Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Monday 3 June 2019

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1. Two particles, P and Q, have masses $3m$ and $2m$ respectively. They are moving in a straight line in opposite directions towards each other on a smooth horizontal plane and collide directly. The speeds of P and Q immediately before the collision are $2u$ and $4u$ respectively. As a result of the collision, the speed of P is halved and its direction of motion is reversed.

(a) Find the speed of Q immediately after the collision.

(3)

(b) Find the magnitude of the impulse exerted on Q by P in the collision.

(2)

(a) Take the original direction of motion of P as positive.

After the collision: velocity of P = $-u$, velocity of Q = v

CLM: $3m(2u) + 2m(-4u) = 3m(-u) + 2mv$

$6mu - 8mu = -3mu + 2mv$

$2mv = mu$

$v = \frac{1}{2}u$

Speed of Q = $\frac{1}{2}u$

(b) Impulse on Q = change in momentum of Q

$I = 2m(\frac{1}{2}u - (-4u))$

$I = 2m \times 4.5u$

Magnitude of impulse = $9mu$

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Question 1 continued

Q1

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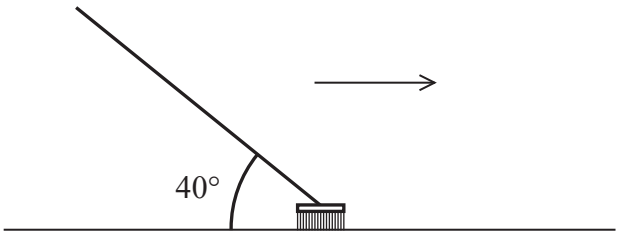
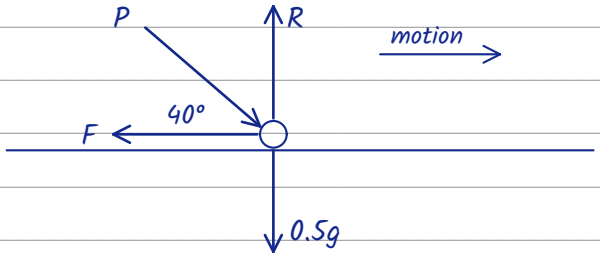


Figure 1

A broom is being used to sweep a rough horizontal floor. The handle of the broom makes a constant angle of 40° with the horizontal, as shown in Figure 1. The broom head is modelled as a particle of mass 0.5 kg and the handle of the broom is modelled as a light rod. The coefficient of friction between the broom head and the floor is $\frac{1}{4}$. The broom head is pushed along the floor in a straight line at constant speed. Find the magnitude of the force that is being applied along the handle of the broom to the broom head.

(6)



Friction is limiting (moving): $F = \frac{1}{4} R$

Resolve horizontally (constant speed, a = 0): $P \cos 40° = F$

Resolve vertically: $R = P \sin 40° + 0.5g$

$P \cos 40° = \frac{1}{4} (P \sin 40° + 0.5g)$

$4P \cos 40° = P \sin 40° + 0.5g$

$P(4 \cos 40° - \sin 40°) = 0.5g$

$P = \frac{4.9}{4 \cos 40° - \sin 40°} = \frac{4.9}{2.4213...}$

$P = 2.0236...$

Force along the handle = 2.02 N

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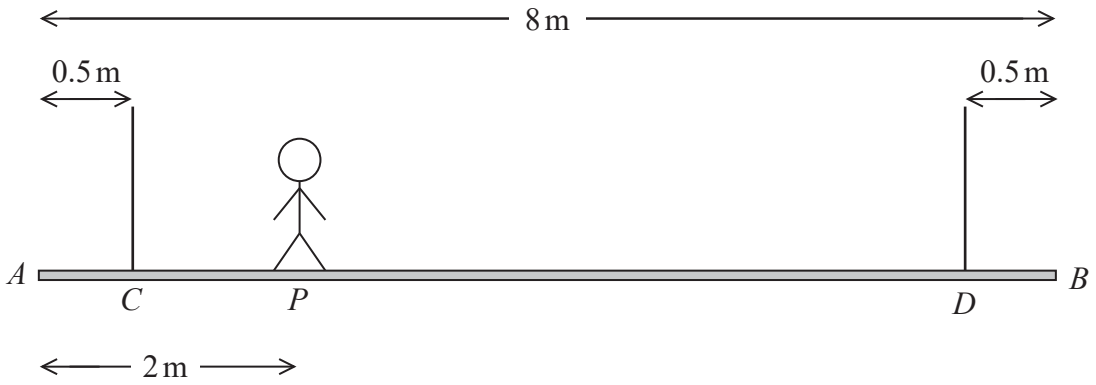


Figure 2

A beam AB has mass 100 kg and length 8 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam at C and D, where $AC = 0.5$ m and $BD = 0.5$ m. A gymnast of mass M kg stands on the beam at the point P, where $AP = 2$ m, as shown in Figure 2. The beam remains horizontal and in equilibrium. The tension in the rope attached to the beam at D is 637 N. The gymnast is modelled as a particle, the beam as a uniform rod and the ropes as light inextensible strings.

(a) Find

(i) the value of M,

(ii) the tension in the rope attached to the beam at C.

(6)

(b) State how you have used the fact that the beam is modelled as a rod.

(1)

The gymnast at P now gets off the beam and is replaced by two gymnasts. One gymnast, of mass 60 kg, stands on the beam at P and the other gymnast, of mass 48 kg, stands on the beam at X, where $AX = x$ metres. The beam remains horizontal and in equilibrium but the tensions in the two ropes are now equal. The two gymnasts are modelled as particles.

(c) Find the value of x.

(6)

(a) (i) M(C): $Mg \times 1.5 + 100g \times 3.5 = 637 \times 7$

$1.5Mg + 3430 = 4459$

$1.5Mg = 1029 \Rightarrow Mg = 686$

$M = 70$

(ii) Resolve vertically (↑): $T + 637 = Mg + 100g$

$T = 686 + 980 - 637 = 1029$

Tension at C = 1030 N

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Question 3 continued

(b) The beam is assumed to remain straight (it does not bend).

(c) Let the tension in each rope be T.

Resolve vertically: $T + T = 60g + 100g + 48g$

$2T = 208g \Rightarrow T = 104g \quad (= 1019.2\ N)$

M(C): $60g \times 1.5 + 100g \times 3.5 + 48g \times (x - 0.5) = T \times 7$

$90g + 350g + 48gx - 24g = 728g$

$48x = 728 - 416 = 312$

$x = \frac{312}{48}$

$x = 6.5$

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Question 3 continued

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Question 3 continued

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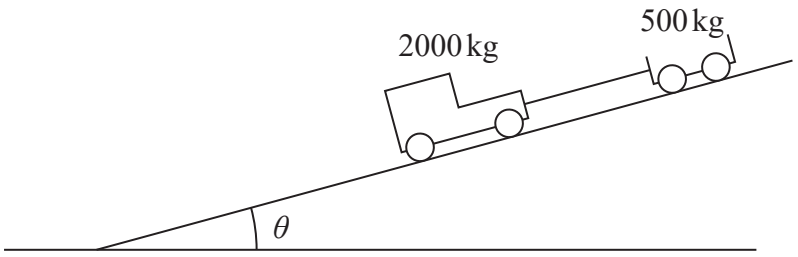


Figure 3

A railway truck of mass 500 kg is pushed up a straight track by a railway engine of mass 2000 kg. The track is inclined to the horizontal at an angle θ, where $\sin\theta = \frac{1}{14}$, as shown in Figure 3. The engine produces a constant driving force of magnitude 3050 N. The truck experiences a constant resistance to motion of magnitude 100 N and the engine experiences a constant resistance to motion of magnitude 200 N. The engine and the truck are connected by a coupling which is modelled as a light rod that is parallel to the track.

Find

(i) the acceleration of the system,

(ii) the magnitude of the force exerted on the truck by the coupling.

(8)

$\sin\theta = \frac{1}{14}$, let a be the acceleration and T the force in the coupling.

(i) N2L for the whole system (2500 kg) up the slope:

$3050 - 300 - 2500g\sin\theta = 2500a$

$3050 - 300 - 2500 \times 9.8 \times \frac{1}{14} = 2500a$

$3050 - 300 - 1750 = 2500a$

$2500a = 1000$

Acceleration $= 0.4\ m\ s^{-2}$

(ii) N2L for the truck (500 kg) up the slope:

$T - 100 - 500g\sin\theta = 500a$

$T - 100 - 500 \times 9.8 \times \frac{1}{14} = 500 \times 0.4$

$T - 100 - 350 = 200$

Force exerted on the truck by the coupling = 650 N

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Question 4 continued

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5. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle. The resultant of the two forces is $\mathbf{R}$.

Given that $\mathbf{R}$ has the same direction as the vector $(3\mathbf{i} - 2\mathbf{j})$,

(a) find the size of the angle between $\mathbf{R}$ and the vector $\mathbf{j}$.

(2)

Given that $\mathbf{F}_1 = (a\mathbf{i} + 3\mathbf{j})$N and $\mathbf{F}_2 = (-4\mathbf{i} + b\mathbf{j})$N,

(b) show that $2a + 3b + 1 = 0$

(4)

(a) R is in the direction of $3i - 2j$.

Angle θ between R and i (below i): $\tan\theta = \frac{2}{3} \Rightarrow \theta = 33.69...°$

Angle between R and j $= \theta + 90° = 123.69...°$

Angle $= 124°$ (3 s.f.)

(b) $R = F_1 + F_2 = (ai + 3j) + (-4i + bj)$

$R = (a - 4)i + (3 + b)j$

R is parallel to $3i - 2j$, so

$\frac{a-4}{3+b} = \frac{3}{-2}$

$-2(a - 4) = 3(3 + b)$

$-2a + 8 = 9 + 3b$

$2a + 3b + 1 = 0$ as required

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Question 5 continued

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6. A small ball is projected vertically upwards with speed $U\,\text{m s}^{-1}$ from a point A that is 12.5 m above horizontal ground. The ball moves freely under gravity until it hits the ground $\frac{25}{7}$ s later. By modelling the ball as a particle,

(a) find the value of U.

(3)

After hitting the ground the ball rebounds vertically and comes to instantaneous rest at the point B, $\frac{5}{7}$ s after hitting the ground.

(b) Find the height of B above the ground.

(3)

(c) Sketch a velocity-time graph for the motion of the ball from the instant when it was first projected from A to the instant when it comes to instantaneous rest at B.

[*No further calculations are needed in order to draw this sketch.*]

(3)

(a) Take upwards as positive: $s = -12.5$, $a = -9.8$, $t = 25/7$

$s = ut + \frac{1}{2}at^2$

$-12.5 = \frac{25}{7}U - 4.9 \times \left(\frac{25}{7}\right)^2$

$-12.5 = \frac{25U}{7} - 62.5$

$\frac{25U}{7} = 50$

$U = 14$

(b) After rebound: $v = 0$, $a = -9.8$, $t = 5/7$ (taking upwards positive)

$s = vt - \frac{1}{2}at^2$

$s = 0 - \frac{1}{2}(-9.8)\left(\frac{5}{7}\right)^2$

$s = 4.9 \times \frac{25}{49} = 2.5$

Height of B above the ground = 2.5 m

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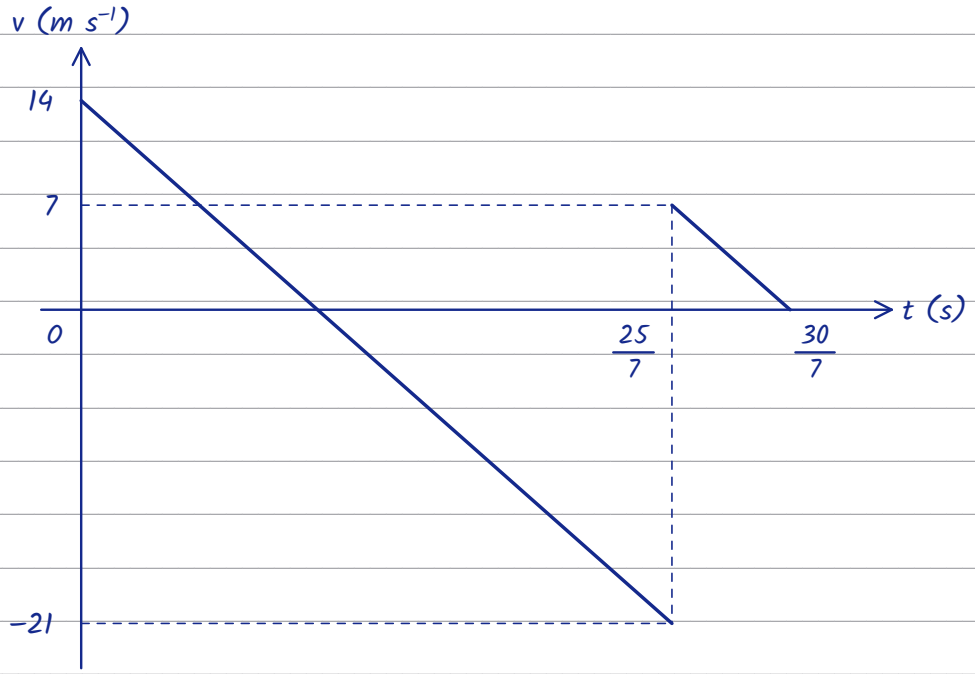
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Question 6 continued

(c) (Upwards positive. Velocity on hitting the ground = $14 - 9.8 \times 25/7 = -21$)



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Question 6 continued

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Question 6 continued

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7. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

A ship A is moving with constant velocity $(2\mathbf{i} - 14\mathbf{j})\,\text{km h}^{-1}$. At 2 pm the position vector of ship A is $(8\mathbf{i} + 7\mathbf{j})$ km.

A ship B is moving with constant velocity $(12\mathbf{i} - 4\mathbf{j})\,\text{km h}^{-1}$. At 2 pm the position vector of ship B is $(\mathbf{i} + 2\mathbf{j})$ km.

(a) Show that at time t hours after 2 pm,

$$\overrightarrow{BA} = [(7 - 10t)\mathbf{i} + (5 - 10t)\mathbf{j}]\,\text{km}$$

(5)

(b) Hence find the length of time for which the ships are within 2 km of each other.

(7)

(a) Using $r = r_0 + vt$:

$r_A = (8i + 7j) + t(2i - 14j) = (8 + 2t)i + (7 - 14t)j$

$r_B = (i + 2j) + t(12i - 4j) = (1 + 12t)i + (2 - 4t)j$

$BA = r_A - r_B$

$BA = (8i + 7j) + t(2i - 14j) - [(i + 2j) + t(12i - 4j)]$

$BA = (8 + 2t - 1 - 12t)i + (7 - 14t - 2 + 4t)j$

$BA = [(7 - 10t)i + (5 - 10t)j]$ km as required

(b) Ships are 2 km apart when $|BA| = 2$:

$(7 - 10t)^2 + (5 - 10t)^2 = 2^2$

$49 - 140t + 100t^2 + 25 - 100t + 100t^2 = 4$

$200t^2 - 240t + 70 = 0$

$20t^2 - 24t + 7 = 0$

$(10t - 7)(2t - 1) = 0$

$t = \frac{1}{2}$ or $t = \frac{7}{10}$

$|BA| < 2$ between these times, so

Time $= \frac{7}{10} - \frac{1}{2} = \frac{1}{5}$ h

Length of time = 0.2 hours = 12 minutes

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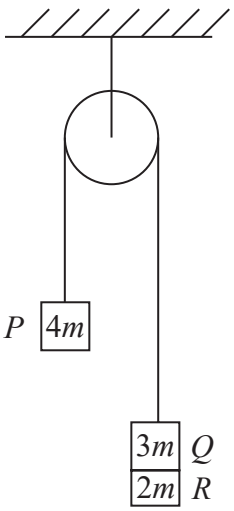


Figure 4

Three particles, P, Q and R, have masses $4m$, $3m$ and $2m$ respectively. Particles P and Q are connected by a light inextensible string that passes over a smooth light fixed pulley. Particle R is attached to particle Q. The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 4. The system is released from rest.

(a) Find

(i) the acceleration of particle P,

(ii) the tension in the string.

(7)

(b) State how you have used the fact that the string is inextensible.

(1)

At the instant when particle P has moved a distance d upwards from its initial position, particle R separates from particle Q and falls away. In the subsequent motion, particles P and Q continue to move and particle P does not reach the pulley.

At the instant when particles R and Q separate, particle Q is at the point A, and it continues to move downwards. Particle Q then comes to instantaneous rest at the point B.

(c) Find, in terms of d, the distance AB.

(8)

(a) Let a be the acceleration and T the tension. Q and R together: 5m.

N2L for P (upwards): $T - 4mg = 4ma$

N2L for Q and R (downwards): $5mg - T = 5ma$

Adding: $mg = 9ma$

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Question 8 continued

(i) $a = \frac{g}{9} = 1.0888...$

Acceleration of P = 1.09 m s^{-2}

(ii) $T = 4mg + 4m \times \frac{g}{9} = \frac{40mg}{9} = 43.555...m$

Tension = $\frac{40mg}{9}$ N (= 43.6m N)

(b) The accelerations of P and of Q (with R) have the same magnitude.

(c) Speed when P has moved d: $v^2 = u^2 + 2as = 0 + 2 \times \frac{g}{9} \times d$

$v^2 = \frac{2gd}{9}$

After R falls away, let the new acceleration of Q downwards be a′ and the new tension T′.

N2L for Q (downwards): $3mg - T' = 3ma'$

N2L for P (upwards): $T' - 4mg = 4ma'$

Adding: $-mg = 7ma'$

$a' = -\frac{g}{7}$ (Q decelerates)

Q comes to rest at B: $v^2 = u^2 + 2as$

$0 = \frac{2gd}{9} - 2 \times \frac{g}{7} \times s$

$\frac{2g}{7} s = \frac{2gd}{9}$

$AB = s = \frac{7d}{9}$

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Question 8 continued

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Q8

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TOTAL FOR PAPER: 75 MARKS

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