Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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1. Adina is saving money to buy a new computer. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she has enough money, in total, to buy the computer.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

(a) (i) state with a reason whether an arithmetic series or a geometric series should be used,

(ii) write down an expression, in terms of n, for the amount, in pounds (£), saved in week n.

(3)

Given that the computer Adina wants to buy costs £350

(b) find the number of weeks it will take for Adina to save enough money to buy the computer.

(4)

(a) (i) $5.25 - 5 = 0.25$ and $5.50 - 5.25 = 0.25$

There is a common difference of 0.25 (the ratios 1.05 and 1.047... differ), so an arithmetic series should be used.

(ii) $u_n = a + (n-1)d = 5 + 0.25(n-1)$ $(= 4.75 + 0.25n)$

(b) $S_n = \frac{n}{2}\{2 \times 5 + (n-1) \times 0.25\} \geqslant 350$

$n(10 + 0.25n - 0.25) \geqslant 700$

$0.25n^2 + 9.75n - 700 \geqslant 0$

$n^2 + 39n - 2800 \geqslant 0$

$n = \frac{-39 + \sqrt{(39^2 + 4 \times 2800)}}{2} = \frac{-39 + \sqrt{12721}}{2} = 36.893...$

n must be a whole number of weeks, so $n = 37$

It takes 37 weeks.

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Question 1 continued

Q1

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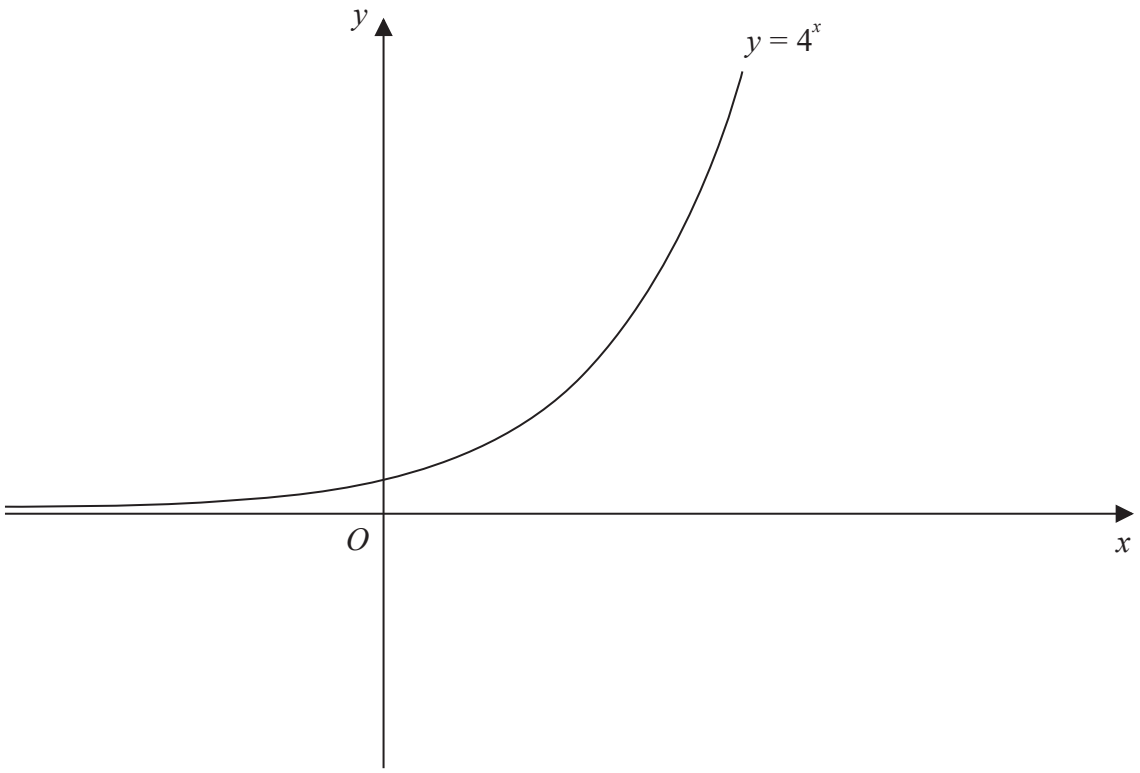


Figure 1

Figure 1 shows a sketch of the curve with equation $y = 4^x$

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(a) On Diagram 1, sketch the curve with equation

(i) $y = 2^x$

(ii) $y = 4^x - 6$

Label clearly the coordinates of any points of intersection with the coordinate axes.

(4)

The curve with equation $y = 2^x$ meets the curve with equation $y = 4^x - 6$ at the point P.

(b) Using algebra, find the exact coordinates of P.

(4)

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Question 2 continued

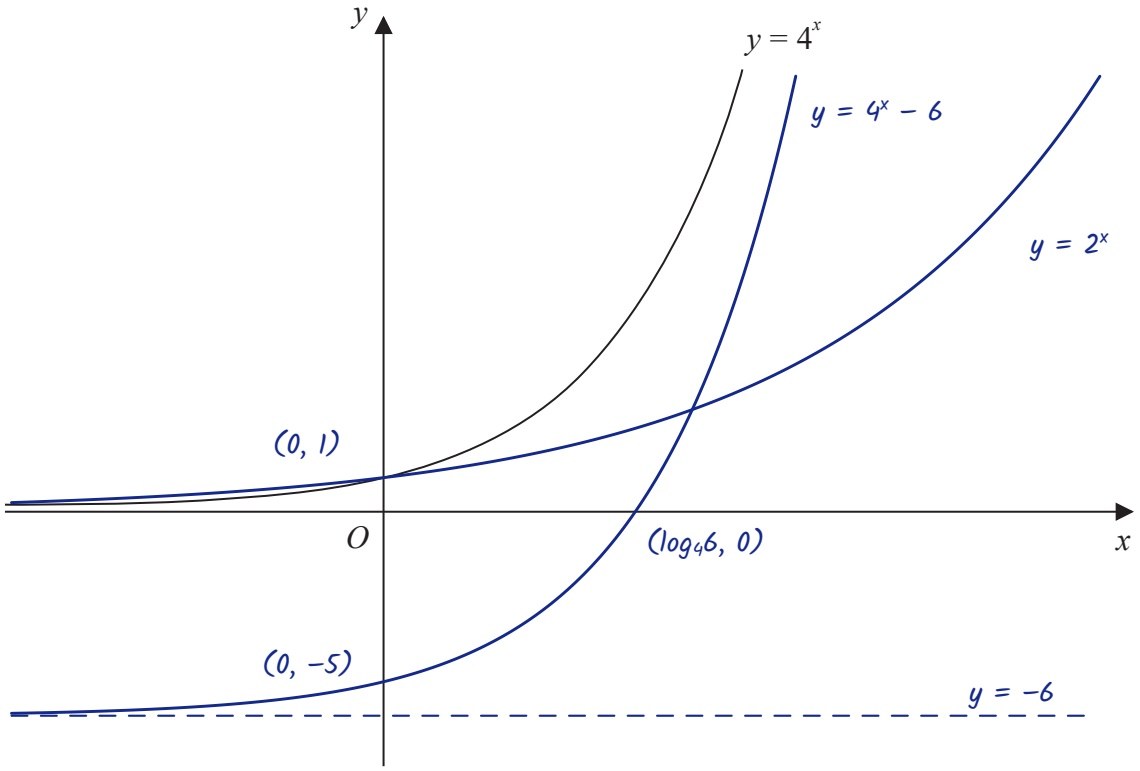


Diagram 1

(b) $2^x = 4^x - 6$ and $4^x = (2^x)^2$

$(2^x)^2 - 2^x - 6 = 0$

$(2^x - 3)(2^x + 2) = 0$

$2^x = 3$ or $2^x = -2$ (rejected since $2^x > 0$)

$x = \log_2 3$, $y = 2^x = 3$

$P(\log_2 3, 3)$

Q2

(Total 8 marks)

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3. (i) Prove that for all single digit prime numbers, p,

$$p^3 + p \text{ is a multiple of } 10$$

(2)

(ii) Show, using algebra, that for $n \in \mathbb{N}$

$$(n + 1)^3 - n^3 \text{ is not a multiple of } 3$$

(3)

(i) $p = 2$: $2^3 + 2 = 8 + 2 = 10 = 1 \times 10$

$p = 3$: $3^3 + 3 = 27 + 3 = 30 = 3 \times 10$

$p = 5$: $5^3 + 5 = 125 + 5 = 130 = 13 \times 10$

$p = 7$: $7^3 + 7 = 343 + 7 = 350 = 35 \times 10$

2, 3, 5 and 7 are the only single digit primes and each gives a multiple of 10, so $p^3 + p$ is a multiple of 10 for all single digit primes p.

(ii) $(n + 1)^3 - n^3 = n^3 + 3n^2 + 3n + 1 - n^3$

$= 3n^2 + 3n + 1$

$= 3(n^2 + n) + 1$

$n^2 + n$ is an integer, so $3(n^2 + n)$ is a multiple of 3

$\therefore (n + 1)^3 - n^3$ is one more than a multiple of 3,

so it is not a multiple of 3 for any $n \in \mathbb{N}$.

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Question 3 continued

Q3

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4. (a) Find, in ascending powers of x, up to and including the term in x^3, the binomial expansion of

$$\left(2+\frac{x}{8}\right)^{13}$$

fully simplifying each coefficient.

(4)

(b) Use the answer to part (a) to find an approximation for 2.0125^{13}

Give your answer to 3 decimal places.

(3)

Without calculating 2.0125^{13}

(c) state, with a reason, whether the answer to part (b) is an overestimate or an underestimate.

(1)

(a) $\left(2+\frac{x}{8}\right)^{13} = 2^{13} + {}^{13}C_1\, 2^{12}\left(\frac{x}{8}\right) + {}^{13}C_2\, 2^{11}\left(\frac{x}{8}\right)^2$

$+ {}^{13}C_3\, 2^{10}\left(\frac{x}{8}\right)^3 + \ldots$

$= 8192 + 13 \times 4096 \times \frac{x}{8} + 78 \times 2048 \times \frac{x^2}{64} + 286 \times 1024 \times \frac{x^3}{512} + \ldots$

$= 8192 + 6656x + 2496x^2 + 572x^3 + \ldots$

(b) $2 + \frac{x}{8} = 2.0125 \Rightarrow \frac{x}{8} = 0.0125 \Rightarrow x = 0.1$

$2.0125^{13} \approx 8192 + 6656(0.1) + 2496(0.1)^2 + 572(0.1)^3$

$= 8192 + 665.6 + 24.96 + 0.572$

$= 8883.132$

(c) Underestimate: with $x = 0.1$ every term in the expansion is positive, so the terms in x^4 onwards that were left out are all positive, and the truncated series is less than the true value.

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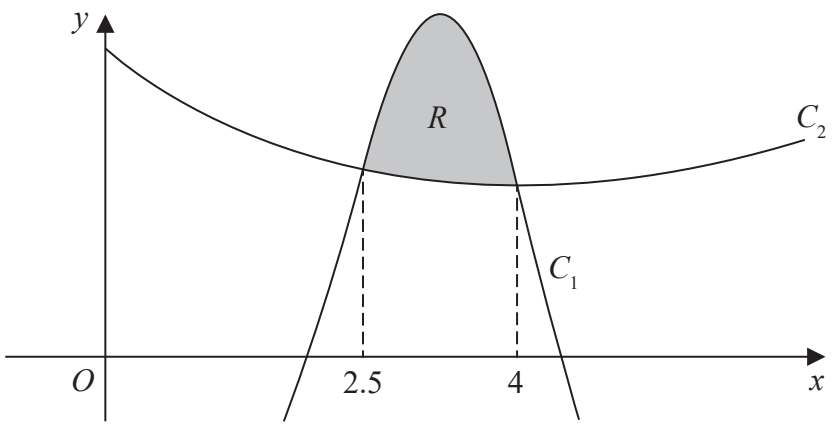


Figure 2

Figure 2 shows a sketch of part of the graph of the curves C_1 and C_2

The curves intersect when $x = 2.5$ and when $x = 4$

A table of values for some points on the curve C_1 is shown below, with y values given to 3 decimal places as appropriate.

x	2.5	2.75	3	3.25	3.5	3.75	4
y	5.453	7.764	9.375	9.964	9.367	7.626	5

Using the trapezium rule with all the values of y in the table,

(a) find, to 2 decimal places, an estimate for the area bounded by the curve C_1, the line with equation $x = 2.5$, the x-axis and the line with equation $x = 4$

(4)

The curve C_2 has equation

$$y = x^{\frac{3}{2}} - 3x + 9 \qquad x > 0$$

(b) Find $\int \left(x^{\frac{3}{2}} - 3x + 9 \right) \mathrm{d}x$

(3)

The region R, shown shaded in Figure 2, is bounded by the curves C_1 and C_2

(c) Use the answers to part (a) and part (b) to find, to one decimal place, an estimate for the area of the region R.

(3)

(a) $h = 0.25$

$A \approx \frac{1}{2} \times 0.25 \times \{5.453 + 5 + 2(7.764 + 9.375 + 9.964 + 9.367 + 7.626)\}$

$= 0.125 \times \{10.453 + 2 \times 44.096\}$

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$= 0.125 \times 98.645 = 12.330625$

$= 12.33$ (2 d.p.)

(b) $\int (x^{3/2} - 3x + 9)\, dx$

$= \frac{2}{5} x^{5/2} - \frac{3}{2} x^2 + 9x + c$

(c) Area under $C_2 = \left[\frac{2}{5} x^{5/2} - \frac{3}{2} x^2 + 9x \right]_{2.5}^{4}$

$= \left(\frac{2}{5} \times 32 - \frac{3}{2} \times 16 + 36 \right) - \left(\frac{2}{5} \times 2.5^{5/2} - \frac{3}{2} \times 2.5^2 + 22.5 \right)$

$= 24.8 - 17.07784... = 7.72215...$

Area of $R \approx 12.330625 - 7.72215... = 4.6084...$

Area of $R \approx 4.6$ (1 d.p.)

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Question 5 continued

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Question 5 continued

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6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

(a) $(x - 3)^2 - 9 + (y + 4)^2 - 16 + k = 0$

$(x - 3)^2 + (y + 4)^2 = 25 - k$, centre $(3, -4)$

x-axis is a tangent $\Rightarrow$ radius = distance from centre to x-axis = 4

$25 - k = 4^2 = 16$

$k = 9$

(b) Circle touches the x-axis at $R(3, 0)$

$x = 0$: $y^2 + 8y + 9 = 0$

$y = \dfrac{-8 \pm \sqrt{(64 - 36)}}{2} = -4 \pm \sqrt{7}$

$S(0, -4 + \sqrt{7})$, $T(0, -4 - \sqrt{7})$, so $ST = 2\sqrt{7}$

Height of triangle = distance from R to the y-axis = 3

Area $RST = \frac{1}{2} \times 2\sqrt{7} \times 3$

$= 3\sqrt{7}$

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Question 6 continued

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7. (a) Given that

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

(4)

(a) $3\log_3(2x - 1) = \log_3(2x - 1)^3$ and $2 = \log_3 9$

$\log_3(2x - 1)^3 = \log_3 9 + \log_3(14x - 25) = \log_3[9(14x - 25)]$

$(2x - 1)^3 = 9(14x - 25)$

$8x^3 - 12x^2 + 6x - 1 = 126x - 225$

$8x^3 - 12x^2 - 120x + 224 = 0$

$\div 4$: $2x^3 - 3x^2 - 30x + 56 = 0$ (as required)

(b) $f(x) = 2x^3 - 3x^2 - 30x + 56$

$f(-4) = 2(-4)^3 - 3(-4)^2 - 30(-4) + 56$

$= -128 - 48 + 120 + 56 = 0$

$f(-4) = 0$, hence -4 is a root of the equation.

(c) $2x^3 - 3x^2 - 30x + 56 = (x + 4)(2x^2 - 11x + 14)$

$(x + 4)(2x - 7)(x - 2) = 0$

$x = -4, \quad x = \frac{7}{2}, \quad x = 2$

$x = -4$ gives $2x - 1 = -9 < 0$, so $\log_3(2x - 1)$ is undefined: reject

$x = 2$ and $x = \frac{7}{2}$ give $2x - 1 > 0$ and $14x - 25 > 0$

Solutions: $x = 2, \quad x = \frac{7}{2}$

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8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < 360°$, the equation

$$3\sin(\theta + 30°) = 7\cos(\theta + 30°)$$

giving your answers to one decimal place.

(4)

(ii) (a) Show that the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

can be written in the form

$$\sin x(a\cos^2 x + b\cos x + c) = 0$$

where a, b and c are constants to be found.

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

(6)

(i) $3\sin(\theta + 30°) = 7\cos(\theta + 30°) \Rightarrow \tan(\theta + 30°) = \frac{7}{3}$

$0° < \theta < 360° \Rightarrow 30° < \theta + 30° < 390°$

$\theta + 30° = \tan^{-1}\left(\frac{7}{3}\right) = 66.801...°$ or $180° + 66.801...° = 246.801...°$

$\theta = 36.801...°$ or $216.801...°$

$\theta = 36.8°$, $\theta = 216.8°$

(ii) (a) $3\sin^3 x = 5\sin x - 7\sin x \cos x$

$3\sin x(1 - \cos^2 x) - 5\sin x + 7\sin x \cos x = 0$

$\sin x(3 - 3\cos^2 x - 5 + 7\cos x) = 0$

$\sin x(-3\cos^2 x + 7\cos x - 2) = 0$

$\sin x(3\cos^2 x - 7\cos x + 2) = 0$, $a = 3$, $b = -7$, $c = 2$

(b) $\sin x = 0 \Rightarrow x = 0$

$3\cos^2 x - 7\cos x + 2 = 0 \Rightarrow (3\cos x - 1)(\cos x - 2) = 0$

$\cos x = 2$: no solutions since $-1 \leq \cos x \leq 1$

$\cos x = \frac{1}{3} \Rightarrow x = \pm\cos^{-1}\left(\frac{1}{3}\right) = \pm 1.2309...$

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Question 8 continued

$x = -1.23, \quad x = 0, \quad x = 1.23 \quad (\text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2})$

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Question 8 continued

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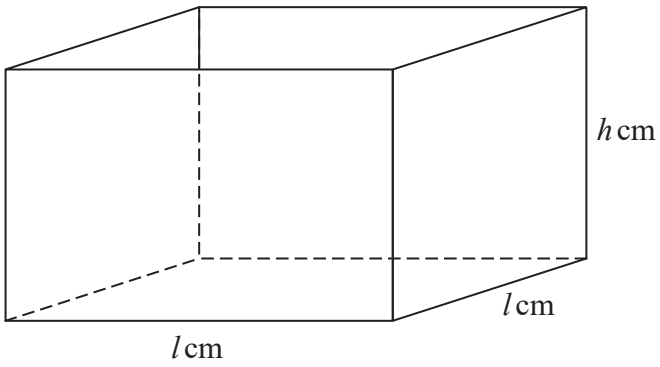


Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is 250 000 cm^3

(a) show that the external surface area, S cm^2, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

(a) $V = l^2h = 250000 \Rightarrow l = \frac{500}{\sqrt{h}}$

Open top: $S = l^2 + 4lh$

$S = \frac{250000}{h} + 4h \times \frac{500}{\sqrt{h}} = \frac{250000}{h} + 2000\sqrt{h}$ (as required)

(b) $S = 250000h^{-1} + 2000h^{1/2}$

$\frac{dS}{dh} = -250000h^{-2} + 1000h^{-1/2}$

$\frac{dS}{dh} = 0 \Rightarrow 1000h^{-1/2} = 250000h^{-2}$

$h^{3/2} = 250$

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$h = 250^{2/3}$, so $k = \frac{2}{3}$ $(h = 39.685...)$

(c) $\frac{d^2S}{dh^2} = 500000h^{-3} - 500h^{-3/2}$

At $h = 250^{2/3}$: $h^3 = 250^2 = 62500$ and $h^{3/2} = 250$

$\frac{d^2S}{dh^2} = \frac{500000}{62500} - \frac{500}{250} = 8 - 2 = 6 > 0$

so this value of h gives the minimum external surface area.

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Question 9 continued

Q9

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TOTAL FOR PAPER IS 75 MARKS

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