Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Wednesday 22 May 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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Answer all questions. Write your answers in the spaces provided.

1. The curve C has equation $y = \frac{1}{8}x^3 - \frac{24}{\sqrt{x}} + 1$

(a) Find $\frac{dy}{dx}$, giving the answer in its simplest form.

(3)

The point $P(4, -3)$ lies on C.

(b) Find the equation of the tangent to C at the point P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(a) $y = \frac{1}{8}x^3 - 24x^{-1/2} + 1$

$\frac{dy}{dx} = \frac{3}{8}x^2 - 24 \times \left(-\frac{1}{2}\right)x^{-3/2}$

$\frac{dy}{dx} = \frac{3}{8}x^2 + 12x^{-3/2}$

(b) At $x = 4$: $\frac{dy}{dx} = \frac{3}{8} \times 16 + 12 \times 4^{-3/2} = 6 + \frac{12}{8} = 7.5$

Tangent at $P(4, -3)$: $y - (-3) = 7.5(x - 4)$

$y + 3 = 7.5x - 30$

$y = 7.5x - 33$

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Question 1 continued

Q1

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2. Answer this question showing each stage of your working.

(a) Simplify $\dfrac{1}{4-2\sqrt{2}}$

giving your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers.

(2)

(b) Hence, or otherwise, solve the equation

$$4x = 2\sqrt{2}x + 20\sqrt{2}$$

giving your answer in the form $p + q\sqrt{2}$ where p and q are rational numbers.

(3)

(a) $\dfrac{1}{4-2\sqrt{2}} \times \dfrac{4+2\sqrt{2}}{4+2\sqrt{2}} = \dfrac{4+2\sqrt{2}}{16-8} = \dfrac{4+2\sqrt{2}}{8}$

$= \dfrac{1}{2} + \dfrac{1}{4}\sqrt{2}$

(b) $4x - 2\sqrt{2}x = 20\sqrt{2}$

$x(4-2\sqrt{2}) = 20\sqrt{2}$

$x = \dfrac{20\sqrt{2}}{4-2\sqrt{2}} = 20\sqrt{2} \times \left(\dfrac{1}{2} + \dfrac{1}{4}\sqrt{2}\right)$ using (a)

$x = 10\sqrt{2} + \dfrac{20 \times 2}{4} = 10\sqrt{2} + 10$

$x = 10 + 10\sqrt{2}$

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Question 2 continued

Q2

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3.

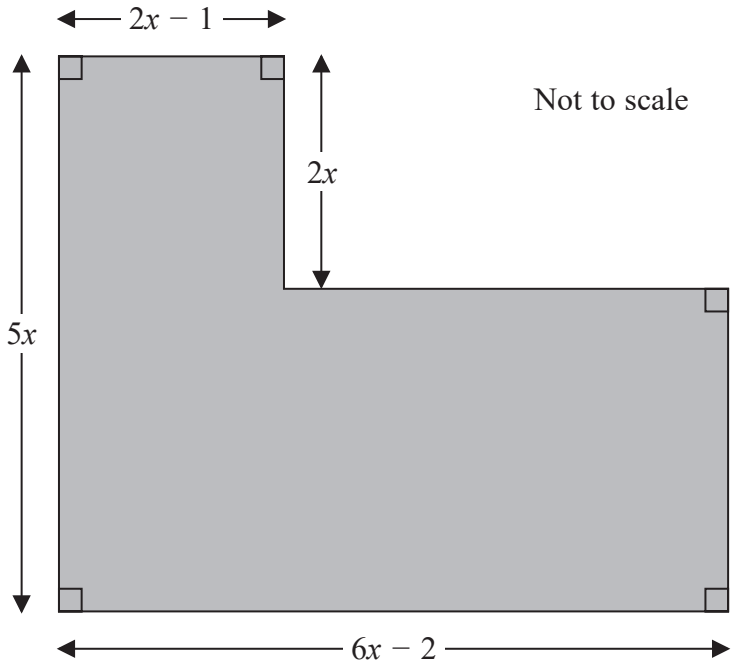


Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 29 m,

(a) show that $x > 1.5$ m

(3)

Given also that the area of the garden is less than 72 m^2,

(b) form and solve a quadratic inequality in x.

(5)

(c) Hence state the range of possible values of x.

(1)

(a) Perimeter = $5x + (6x - 2) + 2x + (4x - 1) + 3x + (2x - 1)$

$= 22x - 4$

$22x - 4 > 29 \Rightarrow 22x > 33$

$x > \frac{33}{22} \Rightarrow x > 1.5$ as required

(b) Area = $2x(2x - 1) + 3x(6x - 2)$

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Question 3 continued

$= 4x^2 - 2x + 18x^2 - 6x = 22x^2 - 8x$

$22x^2 - 8x < 72 \Rightarrow 22x^2 - 8x - 72 < 0$

$11x^2 - 4x - 36 < 0$

Critical values: $(11x + 18)(x - 2) = 0$

$x = -\frac{18}{11}$ or $x = 2$

Quadratic is negative between its roots (inside region):

$-\frac{18}{11} < x < 2$

(c) Combining $x > 1.5$ with $x < 2$: $1.5 < x < 2$

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Question 3 continued

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Question 3 continued

Q3

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4. Find

$$\int \frac{4x^2 + 1}{2\sqrt{x}} \mathrm{d}x$$

giving the answer in its simplest form.

(5)

$\frac{4x^2 + 1}{2\sqrt{x}} = \frac{4x^2}{2x^{1/2}} + \frac{1}{2x^{1/2}} = 2x^{3/2} + \frac{1}{2}x^{-1/2}$

$\int \left(2x^{3/2} + \frac{1}{2}x^{-1/2} \right) dx = \frac{2x^{5/2}}{5/2} + \frac{x^{1/2}}{2 \times 1/2} + c$

$= \frac{4}{5}x^{5/2} + x^{1/2} + c$

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Question 4 continued

Q4

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5. (a) Find, using algebra, all real solutions of

$$2x^3 + 3x^2 - 35x = 0$$

(3)

(b) Hence find all real solutions of

$$2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2 = 0$$

(4)

(a) $2x^3 + 3x^2 - 35x = 0$

$x(2x^2 + 3x - 35) = 0$

$x(2x - 7)(x + 5) = 0$

$x = 0, \quad x = \frac{7}{2}, \quad x = -5$

(b) Let $x = (y-5)^2$: $2x^3 + 3x^2 - 35x = 0$

$x = 0$: $(y-5)^2 = 0 \Rightarrow y = 5$

$x = \frac{7}{2}$: $(y-5)^2 = \frac{7}{2} \Rightarrow y - 5 = \pm\frac{\sqrt{7}}{\sqrt{2}} = \pm\frac{\sqrt{14}}{2}$

$x = -5$: $(y-5)^2 = -5$ has no real solutions

$y = 5, \quad y = 5 + \frac{\sqrt{14}}{2}, \quad y = 5 - \frac{\sqrt{14}}{2}$

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Question 5 continued

Q5

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6. The line with equation $y = 4x + c$, where c is a constant, meets the curve with equation $y = x(x - 3)$ at only one point.

(a) Find the value of c.

(4)

(b) Hence find the coordinates of the point of intersection.

(3)

(a) $4x + c = x(x - 3) \Rightarrow x^2 - 3x - 4x - c = 0$

$x^2 - 7x - c = 0$

One point of intersection: $b^2 - 4ac = 0$

$(-7)^2 - 4 \times 1 \times (-c) = 0 \Rightarrow 49 + 4c = 0$

$c = -\frac{49}{4} = -12.25$

(b) $x^2 - 7x + 12.25 = 0 \Rightarrow (x - 3.5)^2 = 0$

$x = \frac{7}{2}$

$y = 4 \times \frac{7}{2} - 12.25 = 14 - 12.25 = \frac{7}{4}$

Point of intersection $(\frac{7}{2}, \frac{7}{4})$

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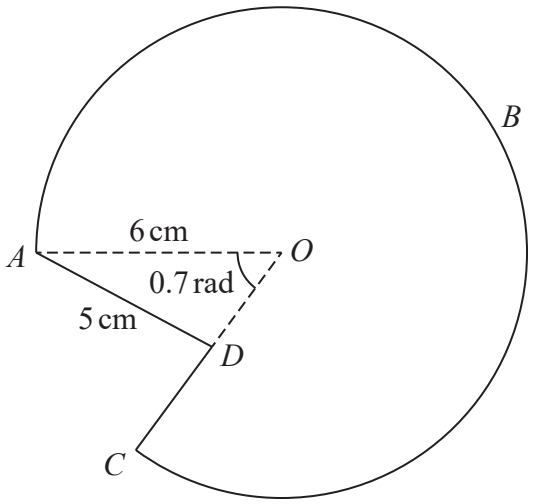
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7.



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Figure 2

The shape *ABCDA* consists of a sector *ABCOA* of a circle, centre *O*, joined to a triangle *AOD*, as shown in Figure 2.

The point *D* lies on *OC*.

The radius of the circle is 6 cm, length *AD* is 5 cm and angle *AOD* is 0.7 radians.

(a) Find the area of the sector *ABCOA*, giving your answer to one decimal place.

(3)

Given angle *ADO* is obtuse,

(b) find the size of angle *ADO*, giving your answer to 3 decimal places.

(3)

(c) Hence find the perimeter of shape *ABCDA*, giving your answer to one decimal place.

(4)

(a) Angle of sector $= 2\pi - 0.7 = 5.5831...$

Area $= \frac{1}{2} r^2\theta = \frac{1}{2} \times 6^2 \times (2\pi - 0.7)$

$= 100.4973... = 100.5 \text{ cm}^2$

(b) $\frac{\sin ADO}{6} = \frac{\sin 0.7}{5}$

$\sin ADO = \frac{6 \sin 0.7}{5} = 0.77306...$

ADO is obtuse: $ADO = \pi - 0.88365... = 2.2579...$

$ADO = 2.258$ rad

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Question 7 continued

(c) Arc ABC = $r\theta = 6(2\pi - 0.7) = 33.4991...$ cm

Angle OAD = $\pi - 0.7 - 2.2579... = 0.18365...$

$$\frac{OD}{\sin 0.18365...} = \frac{5}{\sin 0.7} \Rightarrow OD = \frac{5 \sin 0.18365...}{\sin 0.7} = 1.4173... \text{ cm}$$

DC = $6 - 1.4173... = 4.5826...$ cm

Perimeter = arc ABC + AD + DC

$= 33.4991... + 5 + 4.5826... = 43.0817...$

Perimeter = 43.1 cm

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Question 7 continued

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Question 7 continued

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8. The curve C with equation $y = f(x)$, $x > 0$, passes through the point $P(4, 1)$.

Given that $f'(x) = 4\sqrt{x} - 2 - \dfrac{8}{3x^2}$

(a) find the equation of the normal to C at P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

(b) Find $f(x)$.

(5)

(a) $f'(4) = 4\sqrt{4} - 2 - \dfrac{8}{3 \times 4^2} = 8 - 2 - \dfrac{1}{6} = \dfrac{35}{6}$

Gradient of normal $= -\dfrac{6}{35}$

$y - 1 = -\dfrac{6}{35}(x - 4)$

$35y - 35 = -6x + 24$

$6x + 35y - 59 = 0$

(b) $f'(x) = 4x^{1/2} - 2 - \dfrac{8}{3}x^{-2}$

$f(x) = \dfrac{4x^{3/2}}{3/2} - 2x - \dfrac{8}{3} \times \dfrac{x^{-1}}{-1} + c$

$f(x) = \dfrac{8}{3}x^{3/2} - 2x + \dfrac{8}{3x} + c$

$P(4, 1)$: $1 = \dfrac{8}{3} \times 8 - 8 + \dfrac{8}{12} + c$

$1 = \dfrac{64}{3} - 8 + \dfrac{2}{3} + c = 14 + c \Rightarrow c = -13$

$f(x) = \dfrac{8}{3}x^{3/2} - 2x + \dfrac{8}{3x} - 13$

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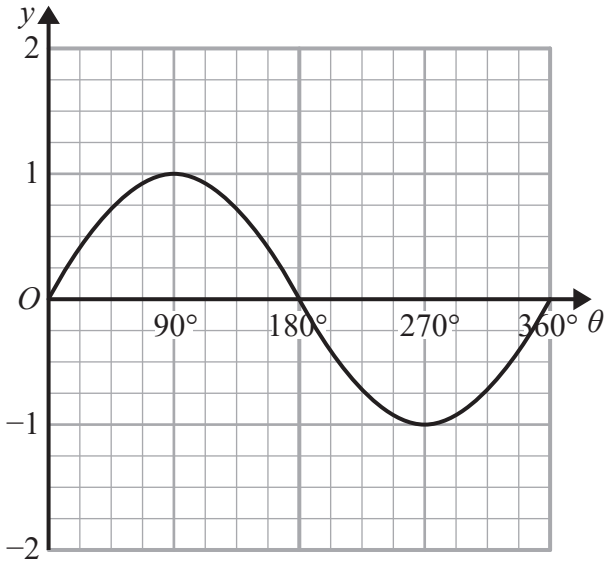


Figure 3

Figure 3 shows a plot of the curve with equation $y = \sin\theta, \quad 0 \leqslant \theta \leqslant 360°$

(a) State the coordinates of the minimum point on the curve with equation

$$y = 4\sin\theta, \quad 0 \leqslant \theta \leqslant 360°$$

(2)

A copy of Figure 3, called Diagram 1, is shown on the next page.

(b) On Diagram 1, sketch and label the curves

(i) $y = 1 + \sin\theta, \quad 0 \leqslant \theta \leqslant 360°$

(ii) $y = \tan\theta, \quad 0 \leqslant \theta \leqslant 360°$

(2)

(c) Hence find the number of solutions of the equation

(i) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leqslant \theta \leqslant 2160°$

(ii) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leqslant \theta \leqslant 1980°$

(3)

(a) (270°, −4)

(c) (i) From Diagram 1 the curves meet twice in $0 \leq \theta \leq 360°$

$\frac{2160}{360} = 6 \quad \Rightarrow \quad 6 \times 2 = 12$ solutions

(ii) $1980° = 5 \times 360° + 180°$: $5 \times 2 = 10$ in $0 \leq \theta \leq 1800°$

plus 1 in $1800°$ to $1980°$ (like $0°$ to $180°$): $10 + 1 = 11$

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Question 9 continued

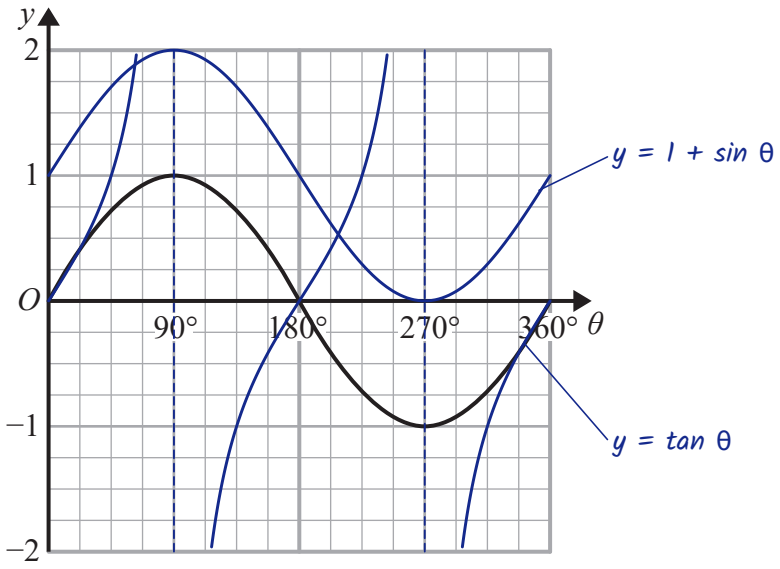


Diagram 1

(b) $y = 1 + \sin\theta$ and $y = \tan\theta$ sketched and labelled on Diagram 1

Intersections: one in $0° < \theta < 90°$, one in $180° < \theta < 270°$

Q9

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10. A curve has equation $y = f(x)$, where

$$f(x) = (x - 4)(2x + 1)^2$$

The curve touches the x-axis at the point P and crosses the x-axis at the point Q.

(a) State the coordinates of the point P.

(1)

(b) Find $f'(x)$.

(4)

(c) Hence show that the equation of the tangent to the curve at the point where $x = \frac{5}{2}$ can be expressed in the form $y = k$, where k is a constant to be found.

(3)

The curve with equation $y = f(x + a)$, where a is a constant, passes through the origin O.

(d) State the possible values of a.

(2)

(a) Touches x-axis where $(2x + 1)^2 = 0$: $P\left(-\frac{1}{2}, 0\right)$

(b) $f(x) = (x - 4)(4x^2 + 4x + 1)$

$= 4x^3 + 4x^2 + x - 16x^2 - 16x - 4$

$= 4x^3 - 12x^2 - 15x - 4$

$f'(x) = 12x^2 - 24x - 15$

(c) $f'(2.5) = 12 \times 2.5^2 - 24 \times 2.5 - 15 = 75 - 60 - 15 = 0$

so the tangent at $x = 2.5$ is horizontal

$f(2.5) = (2.5 - 4)(2 \times 2.5 + 1)^2 = (-1.5)(36) = -54$

Tangent: $y = -54$, so $k = -54$

(d) $f(x + a)$ passes through $0 \Rightarrow f(a) = 0$

$a = 4$ or $a = -\frac{1}{2}$

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Question 10 continued

Q10

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