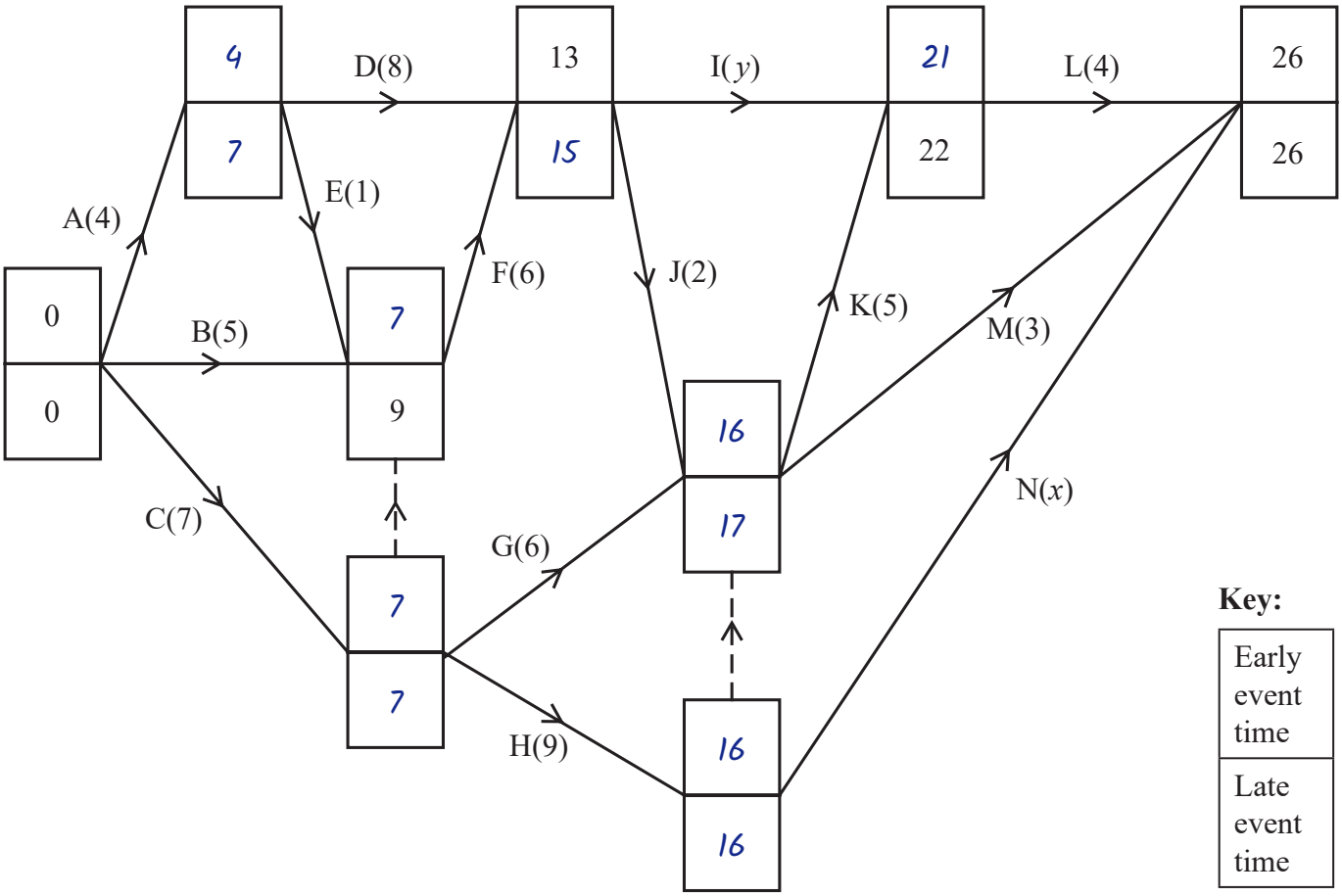


Diagram 1

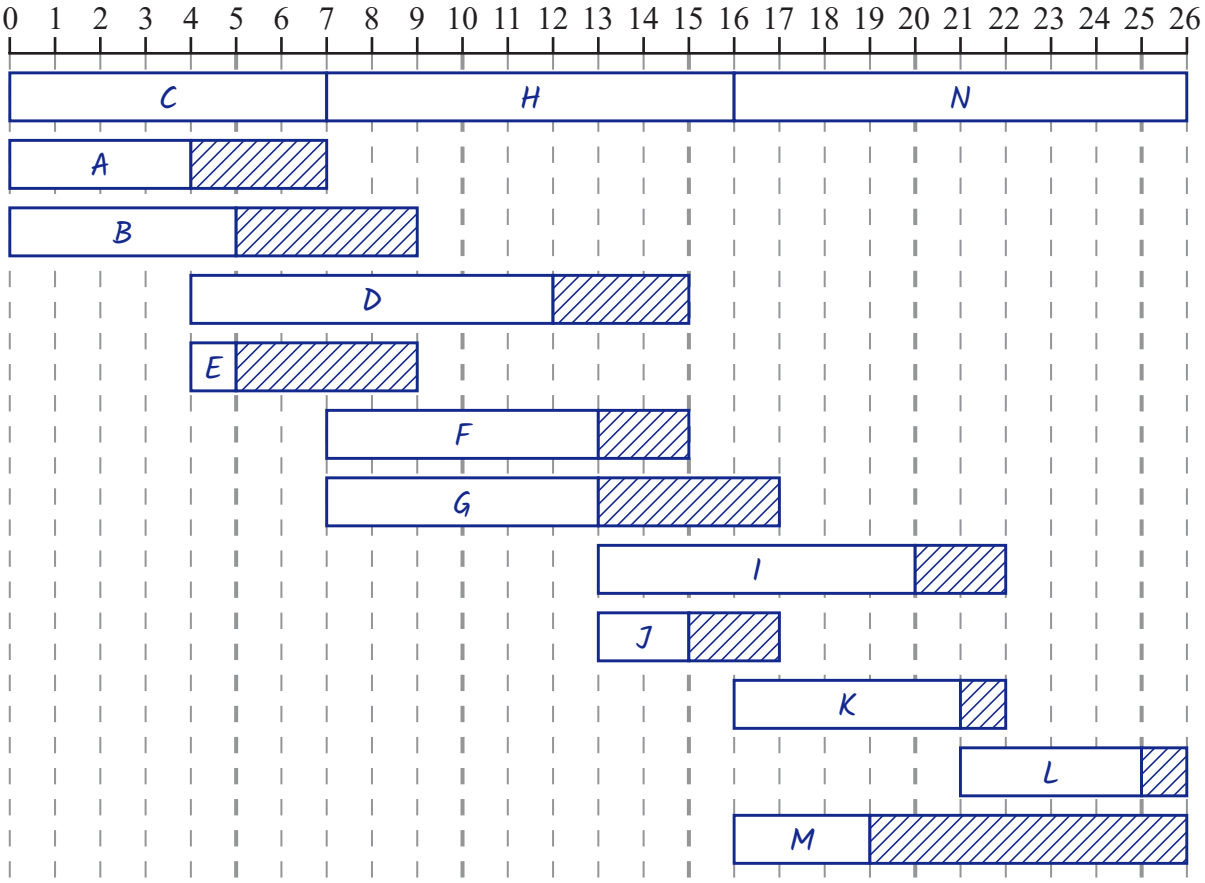
(b)

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Question 2 continued

(c)



Shaded = float

Grid 1

(c) At time 11.5 (between time 11 and time 12) activities H, D, F and G must all be happening at the same time.

So at least 4 workers are needed.

Minimum number of workers = 4

(Total for Question 2 is 11 marks)

3.

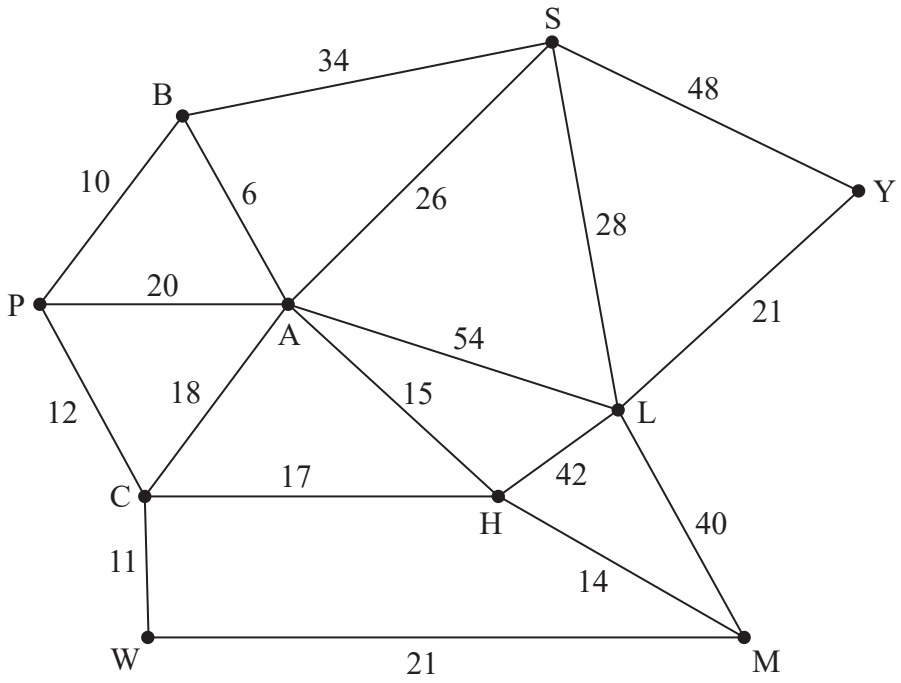


Figure 2

(a) Kruskal:

AB (6) add, BP (10) add, CW (11) add, CP (12) add,

HM (14) add, AH (15) add, CH (17) reject, AC (18) reject,

AP (20) reject, MW (21) reject, LY (21) add, AS (26) add,

LS (28) add (tree complete: 9 arcs for 10 towns)

Arcs in tree: AB, BP, CW, CP, HM, AH, LY, AS, LS

(b) Prim starting at A, arcs in the order selected:

AB, BP, PC, CW, AH, HM, AS, SL, LY

(order of vertices: A, B, P, C, W, H, M, S, L, Y)

(c) Weight = 6 + 10 + 12 + 11 + 15 + 14 + 26 + 28 + 21

= 143 miles

(d) Initial upper bound = 2 × 143 = 286 miles

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Question 3 continued

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

(e) Nearest neighbour starting at W:

W – C (11), C – P (12), P – B (10), B – A (6), A – H (15),

H – M (14), M – L (40), L – Y (21), Y – S (48), S – W (55)

Route: W – C – P – B – A – H – M – L – Y – S – W

Upper bound = 11 + 12 + 10 + 6 + 15 + 14 + 40 + 21 + 48 + 55

= 232 miles

(f) Best upper bound = 212 miles, because it is the smallest of the upper bounds 286, 232 and 212.

(g) Delete W: the MST from (b) without CW is a minimum spanning tree of the remaining towns, weight 143 – 11 = 132

Two least arcs from W: WC (11) and WM (21)

Lower bound = 132 + 11 + 21 = 164 miles

(h) S to W is not a direct road: the shortest way is S – A – C – W (26 + 18 + 11 = 55). Towns visited in order:

W – C – P – B – A – H – M – L – Y – S – A – C – W

Question 3 continues on the next page.

Question 3 continued

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

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Question 3 continued

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

(Total for Question 3 is 14 marks)

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4.

(a)(i) $2x + y + z = 14 \Rightarrow z = 14 - 2x - y$

$x + 2y + (14 - 2x - y) \leq 15$

$\Rightarrow -x + y + 14 \leq 15 \Rightarrow -x + y \leq 1$

$3x - 4y + 2(14 - 2x - y) \geq 1$

$\Rightarrow 3x - 4y + 28 - 4x - 2y \geq 1$

$\Rightarrow -x - 6y \geq -27 \Rightarrow x + 6y \leq 27$

(ii) $P = -x + y \leq 1$, so the maximum possible value of P is 1

(b)(i) $P = 1$: $-x + y = 1 \Rightarrow y = 1 + x$

$x + 6(1 + x) \leq 27 \Rightarrow 7x + 6 \leq 27 \Rightarrow 7x \leq 21 \Rightarrow x \leq 3$

Maximum possible value of $x = 3$

(ii) $x = 3$, $y = 1 + 3 = 4$, $z = 14 - 2(3) - 4 = 4$

Solution: $x = 3$, $y = 4$, $z = 4$ $(P = 1)$

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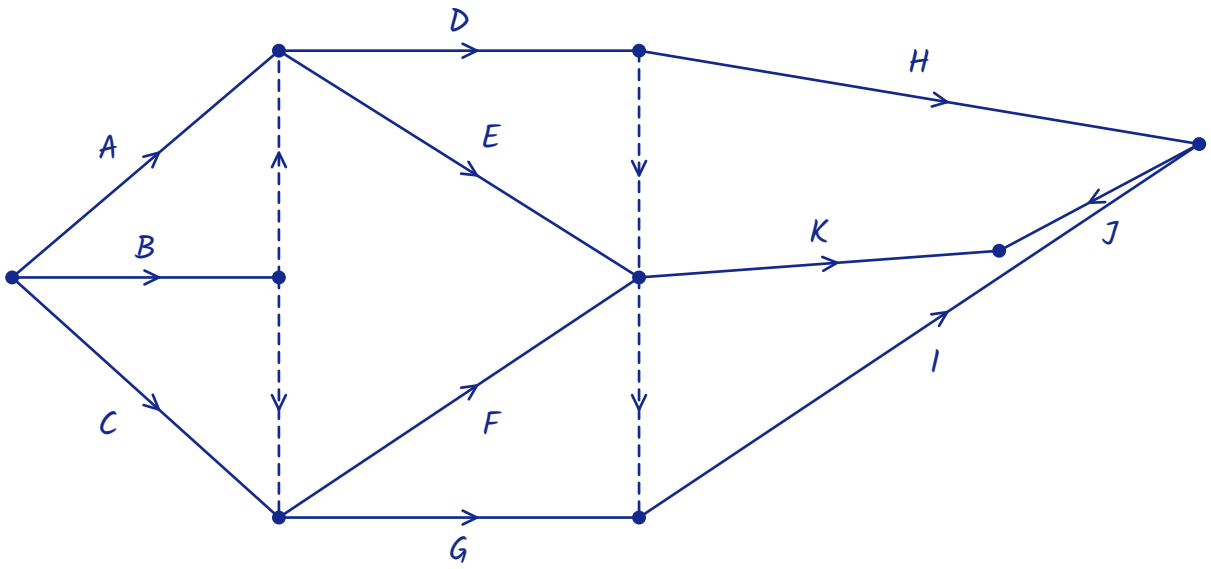
Question 4 continued

(Total for Question 4 is 7 marks)

5.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A, B
E	A, B
F	B, C
G	B, C
H	D
I	D, E, F, G
J	H, I
K	D, E, F

(a)



Dashed lines are dummies

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Question 5 continued

Please redraw your activity network on this page if you need to do so.

(b)(i) Minimum completion time = 24 hours

(ii) Critical activities: C, F, I and J

(iii) G: earliest start 7 (after B and C), duration 3,
latest finish 13 (I starts at 13)
Total float on G = 13 – 7 – 3 = 3 hours

K: earliest start 13 (after F), duration 10, latest finish 24
Total float on K = 24 – 13 – 10 = 1 hour

(Total for Question 5 is 9 marks)

6.

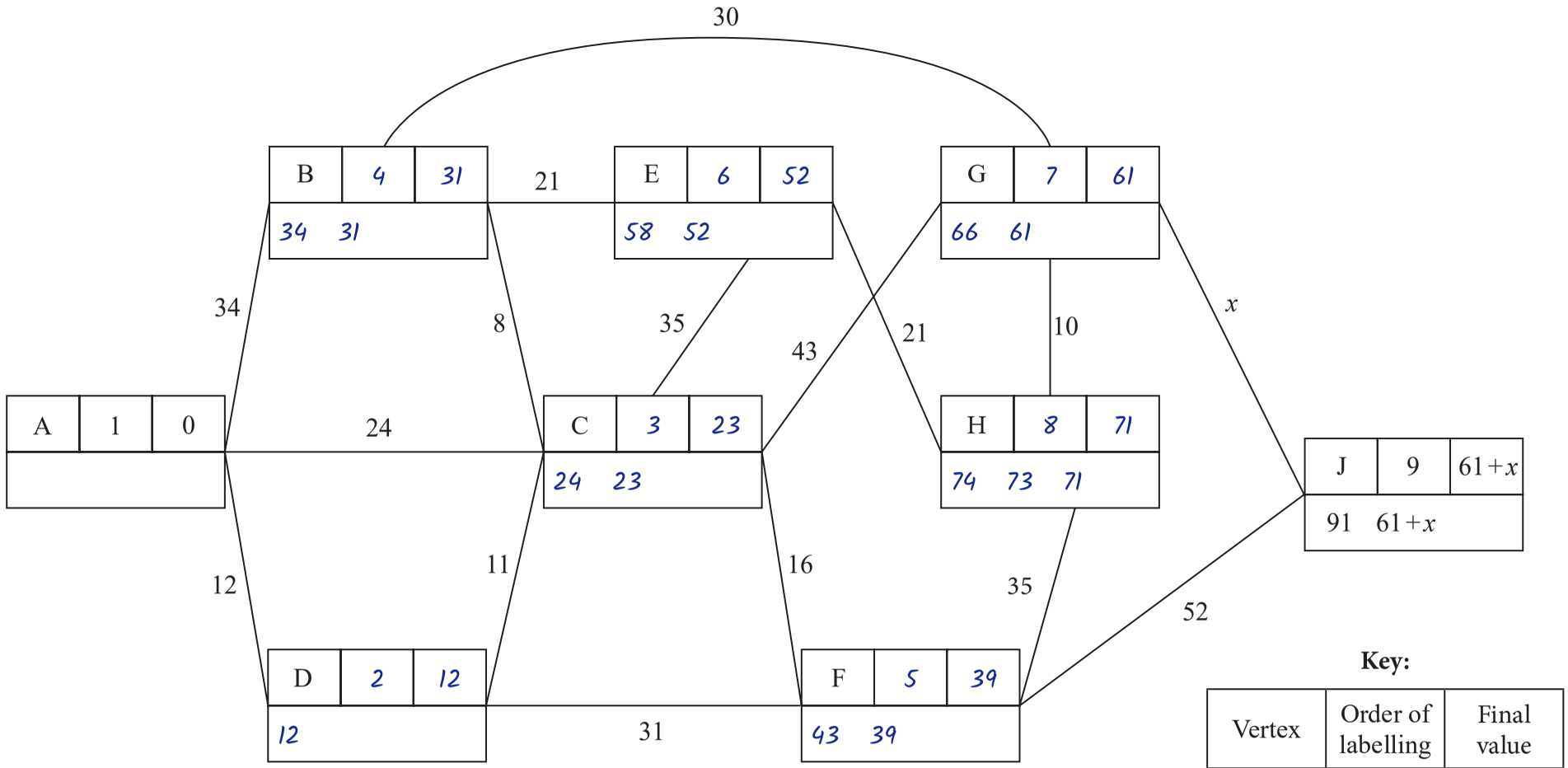


Diagram 1

Fastest time: 71 minutes

Quickest route: A – D – C – B – G – H

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Question 6 continued

(b) Odd vertices: A, D, E and H (each of order 3)

AD + EH = 12 + 21 = 33

AE + DH = 52 + 59 = 111 (ADCBE, DCBGH)

AH + DE = 71 + 40 = 111 (ADCBGH, DCBE)

Least pairing: AD + EH = 33, so AD and EH are repeated

Length of route = (383 + x) + 33 = 416 + x

$416 + x \geqslant 440 \Rightarrow x \geqslant 24$

Final value at J is 61 + x (not 91), so $61 + x < 91 \Rightarrow x < 30$

$\therefore\ 24 \leqslant x < 30$

(c) A – B – C – E – B – G – C – D – A – D – F – J – G – H – E – H – F – C – A

(d) With the new road DH, D and H both become even, so the only odd vertices are A and E. Repeat the shortest path from A to E: A – D – C – B – E = 52

Length = (383 + x) + 25 + 52 = 460 + x

$460 + x = 488 \Rightarrow x = 28$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 15 marks)

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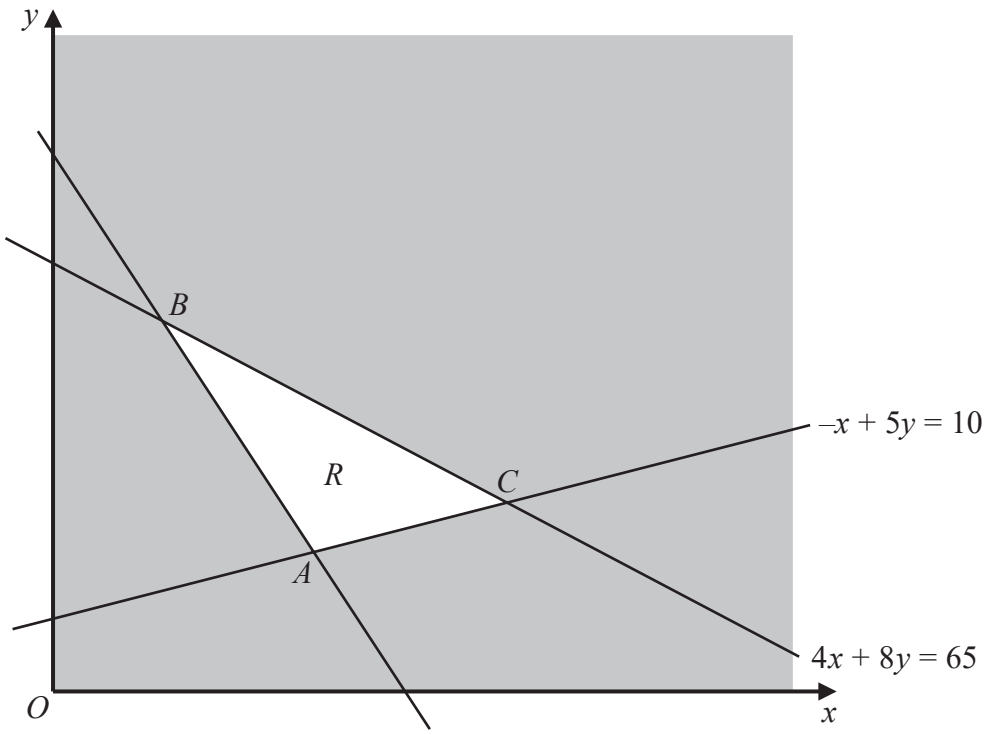


Figure 5

(a)(i) The minimum (P = 10) is at A and the maximum (P = 24) is at B, so A and B lie on $x + 3y = 10$ and $x + 3y = 24$.

A: $x + 3y = 10$ and $-x + 5y = 10$

add: $8y = 20 \Rightarrow y = 5/2$, $x = 10 - 15/2 = 5/2$

A (5/2, 5/2)

B: $x + 3y = 24$ and $4x + 8y = 65$

$4x + 12y = 96$, subtract: $4y = 31 \Rightarrow y = 31/4$

$x = 24 - 93/4 = 3/4$

B (3/4, 31/4)

(ii) Gradient of AB = (31/4 − 5/2) ÷ (3/4 − 5/2)

= (21/4) ÷ (−7/4) = −3

$y - 5/2 = -3(x - 5/2) \Rightarrow y = -3x + 10 \Rightarrow 3x + y = 10$

Inequalities defining R:

$3x + y \geq 10$

$-x + 5y \geq 10$

$4x + 8y \leq 65$

(b) The line $y = kx$ must pass through C (35/4, 15/4), the point of R with the smallest value of y/x:

$k = (15/4) \div (35/4) = 15/35 = 3/7$

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Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER: 75 MARKS

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