Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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Write your answers in the D1 answer book for this paper.

1.

16 23 18 9 4 20 35 5 17 13 6 11

The numbers in the list represent the weights, in kilograms, of twelve parcels. The parcels are to be transported in containers that will each hold a maximum weight of 45 kg.

(a) Calculate a lower bound for the number of containers needed. You must make your method clear.

(2)

(b) Use the first-fit bin packing algorithm to allocate the parcels to the containers.

(3)

(c) Carry out a bubble sort, starting at the left-hand end of the list, to produce a list of the weights in descending order. You should only give the state of the list after each pass.

(4)

(d) Use the first-fit decreasing bin packing algorithm to allocate the parcels to the containers.

(3)

(Total 12 marks)

2.

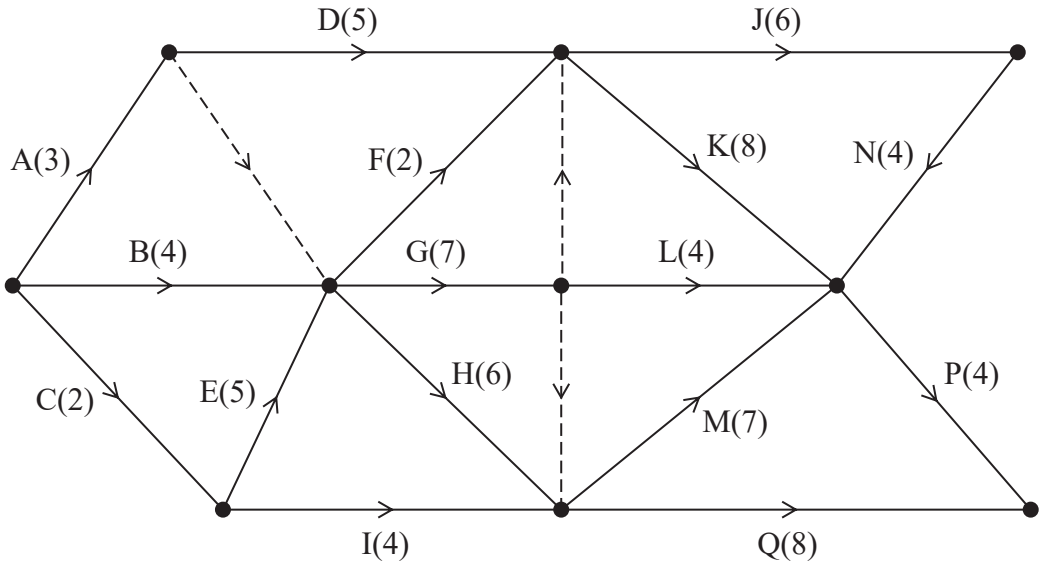


Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(b) Draw a cascade chart for this project on Grid 1 in the answer book.

(4)

(c) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities. (You do not need to provide a schedule of the activities.)

(2)

(Total 10 marks)

3.

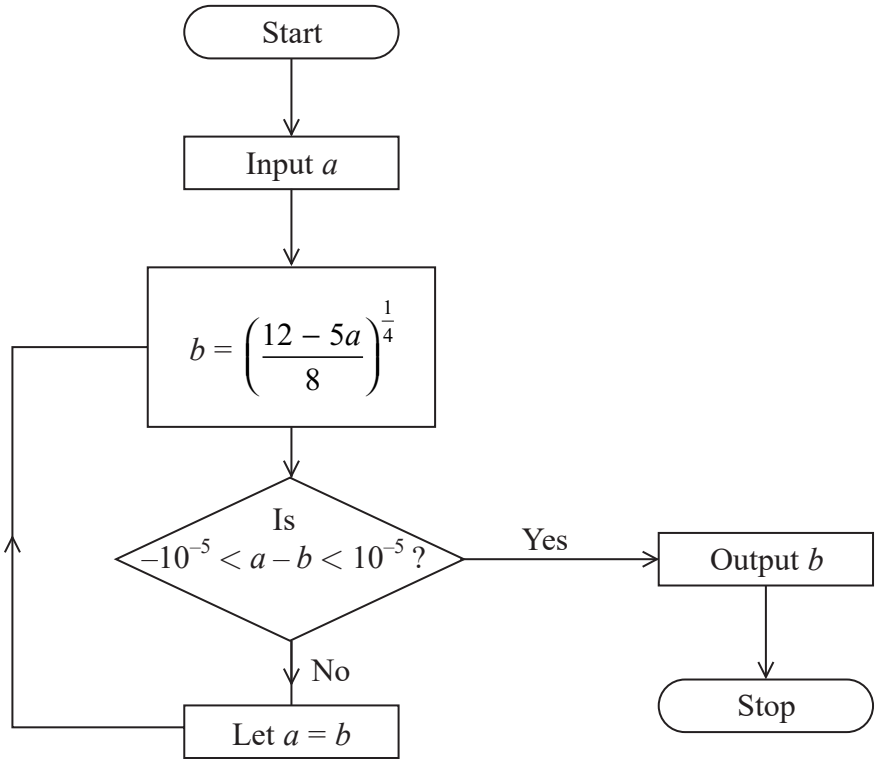


Figure 2

An algorithm for finding the positive real root of the equation $8x^4 + 5x - 12 = 0$ is described by the flow chart shown in Figure 2.

(a) Use the flow chart, with $a = 1$, to complete the table in the answer book, stating values to at least 6 decimal places. Give the final output correct to 5 decimal places.

(4)

Given that the value of the input a is a non-negative real number,

(b) determine the set of values for a that **cannot** be used to find the positive real root of $8x^4 + 5x - 12 = 0$ using this flow chart.

(2)

(Total 6 marks)

4.

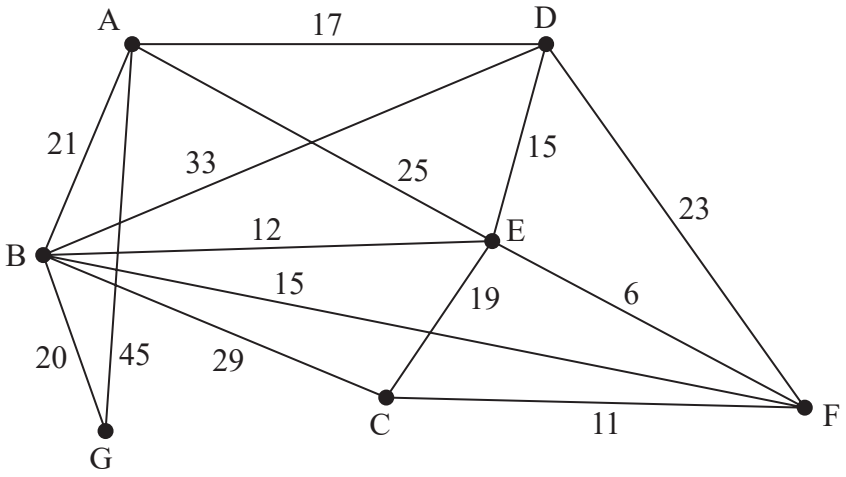


Figure 3

[*The total weight of the network is* 291]

Figure 3 models a network of roads. The number on each edge gives the length, in km, of the corresponding road. The vertices, A, B, C, D, E, F and G, represent seven towns. Derek needs to visit each town. He will start and finish at A and wishes to minimise the total distance travelled.

(a) By inspection, complete the two copies of the table of least distances in the answer book.

(2)

(b) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of Derek's route. Write down the route that gives this upper bound.

(2)

(c) Interpret the route found in (b) in terms of the towns actually visited.

(1)

(d) Starting by deleting A and all of its arcs, find a lower bound for the route length.

(3)

Clive needs to travel along the roads to check that they are in good repair. He wishes to minimise the total distance travelled and must start at A and finish at G.

(e) By considering the pairings of all relevant nodes, find the length of Clive's route. State the edges that need to be traversed twice. You must make your method and working clear.

(5)

(Total 13 marks)

5.

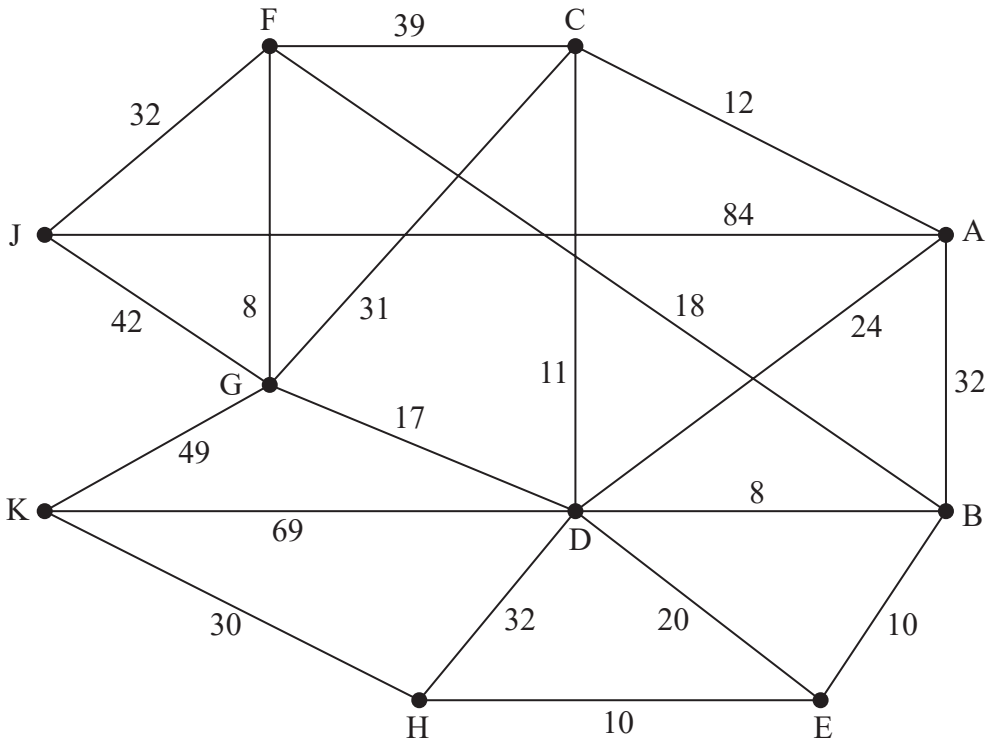


Figure 4

Figure 4 represents a network of roads. The number on each arc represents the length, in miles, of the corresponding road. Tamasi, who lives at A, needs to collect a caravan. Tamasi can collect a caravan from either J or K.

Tamasi decides to use Dijkstra's algorithm once to find the shortest routes between A and J and between A and K.

(a) State, with a reason, which vertex should be chosen as the starting vertex for the algorithm.

(2)

(b) Use Dijkstra's algorithm to find the shortest routes from A to J and from A to K. You should state the routes and their corresponding lengths.

(7)

Tamasi's brother lives at F. He needs to visit Tamasi at A and then visit their mother who lives at H.

(c) Find a route of minimal length that goes from F to H via A.

(1)

(Total 10 marks)

6.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	A
F	A, B, C
G	C
H	G
I	D, E, F, H
J	I
K	I
L	I
M	L

(a) Draw the activity network for the project described in the precedence table above, using activity on arc and the minimum number of dummies.

(5)

(b) State which activity is guaranteed to be critical, giving a reason for your answer.

(2)

It is given that each activity in the table takes two hours to complete.

(c) State the minimum completion time and write down the critical path for the project.

(2)

(Total 9 marks)

7.

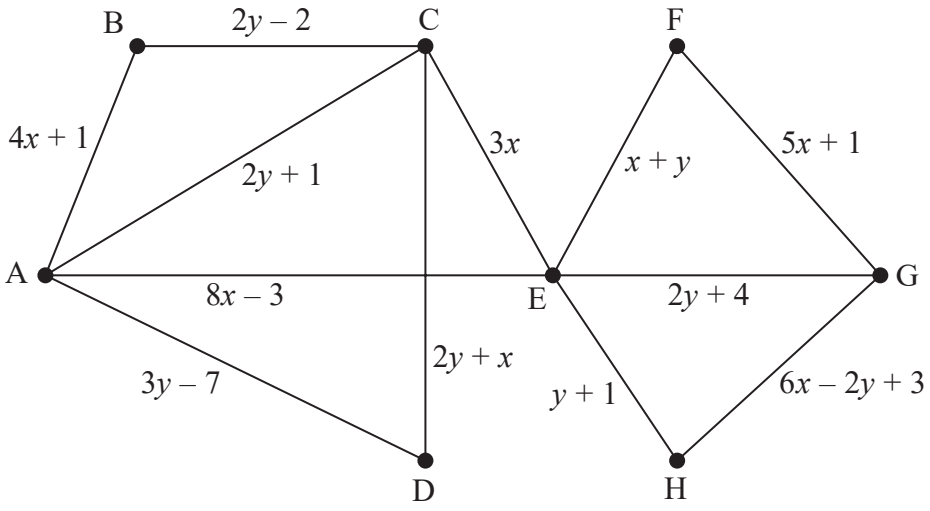


Figure 5

Figure 5 shows a weighted graph that contains 12 arcs and 8 vertices.

It is given that

- no two arcs have the same weight
- x and y are positive integers
- arc CD is not in the minimum spanning tree for the graph

(a) Explain why $y < x + 7$

(2)

It is also given that when Prim's algorithm, starting at A, is applied to the weighted graph, AB is the first arc selected.

(b) Show that $y > 2x$ and write down and simplify two further constraints on the values of x and y.

(3)

(c) Represent these four constraints on Diagram 1 in the answer book.

(4)

(d) Using Diagram 1 **only**, write down the possible pairs of values that x and y can take in the form (x, y).

(2)

The minimum spanning tree for the weighted graph in Figure 5 has total weight 73
Six of the seven arcs in the minimum spanning tree are AB, AD, BC, CE, EF and GH.

(e) Determine the value of x and the value of y. You must make your method and working clear.

(4)

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over ▶

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P 6 6 6 3 9 A 0 1 2 0

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1.

16 23 18 9 4 20 35 5 17 13 6 11

(a) Total weight = 16 + 23 + 18 + 9 + 4 + 20 + 35 + 5 + 17 + 13 + 6 + 11 = 177

177 ÷ 45 = 3.933...

Lower bound = 4 containers

(b) First-fit:

Container 1: 16 23 4

Container 2: 18 9 5 13

Container 3: 20 17 6

Container 4: 35

Container 5: 11

(c) Bubble sort into descending order:

16 23 18 9 4 20 35 5 17 13 6 11

Pass 1: 23 18 16 9 20 35 5 17 13 6 11 4

Pass 2: 23 18 16 20 35 9 17 13 6 11 5 4

Pass 3: 23 18 20 35 16 17 13 9 11 6 5 4

Pass 4: 23 20 35 18 17 16 13 11 9 6 5 4

Pass 5: 23 35 20 18 17 16 13 11 9 6 5 4

Pass 6: 35 23 20 18 17 16 13 11 9 6 5 4

Pass 7: 35 23 20 18 17 16 13 11 9 6 5 4

No swaps in pass 7, so the list is sorted.

(d) First-fit decreasing:

Container 1: 35 9

Container 2: 23 20

Container 3: 18 17 6 4

Container 4: 16 13 11 5

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Question 1 continued

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Question 1 continued

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Question 1 continued

Q1

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2.

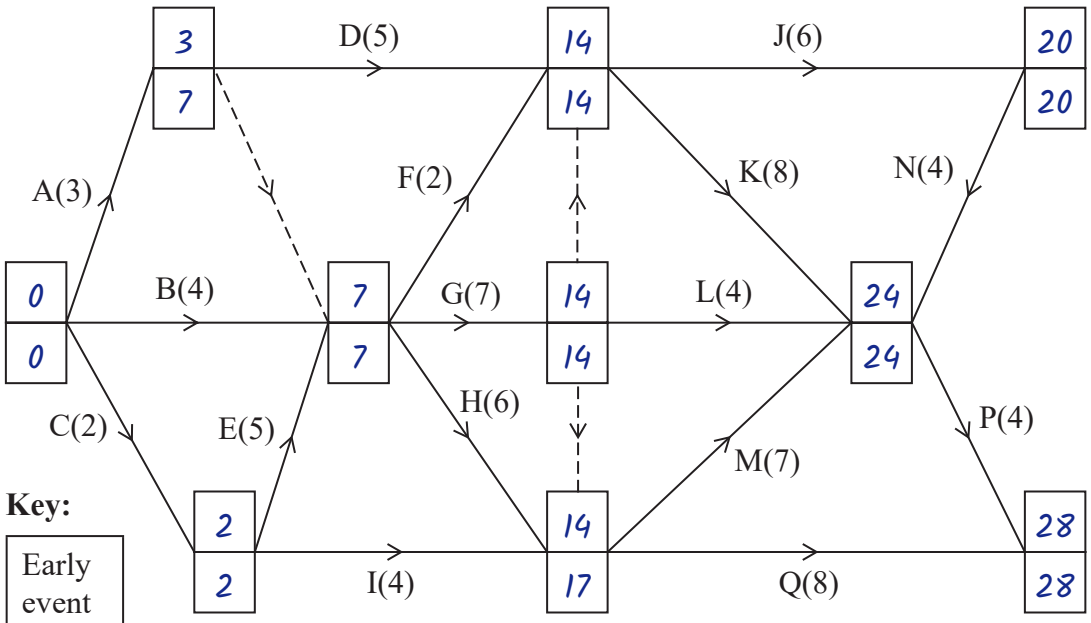


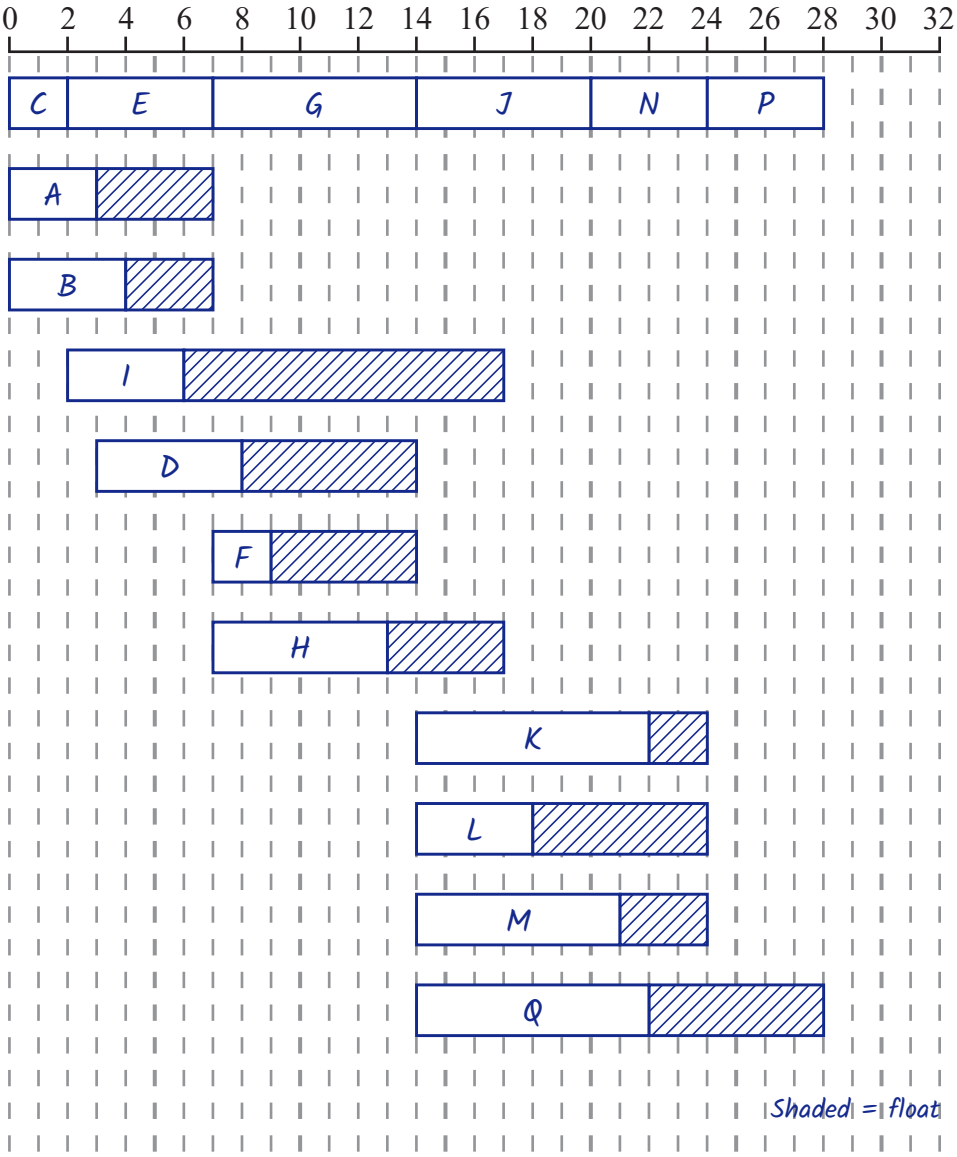
Diagram 1

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Question 2 continued



Grid 1

(c) Between time 20 and time 21 (e.g. at time 20.5), activities K, M, N and Q must all be happening at the same time, so at least 4 workers are needed.

Minimum number of workers = 4

Q2

(Total 10 marks)

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3. *You may not need to use all the rows in this table.*
You should write your final output in the box provided at the bottom of this page.

a	b	Is $-10^{-5} < a - b < 10^{-5}$?
1	0.967168	No
0.967168	0.972789	No
0.972789	0.971834	No
0.971834	0.971996	No
0.971996	0.971969	No
0.971969	0.971974	Yes

Final output	0.97197

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Question 3 continued

(b) $b = ((12 - 5a)/8)^{1/4}$ cannot be found if $12 - 5a < 0$,

since the fourth root of a negative number is not real.

$12 - 5a < 0 \Rightarrow 5a > 12 \Rightarrow a > 2.4$

The values of a that cannot be used are $a > 2.4$

Q3

(Total 6 marks)

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4.

	A	B	C	D	E	F	G
A	–	21	42	17	25	31	41
B	21	–	26	27	12	15	20
C	42	26	–	32	17	11	46
D	17	27	32	–	15	21	47
E	25	12	17	15	–	6	32
F	31	15	11	21	6	–	35
G	41	20	46	47	32	35	–

(a) AC: A – E – C = 25 + 17 = 42, CD: C – E – D = 17 + 15 = 32,
DF: D – E – F = 15 + 6 = 21 (entered in both tables)

(b) Nearest neighbour starting at A:
A – D (17), D – E (15), E – F (6), F – C (11), C – B (26),
B – G (20), G – A (41)
Route: A – D – E – F – C – B – G – A
Upper bound = 17 + 15 + 6 + 11 + 26 + 20 + 41 = 136 km

(c) A – D – E – F – C – F – B – G – B – A

(d) Delete A. Minimum spanning tree on B, C, D, E, F, G:
EF (6), CF (11), BE (12), DE (15), BG (20)
Weight of RMST = 6 + 11 + 12 + 15 + 20 = 64
Two least arcs from A: AD (17) and AB (21)
Lower bound = 64 + 17 + 21 = 102 km

(e) Odd vertices: C (degree 3) and E (degree 5). The route starts
at A and finishes at G, so the relevant nodes are A, C, E and G.
AC + EG = 42 + 32 = 74
AE + CG = 25 + 46 = 71
AG + CE = 41 + 17 = 58 (least)
AG: A – B – G (21 + 20), CE: C – F – E (11 + 6)
Edges traversed twice: AB, BG, CF and EF
Length of Clive's route = 291 + 58 = 349 km

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Question 4 continued

	A	B	C	D	E	F	G
A	–	21	42	17	25	31	41
B	21	–	26	27	12	15	20
C	42	26	–	32	17	11	46
D	17	27	32	–	15	21	47
E	25	12	17	15	–	6	32
F	31	15	11	21	6	–	35
G	41	20	46	47	32	35	–

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Question 4 continued

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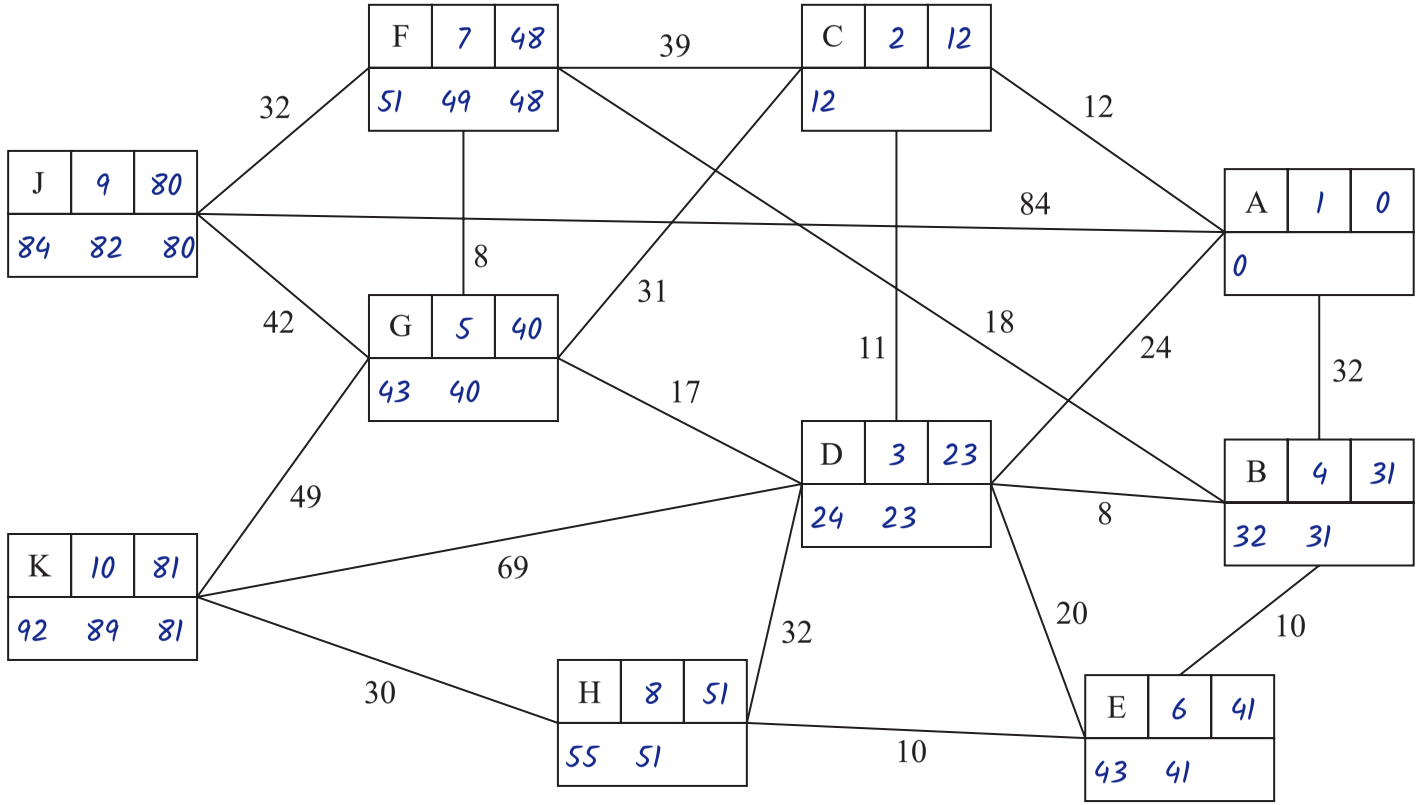
Question 4 continued

Q4

(Total 13 marks)

5.

(a) *Start at A, because A appears in both of the required routes (A to J and A to K). Applying Dijkstra's algorithm from A finds the shortest route from A to every other vertex, so both J and K are found with one application.*



Key:

Vertex	Order of labelling	Final value
Working values		

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Question 5 continued

(b) Trace back from J: 80 − 32 = 48 (F), 48 − 8 = 40 (G),
40 − 17 = 23 (D), 23 − 11 = 12 (C), 12 − 12 = 0 (A)
Shortest route to J: A − C − D − G − F − J, length 80 miles

Trace back from K: 81 − 30 = 51 (H), 51 − 10 = 41 (E),
41 − 10 = 31 (B), 31 − 8 = 23 (D), 23 − 11 = 12 (C), 12 − 12 = 0 (A)
Shortest route to K: A − C − D − B − E − H − K, length 81 miles

(c) F to A: F − G − D − C − A (48)
A to H: A − C − D − B − E − H (51)
Route: F − G − D − C − A − C − D − B − E − H
Length = 48 + 51 = 99 miles

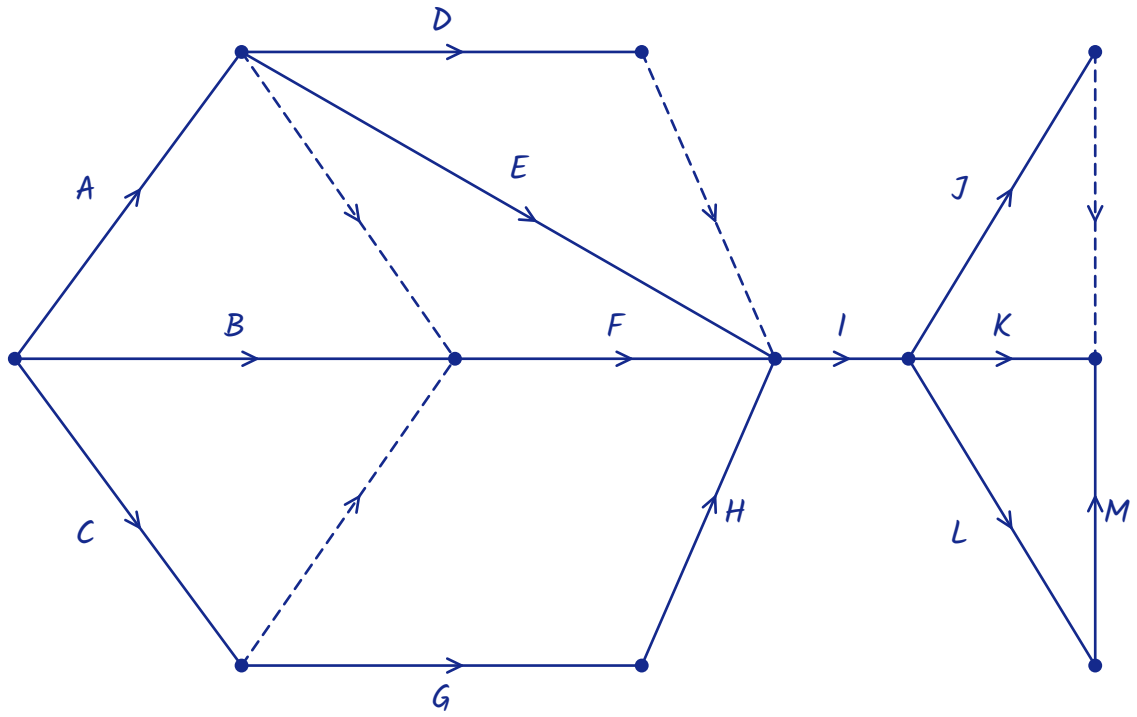
Q5

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6.

(a)



Dashed lines are dummies

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Question 6 continued

(b) Activity I is guaranteed to be critical, because every path from the source to the sink passes through activity I.

(c) Each activity takes 2 hours.

Longest path: C – G – H – I – L – M, 6 activities × 2 = 12

Minimum completion time = 12 hours

Critical path: C G H I L M

Q6

(Total 9 marks)

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7.

(a) The only arcs incident to D are AD and CD. Every vertex must be in the minimum spanning tree, so as CD is not in the tree, AD must be in it, and so the weight of CD is greater than the weight of AD (otherwise CD would be chosen to connect D).

$2y + x > 3y - 7$

$\Rightarrow y < x + 7$

(b) AB is selected first, so AB is the least arc from A:

AB < AC: $4x + 1 < 2y + 1 \Rightarrow 4x < 2y \Rightarrow y > 2x$

AB < AE: $4x + 1 < 8x - 3 \Rightarrow 4 < 4x \Rightarrow x > 1$

AB < AD: $4x + 1 < 3y - 7 \Rightarrow 3y > 4x + 8$

Constraints: $y < x + 7$, $y > 2x$, $x > 1$, $3y > 4x + 8$

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Question 7 continued

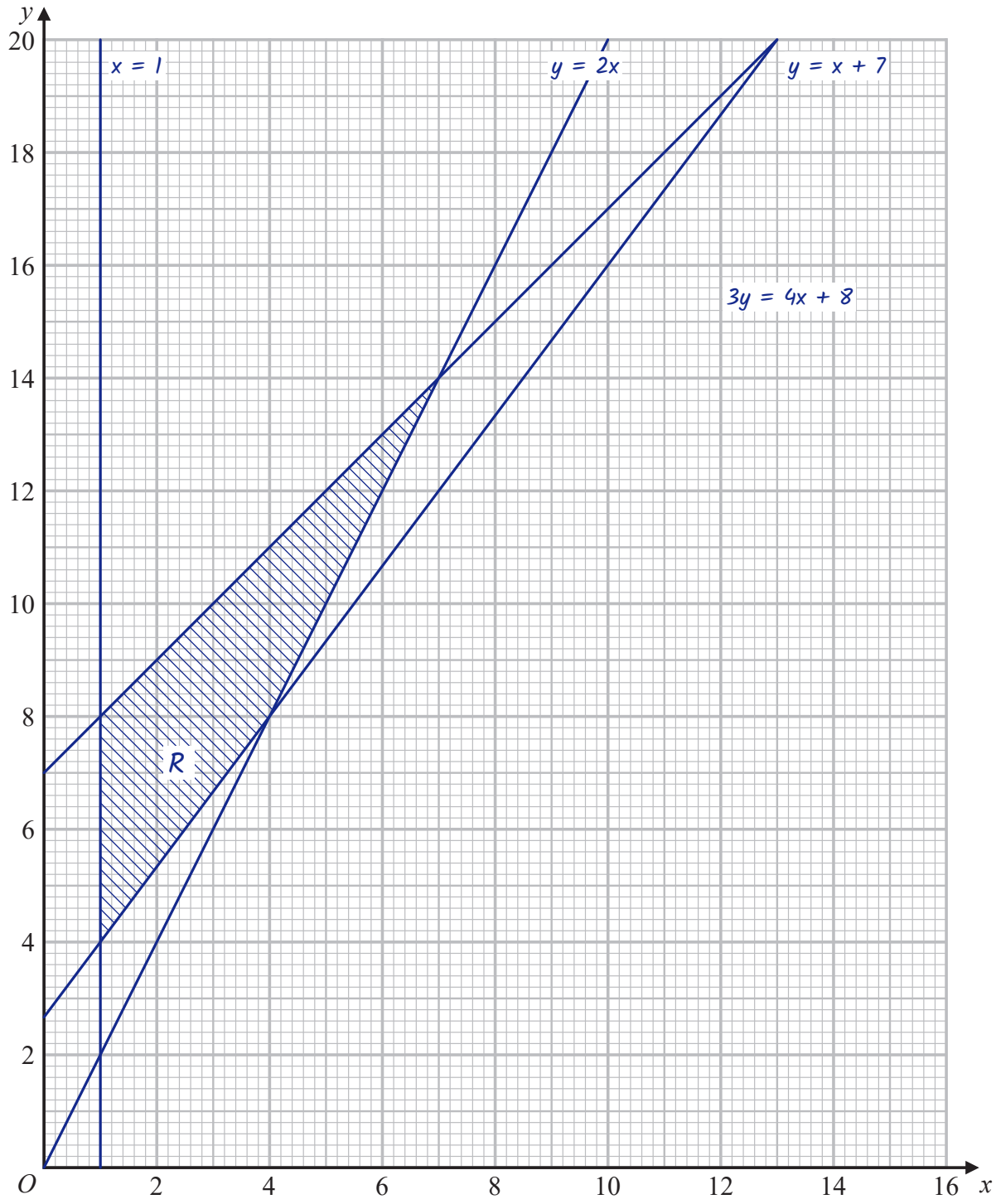


Diagram 1

(c) Feasible region R hatched.

(d) (2, 6), (2, 7), (2, 8), (3, 7), (3, 8), (3, 9),
(4, 9), (4, 10), (5, 11)

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Question 7 continued

(e) AB, AD, BC, CE, EF connect A, B, C, D, E, F and GH connects G and H, so the remaining arc joins {G, H} to the rest:
EG (2y + 4), FG (5x + 1) or EH (y + 1).
EH < EG since y + 1 < 2y + 4, and EH < FG since y < 5x for all points in R, so the seventh arc is EH.

Weight of MST = (4x + 1) + (3y − 7) + (2y − 2) + 3x + (x + y)
+ (6x − 2y + 3) + (y + 1)
= 14x + 5y − 4
$14x + 5y - 4 = 73 \Rightarrow 14x + 5y = 77$

Testing the points from (d):
x = 2: 5y = 77 − 28 = 49, y = 9.8 (not an integer)
x = 3: 5y = 77 − 42 = 35, y = 7 and (3, 7) is in R
x = 4: 5y = 77 − 56 = 21, y = 4.2 (not an integer)
x = 5: 5y = 77 − 70 = 7, y = 1.4 (not an integer)

$\therefore$ x = 3 and y = 7

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

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