Please check the examination details below before entering your candidate information

Candidate surname	StemUp Model Solution	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Tuesday 18 June 2019

Morning (Time: 1 hour 30 minutes)	Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Answer all questions. Write your answers in the spaces provided.

1. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4 - a_n$$
$$a_1 = 3$$

Find the value of

(a) (i) a_2

(ii) a_{107}

(2)

(b) $\sum_{n=1}^{200}(2a_n - 1)$

(2)

(a) (i) $a_2 = 4 - a_1 = 4 - 3 = 1$

(ii) $a_3 = 4 - 1 = 3$, $a_4 = 1$, ... so $a_n = 3$ for n odd, 1 for n even

107 is odd, so $a_{107} = 3$

(b) $2a_n - 1 = 5$ (n odd), $2a_n - 1 = 1$ (n even)

$\Sigma (2a_n - 1) = 5 + 1 + 5 + 1 + \ldots + 5 + 1$ (100 pairs)

$= 100 \times (5 + 1) = 600$

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Question 1 continued

Q1

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2. A circle C has equation

$$x^2 + y^2 + 4x - 10y - 21 = 0$$

Find

(a) (i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The point $P(5, 4)$ lies on C.

(b) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

(a) $x^2 + 4x + y^2 - 10y = 21$

$(x + 2)^2 - 4 + (y - 5)^2 - 25 = 21$

$(x + 2)^2 + (y - 5)^2 = 50$

(i) Centre $(-2, 5)$

(ii) Radius $= \sqrt{50} = 5\sqrt{2}$

(b) Gradient of radius $CP = \frac{5 - 4}{-2 - 5} = -\frac{1}{7}$

Tangent is perpendicular to radius: $m = -1 \div \left(-\frac{1}{7}\right) = 7$

$y - 4 = 7(x - 5)$

$y = 7x - 31$

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Question 2 continued

Q2

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3. (i) Use algebra to prove that for all real values of x

$$(x-4)^2 \geqslant 2x-9$$

(3)

(ii) Show that the following statement is untrue.

$2^n + 1$ is a prime number for all values of n, $n \in \mathbb{N}$

(1)

(i) $(x-4)^2 - (2x-9) = x^2 - 8x + 16 - 2x + 9$

$= x^2 - 10x + 25$

$= (x-5)^2$

Square numbers are greater than or equal to zero, so as $x \in \mathbb{R}$,

$(x-5)^2 \geq 0 \Rightarrow (x-4)^2 - (2x-9) \geq 0$

$\Rightarrow (x-4)^2 \geq 2x - 9$ for all real x

(ii) When $n = 3$: $2^3 + 1 = 8 + 1 = 9 = 3 \times 3$

9 is not prime, so the statement is untrue.

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Question 3 continued

Q3

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4. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2-\frac{1}{4}x\right)^6$$

(4)

(b) Given that x is small, so terms in x^4 and higher powers of x may be ignored, show

$$\left(2-\frac{1}{4}x\right)^6+\left(2+\frac{1}{4}x\right)^6=a+bx^2$$

where a and b are constants to be found.

(3)

(a) $\left(2-\frac{1}{4}x\right)^6 = 2^6+\binom{6}{1}2^5\left(-\frac{1}{4}x\right)+\binom{6}{2}2^4\left(-\frac{1}{4}x\right)^2$

$+\binom{6}{3}2^3\left(-\frac{1}{4}x\right)^3+\ldots$

$= 64+6\times 32\left(-\frac{x}{4}\right)+15\times 16\left(\frac{x^2}{16}\right)+20\times 8\left(-\frac{x^3}{64}\right)+\ldots$

$= 64-48x+15x^2-2.5x^3+\ldots$

(b) $\left(2+\frac{1}{4}x\right)^6 = 64+48x+15x^2+2.5x^3+\ldots$ (replace x by −x)

$\left(2-\frac{1}{4}x\right)^6+\left(2+\frac{1}{4}x\right)^6$

$\approx (64-48x+15x^2-2.5x^3)+(64+48x+15x^2+2.5x^3)$

$= 128+30x^2$

$a = 128, \quad b = 30$

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Question 4 continued

Q4

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5. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and £x is the selling price of the watch.

According to this model,

(a) find, using calculus, the maximum possible annual profit.

(6)

(b) Justify, also using calculus, that the profit you have found is a maximum.

(2)

(a) $P = 12x - x^{3/2} - 120 \Rightarrow \frac{dP}{dx} = 12 - \frac{3}{2}x^{1/2}$

For a maximum, $\frac{dP}{dx} = 0$: $12 - \frac{3}{2}x^{1/2} = 0$

$x^{1/2} = 8 \Rightarrow x = 64$

$P = 12 \times 64 - 64^{3/2} - 120 = 768 - 512 - 120 = 136$

Maximum possible annual profit = £136 000

(b) $\frac{d^2P}{dx^2} = -\frac{3}{4}x^{-1/2}$

At $x = 64$: $\frac{d^2P}{dx^2} = -\frac{3}{4} \times \frac{1}{8} = -\frac{3}{32}$ $(= -0.09375)$

$\frac{d^2P}{dx^2} < 0$, so the profit of £136 000 is a maximum

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Question 5 continued

Q5

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6. $f(x) = kx^3 - 15x^2 - 32x - 12$ where k is a constant

Given $(x - 3)$ is a factor of $f(x)$,

(a) show that $k = 9$

(2)

(b) Using algebra and showing each step of your working, fully factorise $f(x)$.

(4)

(c) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$$

giving your answers to one decimal place.

(2)

(a) $(x - 3)$ is a factor, so $f(3) = 0$

$k(3)^3 - 15(3)^2 - 32(3) - 12 = 0$

$27k - 135 - 96 - 12 = 0$

$27k = 243 \Rightarrow k = 9$

(b) $f(x) = 9x^3 - 15x^2 - 32x - 12$

$= (x - 3)(9x^2 + ax + 4)$

x^2 terms: $a - 27 = -15 \Rightarrow a = 12$

Check x terms: $4 - 3a = 4 - 36 = -32$ ✓

$f(x) = (x - 3)(9x^2 + 12x + 4)$

$f(x) = (x - 3)(3x + 2)^2$

(c) Let $x = \cos\theta$: $(\cos\theta - 3)(3\cos\theta + 2)^2 = 0$

$\cos\theta = 3$ (no solutions since $-1 \leqslant \cos\theta \leqslant 1$) or $\cos\theta = -\frac{2}{3}$

$\theta = \cos^{-1}(-2/3) = 131.810...°$ or $\theta = 360° - 131.810...° = 228.189...°$

$\theta = 131.8°,\ 228.2°$

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Question 6 continued

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7. Kim starts working for a company.

- In year 1 her annual salary will be £16 200
- In year 10 her annual salary is predicted to be £31 500

Model *A* assumes that her annual salary will increase by the same amount each year.

(a) According to model *A*, determine Kim's annual salary in year 2.

(3)

Model *B* assumes that her annual salary will increase by the same percentage each year.

(b) According to model *B*, determine Kim's annual salary in year 2. Give your answer to the nearest £10

(3)

(c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10

(3)

(a) AP with $a = 16\,200$, $u_{10} = a + 9d = 31\,500$

$9d = 31\,500 - 16\,200 = 15\,300 \Rightarrow d = \frac{15\,300}{9} = 1700$

Year 2 salary $= 16\,200 + 1700 = £17\,900$

(b) GP with $a = 16\,200$, $u_{10} = ar^9 = 31\,500$

$r^9 = \frac{31\,500}{16\,200} = \frac{35}{18} \Rightarrow r = \left(\frac{35}{18}\right)^{1/9} = 1.076684...$

Year 2 salary $= 16\,200 \times 1.076684... = 17\,442.28...$

$= £17\,440$ (nearest £10)

(c) Model A: $S_{10} = \frac{10}{2}(16\,200 + 31\,500) = 5 \times 47\,700 = £238\,500$

Model B: $S_{10} = \frac{16\,200(1.076684...^{10} - 1)}{1.076684... - 1} = \frac{16\,200 \times 1.093552...}{0.076684...}$

$= £231\,019.24...$

Difference $= 238\,500 - 231\,019.24... = 7480.75...$

$= £7480$ (nearest £10)

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Question 7 continued

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8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b\log_2 3$, where a and b are constants to be found.

(4)

(ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2\log_5(y + 1) - 1$$

(5)

(i) $8^{2x+1} = 6$

$\log_2 8^{2x+1} = \log_2 6 \Rightarrow (2x + 1)\log_2 8 = \log_2 6$

$3(2x + 1) = \log_2(2 \times 3) = \log_2 2 + \log_2 3 = 1 + \log_2 3$

$6x + 3 = 1 + \log_2 3$

$6x = -2 + \log_2 3$

$x = -\frac{1}{3} + \frac{1}{6}\log_2 3$

(ii) $\log_5(7 - 2y) = 2\log_5(y + 1) - 1$

$\log_5(7 - 2y) = \log_5(y + 1)^2 - \log_5 5$

$\log_5(7 - 2y) = \log_5\left(\frac{(y + 1)^2}{5}\right)$

$7 - 2y = \frac{(y + 1)^2}{5}$

$35 - 10y = y^2 + 2y + 1$

$y^2 + 12y - 34 = 0$

$y = \frac{-12 \pm \sqrt{(144 + 136)}}{2} = \frac{-12 \pm \sqrt{280}}{2} = -6 \pm \sqrt{70}$

$y = -6 - \sqrt{70}$ gives $y + 1 < 0$, so $\log_5(y + 1)$ is undefined: reject

$y = -6 + \sqrt{70}$

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9. (a) Show that the equation

$$\cos\theta - 1 = 4\sin\theta\tan\theta$$

can be written in the form

$$5\cos^2\theta - \cos\theta - 4 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < \frac{\pi}{2}$

$$\cos 2x - 1 = 4\sin 2x\tan 2x$$

giving your answers, where appropriate, to 2 decimal places.

(4)

(a) $\cos\theta - 1 = 4\sin\theta\tan\theta \Rightarrow \cos\theta - 1 = 4\sin\theta \times \frac{\sin\theta}{\cos\theta}$

$\cos^2\theta - \cos\theta = 4\sin^2\theta$

$\cos^2\theta - \cos\theta = 4(1 - \cos^2\theta)$

$\cos^2\theta - \cos\theta = 4 - 4\cos^2\theta$

$5\cos^2\theta - \cos\theta - 4 = 0$ (as required)

(b) Let $\theta = 2x$: $5\cos^2 2x - \cos 2x - 4 = 0$

$(5\cos 2x + 4)(\cos 2x - 1) = 0$

$\cos 2x = 1$ or $\cos 2x = -\frac{4}{5}$

$0 \leq x < \pi/2 \Rightarrow 0 \leq 2x < \pi$

$\cos 2x = 1 \Rightarrow 2x = 0 \Rightarrow x = 0$

$\cos 2x = -4/5 \Rightarrow 2x = 2.498091\ldots \Rightarrow x = 1.249045\ldots$

$x = 0$, $x = 1.25$ (2 d.p.)

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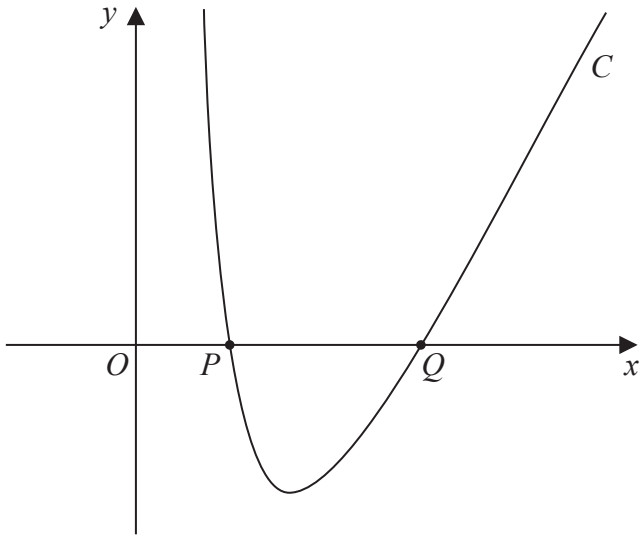


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{36}{x^2} + 2x - 13 \qquad x > 0$$

Using calculus,

(a) find the range of values of x for which $f(x)$ is increasing,

(4)

(b) show that $\int_2^9 \left(\frac{36}{x^2} + 2x - 13\right) dx = 0$

(4)

The point $P(2, 0)$ and the point $Q(6, 0)$ lie on C.

Given $\int_2^6 \left(\frac{36}{x^2} + 2x - 13\right) dx = -8$

(c) (i) state the value of $\int_6^9 \left(\frac{36}{x^2} + 2x - 13\right) dx$

(ii) find the value of the constant k such that $\int_2^6 \left(\frac{36}{x^2} + 2x + k\right) dx = 0$

(3)

(a) $f(x) = 36x^{-2} + 2x - 13 \Rightarrow f'(x) = -72x^{-3} + 2 = -\frac{72}{x^3} + 2$

$f(x)$ increasing when $f'(x) > 0$: $2 - \frac{72}{x^3} > 0$

$2x^3 > 72 \quad (x > 0) \Rightarrow x^3 > 36$

$x > \sqrt[3]{36}$

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Question 10 continued

(b) $\int \left(\frac{36}{x^2} + 2x - 13\right) dx = -\frac{36}{x} + x^2 - 13x \quad (+ c)$

$\int_2^9 \left(\frac{36}{x^2} + 2x - 13\right) dx = \left[-\frac{36}{x} + x^2 - 13x\right]_2^9$

$= \left(-\frac{36}{9} + 9^2 - 13 \times 9\right) - \left(-\frac{36}{2} + 2^2 - 13 \times 2\right)$

$= (-4 + 81 - 117) - (-18 + 4 - 26)$

$= (-40) - (-40) = 0$ (as required)

(c) (i) $\int_6^9 \left(\frac{36}{x^2} + 2x - 13\right) dx = 0 - (-8) = 8$

(ii) $\int_2^6 \left(\frac{36}{x^2} + 2x + k\right) dx = \left[-\frac{36}{x} + x^2 + kx\right]_2^6 = 0$

$(-6 + 36 + 6k) - (-18 + 4 + 2k) = 0$

$(30 + 6k) - (-14 + 2k) = 0$

$44 + 4k = 0 \Rightarrow k = -11$

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Question 10 continued

Q10

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