Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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1. The curve C has equation

$$y = \frac{x^2}{3} + \frac{4}{\sqrt{x}} + \frac{8}{3x} - 5 \qquad x > 0$$

(a) Find $\dfrac{dy}{dx}$, giving your answer in simplest form.

(4)

The point $P(4,3)$ lies on C.

(b) Find the equation of the normal to C at the point P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

(a) $y = \frac{1}{3}x^2 + 4x^{-1/2} + \frac{8}{3}x^{-1} - 5$

$\frac{dy}{dx} = \frac{2}{3}x - 2x^{-3/2} - \frac{8}{3}x^{-2}$

(b) $x = 4$: $\frac{dy}{dx} = \frac{2}{3}(4) - 2(4)^{-3/2} - \frac{8}{3}(4)^{-2}$

$= \frac{8}{3} - \frac{1}{4} - \frac{1}{6} = \frac{9}{4}$

Gradient of normal $= -1 \div \frac{9}{4} = -\frac{4}{9}$

$y - 3 = -\frac{4}{9}(x - 4)$

$9y - 27 = -4x + 16$

$4x + 9y - 43 = 0$

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Question 1 continued

Q1

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2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^3 + (6a + 8)x^2 - a^2x$$

where a is a positive constant.

Given $f(-1) = 32$

(a) (i) show that the only possible value for a is 3

(ii) Using $a = 3$ solve the equation

$$f(x) = 0$$

(5)

(b) Hence find all real solutions of

(i) $3y + 26y^{\frac{2}{3}} - 9y^{\frac{1}{3}} = 0$

(ii) $3\left(9^{3z}\right) + 26\left(9^{2z}\right) - 9\left(9^{z}\right) = 0$

(5)

(a) (i) $f(-1) = a(-1)^3 + (6a + 8)(-1)^2 - a^2(-1) = 32$

$-a + 6a + 8 + a^2 = 32 \Rightarrow a^2 + 5a - 24 = 0$

$(a + 8)(a - 3) = 0 \Rightarrow a = 3$ or $a = -8$

a is a positive constant, so $a = -8$ is rejected: $a = 3$ only

(ii) $f(x) = 3x^3 + 26x^2 - 9x = 0$

$x(3x^2 + 26x - 9) = 0$

$x(3x - 1)(x + 9) = 0$

$x = 0, \quad x = \frac{1}{3}, \quad x = -9$

(b) (i) Let $x = y^{1/3}$: $3y + 26y^{2/3} - 9y^{1/3} = 3x^3 + 26x^2 - 9x = 0$

So $y^{1/3} = 0, \quad y^{1/3} = \frac{1}{3}, \quad y^{1/3} = -9$

$y = 0^3, \quad y = \left(\frac{1}{3}\right)^3, \quad y = (-9)^3$

$y = 0, \quad y = \frac{1}{27}, \quad y = -729$

(ii) Let $x = 9^z$: $3x^3 + 26x^2 - 9x = 0$

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Question 2 continued

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So $9^z = 0$, $9^z = \frac{1}{3}$, $9^z = -9$

$9^z > 0$ for all z, so $9^z = 0$ and $9^z = -9$ have no solutions

$9^z = \frac{1}{3} \Rightarrow 3^{2z} = 3^{-1} \Rightarrow 2z = -1$

$z = -\frac{1}{2}$ only

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Question 2 continued

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Question 2 continued

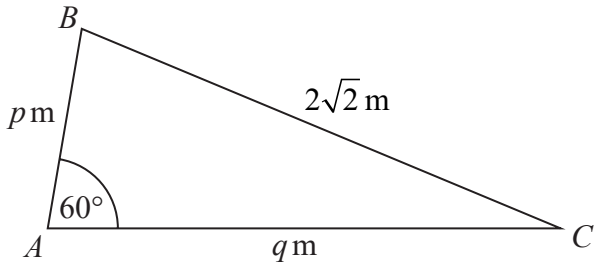
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Q2

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3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.



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Figure 1

Figure 1 shows the plan view of a flower bed.
The flowerbed is in the shape of a triangle ABC with

- $AB = p$ metres
- $AC = q$ metres
- $BC = 2\sqrt{2}$ metres
- angle $BAC = 60°$

(a) Show that

$$p^2 + q^2 - pq = 8$$

(2)

Given that side AC is 2 metres longer than side AB, use algebra to find

(b) (i) the exact value of p,

(ii) the exact value of q.

(5)

Using the answers to part (b),

(c) calculate the exact area of the flower bed.

(2)

(a) Cosine rule: $BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos 60°$

$(2\sqrt{2})^2 = p^2 + q^2 - 2pq \times \frac{1}{2}$

$8 = p^2 + q^2 - pq \Rightarrow p^2 + q^2 - pq = 8$

(b) (i) $q = p + 2$: $p^2 + (p+2)^2 - p(p+2) = 8$

$p^2 + p^2 + 4p + 4 - p^2 - 2p = 8$

$p^2 + 2p - 4 = 0$

$p = \frac{-2 \pm \sqrt{(2^2 - 4 \times 1 \times (-4))}}{2 \times 1} = \frac{-2 \pm \sqrt{20}}{2} = -1 \pm \sqrt{5}$

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Question 3 continued

p is a length so $p > 0$: $p = -1 - \sqrt{5}$ is rejected

$p = -1 + \sqrt{5}$ $(= \sqrt{5} - 1)$

(ii) $q = p + 2 = -1 + \sqrt{5} + 2$

$q = 1 + \sqrt{5}$

(c) Area $= \frac{1}{2} pq \sin 60° = \frac{1}{2} (\sqrt{5} - 1)(\sqrt{5} + 1) \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2} \times (5 - 1) \times \frac{\sqrt{3}}{2} = \frac{4\sqrt{3}}{4}$

Area $= \sqrt{3}$ m^2

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Question 3 continued

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Question 3 continued

Q3

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4. Find

$$\int \frac{(3\sqrt{x}+2)(x-5)}{4\sqrt{x}}\,dx$$

writing each term in simplest form.

(6)

$(3\sqrt{x}+2)(x-5) = 3x^{3/2} - 15x^{1/2} + 2x - 10$

$\frac{3x^{3/2} - 15x^{1/2} + 2x - 10}{4x^{1/2}} = \frac{3}{4}x - \frac{15}{4} + \frac{1}{2}x^{1/2} - \frac{5}{2}x^{-1/2}$

$\int \left(\frac{3}{4}x - \frac{15}{4} + \frac{1}{2}x^{1/2} - \frac{5}{2}x^{-1/2}\right) dx$

$= \frac{3}{4} \times \frac{x^2}{2} - \frac{15}{4}x + \frac{1}{2} \times \frac{x^{3/2}}{3/2} - \frac{5}{2} \times \frac{x^{1/2}}{1/2} + C$

$= \frac{3}{8}x^2 - \frac{15}{4}x + \frac{1}{3}x^{3/2} - 5x^{1/2} + C$

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Question 4 continued

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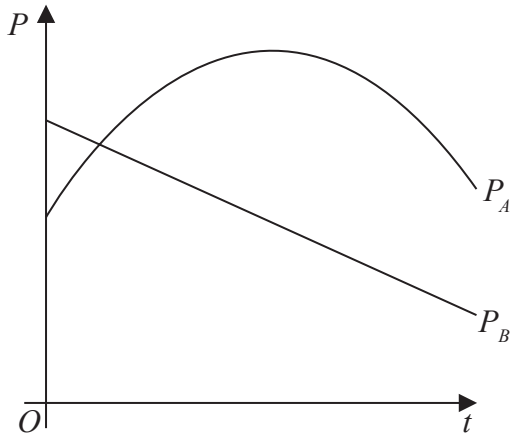


Figure 2

The share value of two companies, company A and company B, has been monitored over a 15-year period.

The share value P_A of **company *A***, in millions of pounds, is modelled by the equation

$$P_A = 53 - 0.4(t - 8)^2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

The share value P_B of **company *B***, in millions of pounds, is modelled by the equation

$$P_B = -1.6t + 44.2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

Figure 2 shows a graph of both models.

Use the equations of one or both models to answer parts (a) to (d).

(a) Find the difference between the share value of **company *A*** and the share value of **company *B*** at the point monitoring began.

(2)

(b) State the maximum share value of **company *A*** during the 15-year period.

(1)

(c) Find, using algebra and showing your working, the times during this 15-year period when the share value of **company *A*** was greater than the share value of **company *B***.

(4)

(d) Explain why the model for **company *A*** should not be used to predict its share value when $t = 20$

(1)

(a) $t = 0$: $P_A = 53 - 0.4(0 - 8)^2 = 53 - 25.6 = 27.4$

$P_B = -1.6(0) + 44.2 = 44.2$

Difference $= 44.2 - 27.4 = £16.8$ million

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Question 5 continued

(b) Maximum of P_A when $t = 8$: £53 million

(c) $P_A = P_B$: $53 - 0.4(t - 8)^2 = -1.6t + 44.2$

$53 - 0.4t^2 + 6.4t - 25.6 = -1.6t + 44.2$

$0.4t^2 - 8t + 16.8 = 0 \Rightarrow t^2 - 20t + 42 = 0$

$t = \frac{20 \pm \sqrt{(20^2 - 4 \times 1 \times 42)}}{2} = \frac{20 \pm \sqrt{232}}{2} = 10 \pm \sqrt{58}$

$t = 10 - \sqrt{58} = 2.384...$ or $t = 10 + \sqrt{58} = 17.6...$ (> 15, reject)

$P_A > P_B$ between these times (e.g. $t = 8$: $53 > 31.4$)

$2.38 < t \leq 15$ (years)

(d) $t = 20$: $P_A = 53 - 0.4(20 - 8)^2 = 53 - 57.6 = -4.6$

The share value would be negative, which is impossible

(also the model is only known to hold for the 15 years).

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Question 5 continued

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Question 5 continued

Q5

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6. The curve C has equation $y = f(x)$, $x > 0$

Given that

- C passes through the point $P(8, 2)$
- $f'(x) = \dfrac{32}{3x^2} + 3 - 2\left(\sqrt[3]{x}\right)$

(a) find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(b) Find, in simplest form, $f(x)$.

(5)

(a) $f'(8) = \dfrac{32}{3 \times 8^2} + 3 - 2\sqrt[3]{8} = \dfrac{1}{6} + 3 - 4 = -\dfrac{5}{6}$

$y - 2 = -\dfrac{5}{6}(x - 8)$

$y = -\dfrac{5}{6}x + \dfrac{20}{3} + 2$

$y = -\dfrac{5}{6}x + \dfrac{26}{3}$

(b) $f'(x) = \dfrac{32}{3}x^{-2} + 3 - 2x^{1/3}$

$f(x) = \int \left(\dfrac{32}{3}x^{-2} + 3 - 2x^{1/3}\right) dx$

$= \dfrac{32}{3} \times \dfrac{x^{-1}}{-1} + 3x - 2 \times \dfrac{x^{4/3}}{4/3} + c$

$= -\dfrac{32}{3}x^{-1} + 3x - \dfrac{3}{2}x^{4/3} + c$

$(8, 2)$: $2 = -\dfrac{32}{3 \times 8} + 3(8) - \dfrac{3}{2}(8)^{4/3} + c$

$2 = -\dfrac{4}{3} + 24 - 24 + c \Rightarrow c = \dfrac{10}{3}$

$f(x) = -\dfrac{32}{3}x^{-1} + 3x - \dfrac{3}{2}x^{4/3} + \dfrac{10}{3}$

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Question 6 continued

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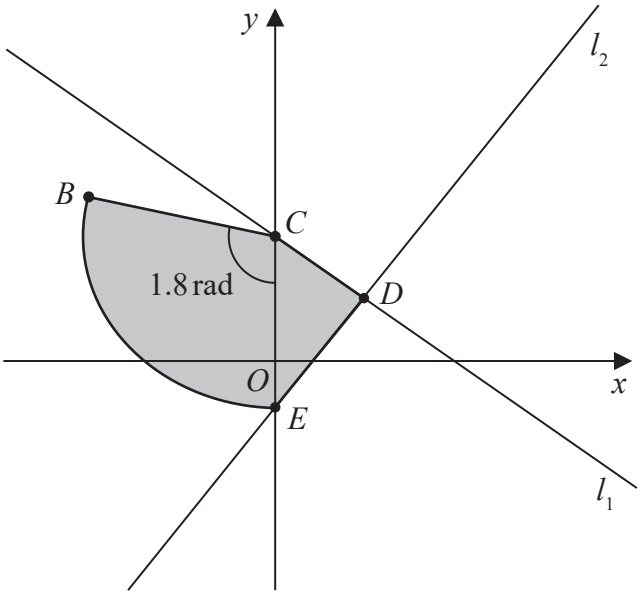


Figure 3

The line l_1 has equation $4y + 3x = 48$

The line l_1 cuts the y-axis at the point C, as shown in Figure 3.

(a) State the y coordinate of C.

(1)

The point $D(8, 6)$ lies on l_1

The line l_2 passes through D and is perpendicular to l_1

The line l_2 cuts the y-axis at the point E as shown in Figure 3.

(b) Show that the y coordinate of E is $-\frac{14}{3}$

(3)

A sector BCE of a circle with centre C is also shown in Figure 3.

Given that angle BCE is 1.8 radians,

(c) find the length of arc BE.

(3)

The region $CBED$, shown shaded in Figure 3, consists of the sector BCE joined to the triangle CDE.

(d) Calculate the exact area of the region $CBED$.

(3)

(a) $x = 0$: $4y = 48 \Rightarrow y = 12$

(b) l_1: $y = 12 - \frac{3}{4}x$, gradient of $l_1 = -\frac{3}{4}$

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Question 7 continued

$l_2 \perp l_1$: gradient of $l_2 = \frac{4}{3}$

$y - 6 = \frac{4}{3}(x - 8)$

$x = 0$: $y = 6 - \frac{32}{3} = -\frac{14}{3}$ so y coordinate of E is $-\frac{14}{3}$

(c) Radius CE $= 12 - \left(-\frac{14}{3}\right) = \frac{50}{3}$

Arc BE $= r\theta = \frac{50}{3} \times 1.8$

Arc BE $= 30$

(d) Sector BCE $= \frac{1}{2} r^2\theta = \frac{1}{2} \times \left(\frac{50}{3}\right)^2 \times 1.8 = 250$

Triangle CDE $= \frac{1}{2} \times$ CE $\times$ (x coord of D) $= \frac{1}{2} \times \frac{50}{3} \times 8 = \frac{200}{3}$

Area CBED $= 250 + \frac{200}{3} = \frac{950}{3}$

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Question 7 continued

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Question 7 continued

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Q7

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8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

(a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point P is the minimum point of C_1

(b) Deduce the coordinates of P.

(1)

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A, B, C and D are constants.

Given that C_2

- passes through P
- intersects the x-axis at -4, -2 and 3

(c) find, making your method clear, the values of A, B, C and D.

(5)

(a) $3x^2 + 6x + 9 = 3(x^2 + 2x) + 9$

$= 3[(x + 1)^2 - 1] + 9$

$= 3(x + 1)^2 + 6$ $\quad a = 3,\ b = 1,\ c = 6$

(b) $P\ (-1, 6)$

(c) Roots $-4, -2, 3$: $y = k(x + 4)(x + 2)(x - 3)$

Through $P\ (-1, 6)$: $6 = k(-1 + 4)(-1 + 2)(-1 - 3)$

$6 = k(3)(1)(-4) = -12k \Rightarrow k = -\frac{1}{2}$

$(x + 4)(x + 2)(x - 3) = (x^2 + 6x + 8)(x - 3)$

$= x^3 - 3x^2 + 6x^2 - 18x + 8x - 24 = x^3 + 3x^2 - 10x - 24$

$y = -\frac{1}{2}(x^3 + 3x^2 - 10x - 24) = -\frac{1}{2}x^3 - \frac{3}{2}x^2 + 5x + 12$

$A = -\frac{1}{2},\ B = -\frac{3}{2},\ C = 5,\ D = 12$

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Question 8 continued

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Question 8 continued

Q8

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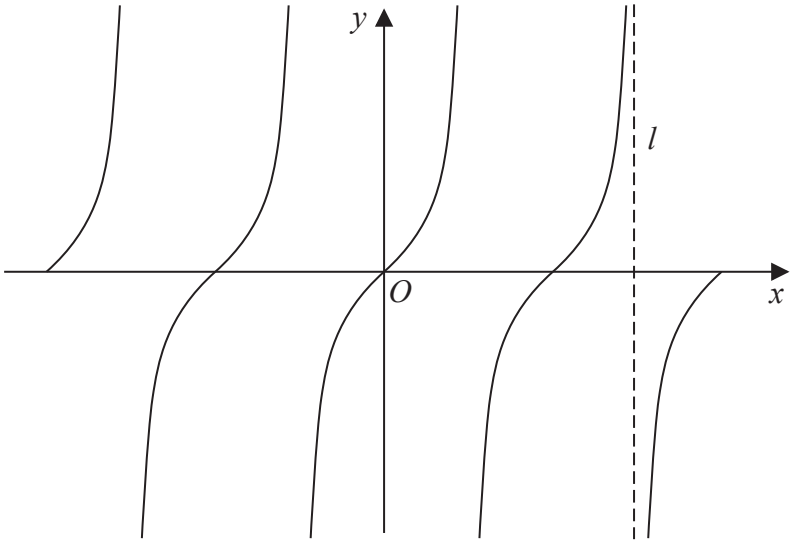


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

The line l, shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l.

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \qquad -2\pi \leqslant x \leqslant 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leqslant x \leqslant 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leqslant x \leqslant 40\pi$

(ii) $-10\pi \leqslant x \leqslant \frac{5}{2}\pi$

(2)

(a) $x = \frac{3\pi}{2}$

(b) (i) Sketch on Diagram 1. Horizontal asymptote: $y = 1$

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Question 9 continued

(ii) 5 solutions, because the curve $y = 1/x + 1$ meets $y = \tan x$ at 5 points in $-2\pi \leq x \leq 2\pi$ (number of intersections of the graphs)

(c) (i) 40

(ii) 14

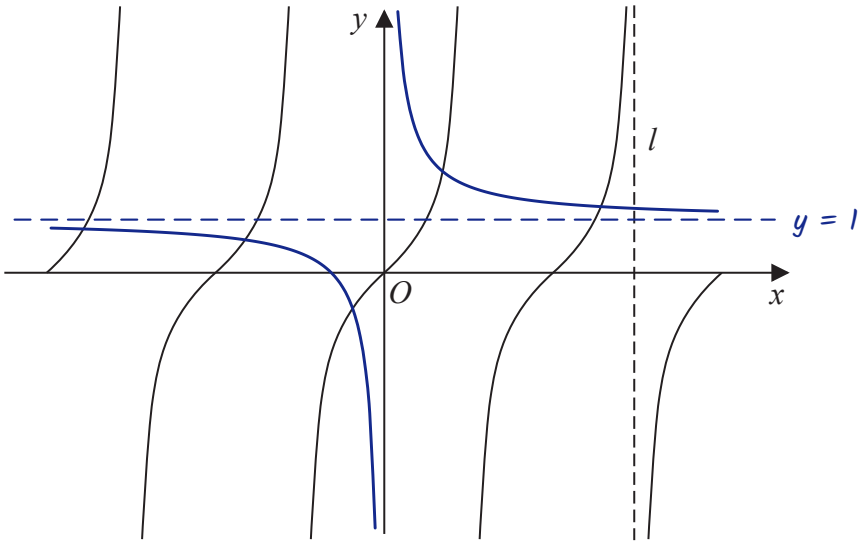


Diagram 1

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Question 9 continued

Q9

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