Pearson Edexcel International Advanced Level

Thursday 20 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Write your answers in the D1 answer book for this paper.

1.

	A	B	C	D	E	F
A	–	73	56	27	38	48
B	73	–	58	59	43	34
C	56	58	–	46	38	42
D	27	59	46	–	25	32
E	38	43	38	25	–	21
F	48	34	42	32	21	–

The table above shows the least distances, in km, between six cities, A, B, C, D, E and F. Mohsen needs to visit each city, starting and finishing at A, and wishes to minimise the total distance he will travel.

(a) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the length of Mohsen's route. You must state your route and its length.

(3)

(b) Starting by deleting A and all of its arcs, find a lower bound for the length of Mohsen's route.

(3)

(c) Use your answers from (a) and (b) to write down the smallest interval that you can be confident contains the optimal length of the route.

(2)

(Total 8 marks)

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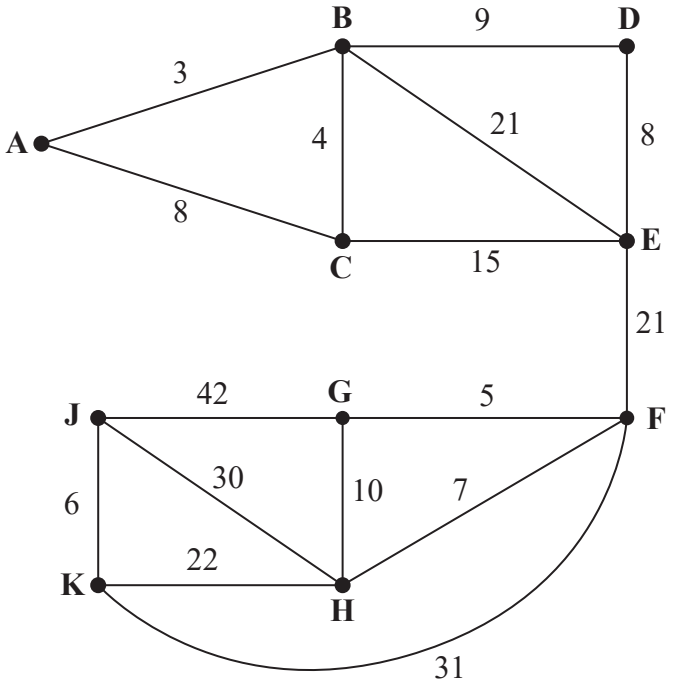


Figure 1

Figure 1 represents a network of roads between ten villages, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in kilometres, of the corresponding road. The local council needs to find the shortest route from A to J.

(a) Use Dijkstra's algorithm to find the shortest route from A to J. State the route and its length.

(6)

During the winter, the council needs to ensure that all ten villages are accessible by road even if there is heavy snow. The council wishes to minimise the total length of road it needs to keep clear.

(b) Use Prim's algorithm, starting at A, to find a minimum connector for the five villages A, B, C, D and E. You must clearly state the order in which you select the edges of your minimum connector.

(2)

(c) Use Kruskal's algorithm to find a minimum connector for the five villages F, G, H, J and K. You must clearly show the order in which you consider the edges. For each edge, state whether or not you are including it in your minimum connector.

(2)

(d) Calculate the total length of road that the council must keep clear of snow to ensure that all ten villages are accessible.

(1)

(Total 11 marks)

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3. Pupils from ten schools are visiting a museum on the same day. The museum needs to allocate each school to a tour group. The maximum size of each tour group is 42 pupils. A group may include pupils from more than one school. Pupils from each school must be kept in the same tour group. The numbers of pupils visiting from each school are given below.

8 17 9 14 18 12 22 10 15 7

(a) Calculate a lower bound for the number of tour groups required. You must make your method clear.

(2)

(b) Using the above list, apply the first-fit bin packing algorithm to allocate the pupils visiting from each school to tour groups.

(2)

The above list of numbers is to be sorted into descending order.

(c) Perform a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Using your sorted list from (c), apply the first-fit decreasing bin packing algorithm to obtain a second allocation of pupils to tour groups.

(2)

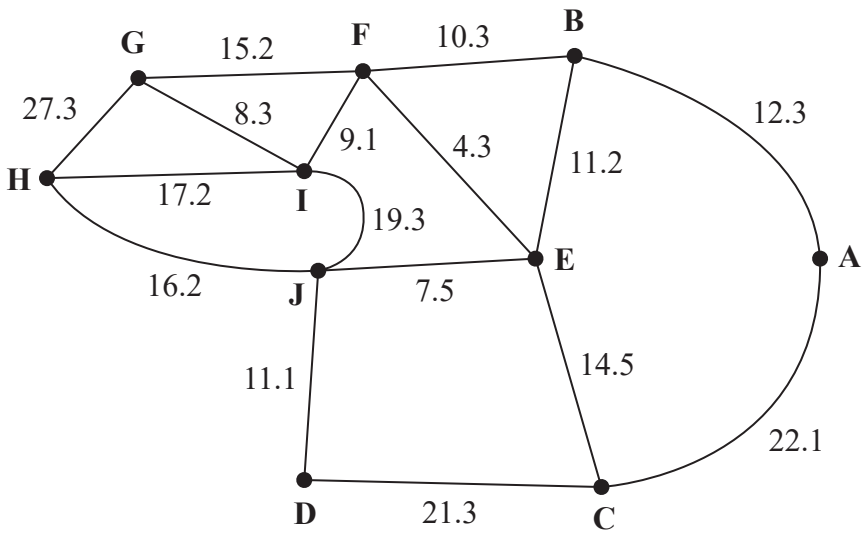


Figure 2

[*The total weight of the network is 227.2*]

Figure 2 represents the corridors in the museum. The number on each arc is the length, in metres, of the corresponding corridor. Sally is a tour guide in the museum and she must travel along each corridor at least once during each tour. Sally wishes to minimise the length of her route. She must start and finish at the museum's entrance at A.

(e) Use an appropriate algorithm to find the corridors that Sally will need to traverse twice. You should make your method and working clear.

(4)

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(f) Write down a possible shortest route, giving its length.

(2)

Sally is now allowed to start at H and finish her route at a different vertex. A route of minimum length that includes each corridor at least once needs to be found.

(g) State the finishing vertex of Sally's new route and calculate the difference in length between this new route and the route found in (f).

(2)

(Total 18 marks)

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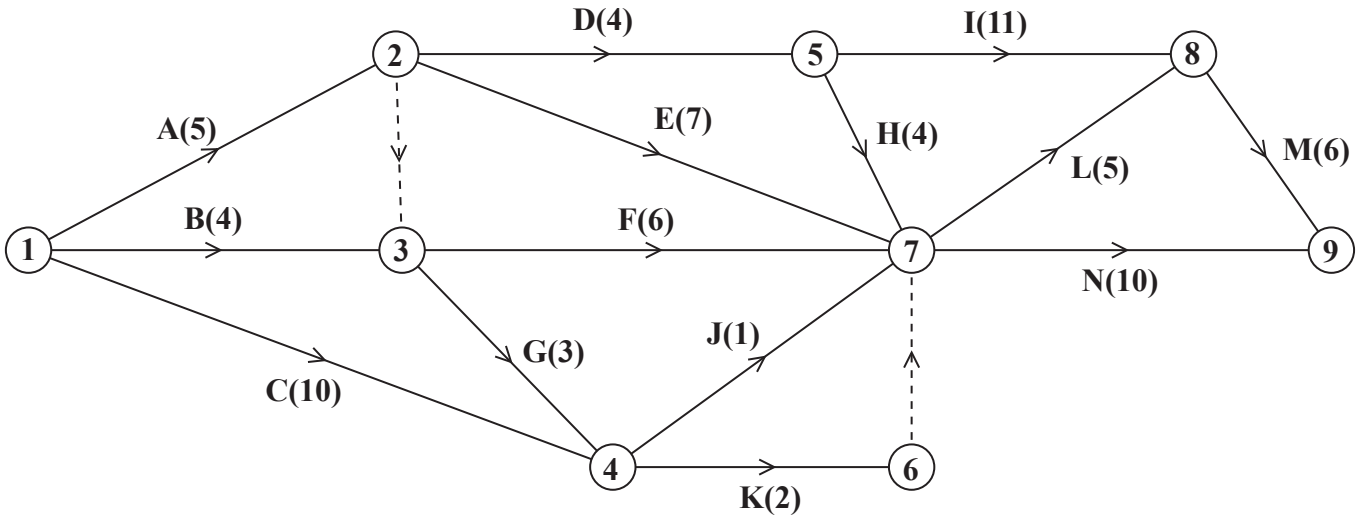


Figure 3

A project is modelled by the activity network shown in Figure 3. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the corresponding activity. The numbers in circles are the event numbers.

(a) Explain the significance of the dummy activity

(i) from event 2 to event 3

(ii) from event 6 to event 7

(2)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(c) State the minimum project completion time and list the critical activities.

(2)

The duration of activity H changes to x hours.

(d) Find, in terms of x where necessary,

(i) the possible new early event time for event 7

(ii) the possible new late event time for event 7

(3)

Given that the duration of activity H is such that the minimum project completion time is four hours greater than the time found in (c),

(e) determine the value of x.

(1)

(Total 12 marks)

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5. A clothing shop sells a particular brand of shirt, which comes in three different sizes, small, medium and large.

Each month the manager of the shop orders x small shirts, y medium shirts and z large shirts.

The manager forms constraints on the number of each size of shirts he will have to order.

One constraint is that for every 3 medium shirts he will order at least 5 large shirts.

(a) Write down an inequality, with integer coefficients, to model this constraint.

(2)

Two further constraints are

$$x + y + z \geqslant 250 \text{ and } x \leqslant 0.2(x + y + z)$$

(b) Use these two constraints to write down statements, in context, that describe the number of different sizes of shirt the manager will order.

(3)

The cost of each small shirt is £6, the cost of each medium shirt is £10 and the cost of each large shirt is £15

The manager must minimise the total cost of all the shirts he will order.

(c) Write down the objective function.

(1)

Initially, the manager decides to order exactly 150 large shirts.

(d) (i) Rewrite the constraints, as simplified inequalities with integer coefficients, in terms of x and y only.

(ii) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region R.

(6)

(e) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label V.

(2)

(f) Write down the number of each size of shirt the manager should order. Calculate the total cost of this order.

(2)

Later, the manager decides to order exactly 50 small shirts and exactly 75 medium shirts instead of 150 large shirts.

(g) Find the minimum number of large shirts the manager should order and show that this leads to a lower cost than the cost found in (f).

(2)

(Total 18 marks)

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6.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	–
E	A
F	A, B, C
G	C
H	C
I	D, H
J	E
K	E
L	F, G, I
M	G, I

(a) Draw the activity network described in the precedence table, using activity on arc and exactly four dummies.

(5)

Given that there is a unique critical path for the network and that K is a critical activity,

(b) state the critical path for the network.

(1)

Given instead that all the activities shown in the precedence table have the same duration and K is not necessarily critical,

(c) state all the possible critical paths for the network.

(2)

(Total 8 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Thursday 20 June 2019

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Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

Answer Book
Do not return the question paper with the answer book.

Total Marks

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1.

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F	48	34	42	32	21	–

(a) Nearest neighbour starting at A:

A → D (27), D → E (25), E → F (21),

F → B (34), B → C (58), C → A (56)

Route: A – D – E – F – B – C – A

Length = 27 + 25 + 21 + 34 + 58 + 56 = 221 km

Upper bound = 221 km

(b) Delete A. Residual minimum spanning tree on B, C, D, E, F

(Kruskal): EF 21 accept, DE 25 accept, DF 32 reject,

BF 34 accept, CE 38 accept

RMST weight = 21 + 25 + 34 + 38 = 118 km

Two least arcs from A: AD = 27, AE = 38

Lower bound = 118 + 27 + 38 = 183 km

(c) 183 km ≤ optimal route length ≤ 221 km

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Question 1 continued

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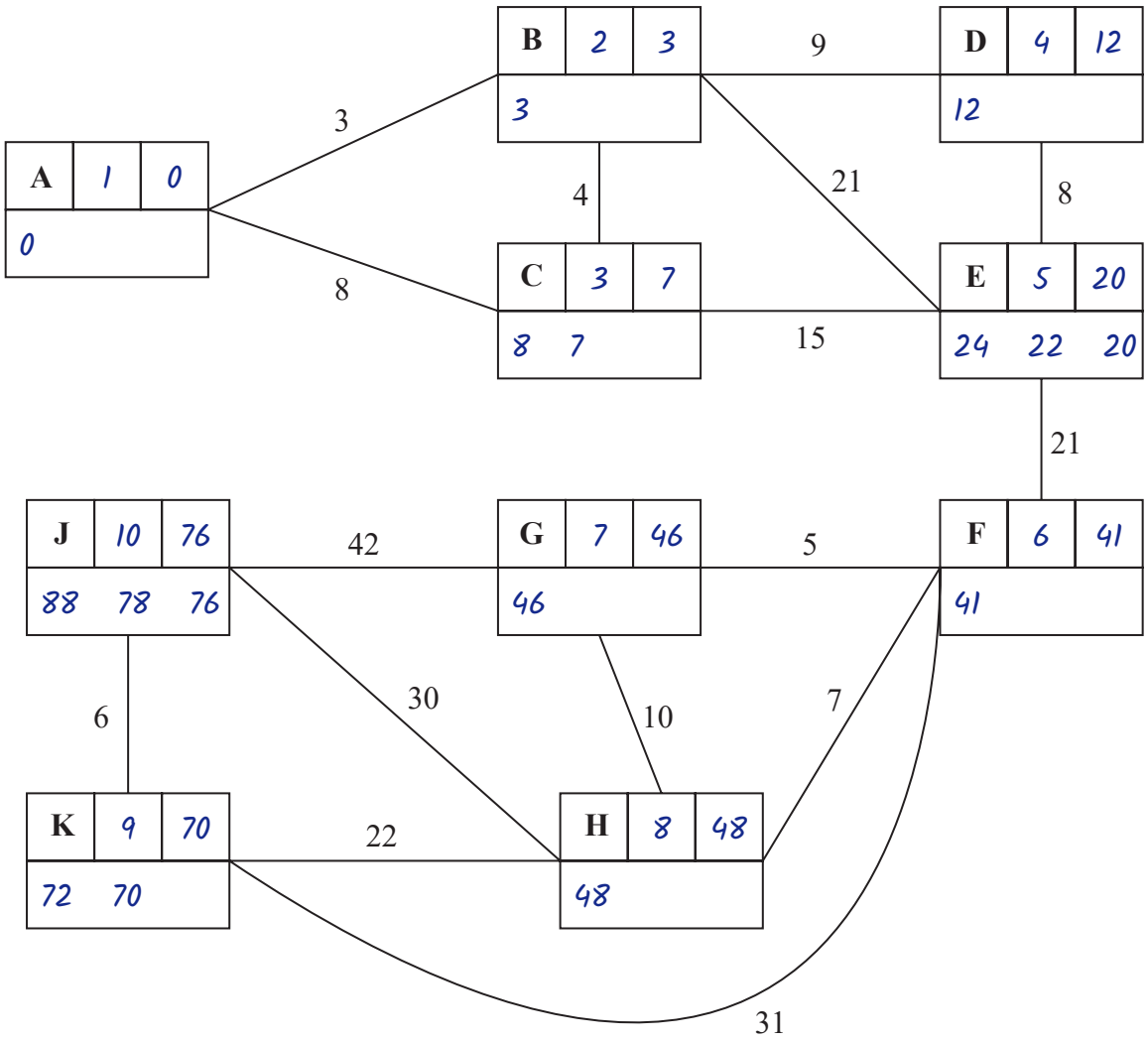
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Key:

Vertex	Order of labelling	Final value
Working values		

Shortest route: A – B – D – E – F – H – K – J

Length of shortest route: 76 km

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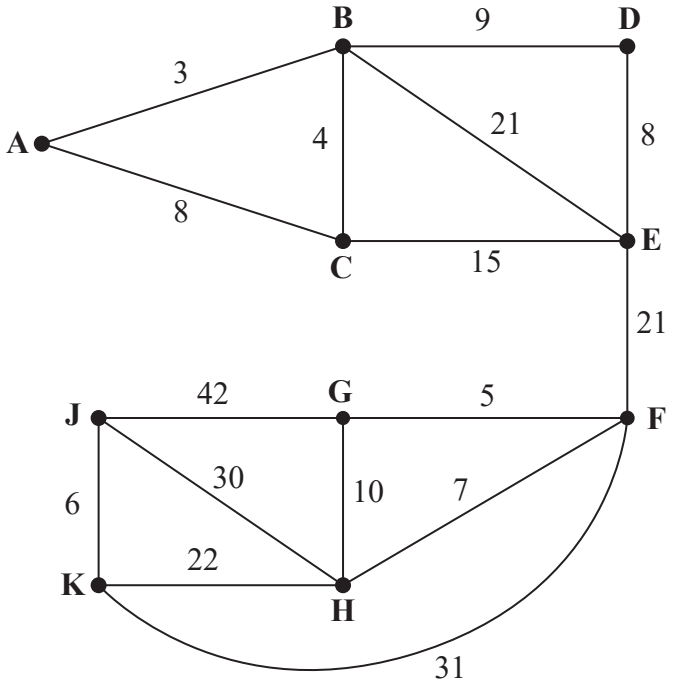


Figure 1

(b) Prim from A on A, B, C, D, E:

Order of selecting arcs: AB (3), BC (4), BD (9), DE (8)

(weight 24 km)

(c) Kruskal on F, G, H, J, K (arcs in order of weight):

FG 5 include, JK 6 include, FH 7 include,

GH 10 reject, HK 22 include

(all five villages now connected: HJ 30, FK 31 and GJ 42 rejected)

Minimum connector: FG, JK, FH, HK (weight 40 km)

(d) Total = 24 + 40 + 21 (EF joins the two parts)

= 85 km

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Question 2 continued

Q2

(Total 11 marks)

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3.

8 17 9 14 18 12 22 10 15 7

(a) Total = 8 + 17 + 9 + 14 + 18 + 12 + 22 + 10 + 15 + 7 = 132

132 ÷ 42 = 3.142... so lower bound = 4 tour groups

(b) First-fit:

Group 1: 8 17 9 7

Group 2: 14 18 10

Group 3: 12 22

Group 4: 15

(c) Quick sort into descending order (pivot = middle item, right of centre when even; pivots underlined, fixed ones boxed):

8 17 9 14 18 <u>12</u> 22 10 15 7 pivot 12

17 14 <u>18</u> 22 15 [12] 8 9 <u>10</u> 7 pivots 18, 10

22 [18] 17 <u>14</u> 15 [12] [10] 8 <u>9</u> 7 pivots 14, 9

[22] [18] 17 <u>15</u> [14] [12] [10] [9] 8 <u>7</u> pivots 15, 7

[22] [18] [17] [15] [14] [12] [10] [9] [8] [7] sort complete

Sorted list: 22 18 17 15 14 12 10 9 8 7

(d) First-fit decreasing:

Group 1: 22 18

Group 2: 17 15 10

Group 3: 14 12 9 7

Group 4: 8

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Question 3 continued

8 17 9 14 18 12 22 10 15 7

(e) Valencies: B 3, C 3, G 3, H 3 (all others even)

Odd vertices: B, C, G, H. Possible pairings:

BC + GH = (BE + EC) + (GI + IH)

= (11.2 + 14.5) + (8.3 + 17.2) = 25.7 + 25.5 = 51.2

BG + CH = (BF + FG) + (CE + EJ + JH)

= (10.3 + 15.2) + (14.5 + 7.5 + 16.2) = 25.5 + 38.2 = 63.7

BH + CG = (BE + EJ + JH) + (CE + EF + FG)

= (11.2 + 7.5 + 16.2) + (14.5 + 4.3 + 15.2) = 34.9 + 34.0 = 68.9

Least total is 51.2, so the corridors traversed twice are

BE, CE, GI and HI

(f) Route: A B E B F E C E J I F G I G H I H J D C A

Length = 227.2 + 51.2 = 278.4 m

(g) Start at H, so H is an end; the other three odd vertices

B, C, G: one is the finish and the other two are paired:

finish B: CG = 34.0, finish C: BG = 25.5, finish G: BC = 25.7

Least is 25.5, so the finishing vertex is C

Difference = 51.2 − (10.3 + 15.2) = 51.2 − 25.5 = 25.7 m

(new route is 25.7 m shorter)

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Question 3 continued

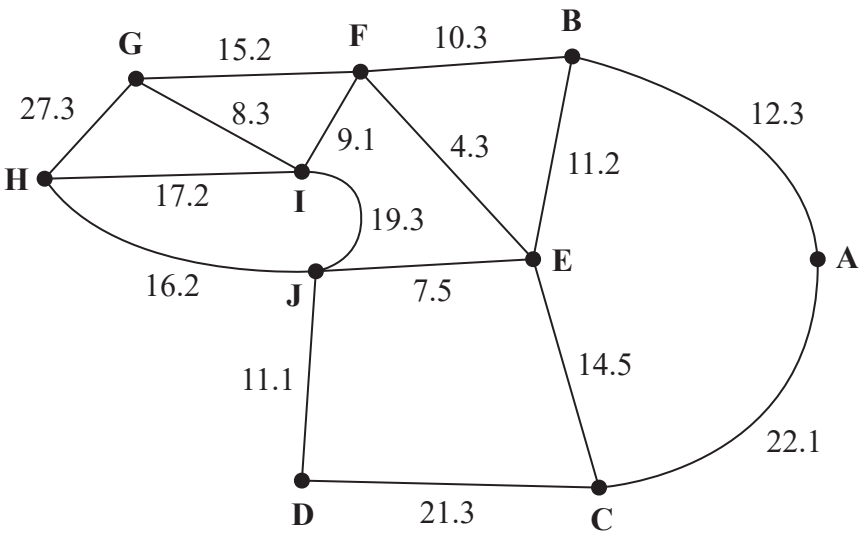


Figure 2

[*The total weight of the network is 227.2*]

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Question 3 continued

Q3

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4.

(a)(i) The dummy from event 2 to event 3 is needed because activities F and G depend on both A and B, but activities D and E depend on A only.

(ii) The dummy from event 6 to event 7 is needed because activities J and K both start at event 4 and would otherwise both end at event 7: each activity must be uniquely described by its start and end events.

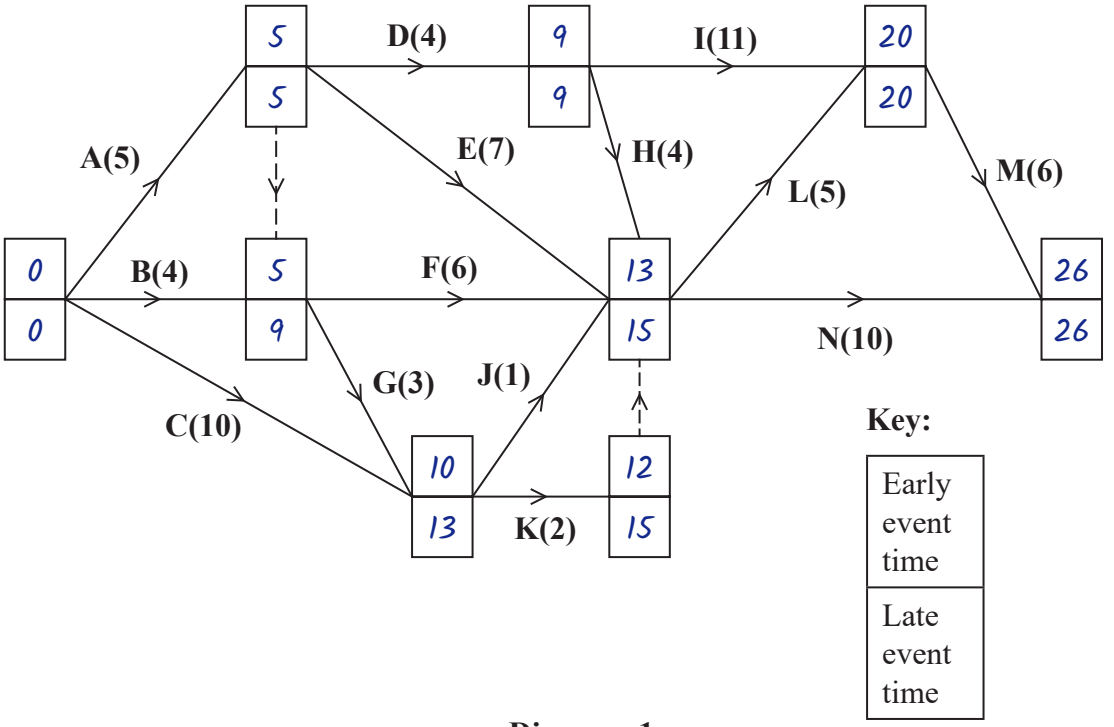


Diagram 1

(c) Minimum completion time = 26 hours

Critical activities: A, D, I and M

(d)(i) Early event time for event 7

= 12 or 9 + x (whichever is larger: 5 + 7 = 12 or 5 + 4 + x)

(ii) Late event time for event 7

= 15 or 9 + x (whichever is larger)

(e) Longest path through H: A – D – H – L – M

5 + 4 + x + 5 + 6 = 20 + x = 26 + 4 = 30

x = 10

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Question 4 continued

Q4

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5.

(a) For every 3 medium shirts, at least 5 large: $z/y \geq 5/3$

$5y \leq 3z$

(b) $x + y + z \geq 250$: the total number of shirts ordered must be at least 250.

$x \leq 0.2(x + y + z)$: at most 20% of all the shirts ordered should be small.

(c) Minimise $C = 6x + 10y + 15z$

(d)(i) $z = 150$:

$x + y + 150 \geq 250 \Rightarrow x + y \geq 100$

$5y \leq 3(150) = 450 \Rightarrow y \leq 90$

$x \leq 0.2(x + y + 150) \Rightarrow 5x \leq x + y + 150$

$\Rightarrow 4x - y \leq 150$

(and $x \geq 0$, $y \geq 0$)

(ii) See Diagram 1 (unwanted regions shaded, R unshaded).

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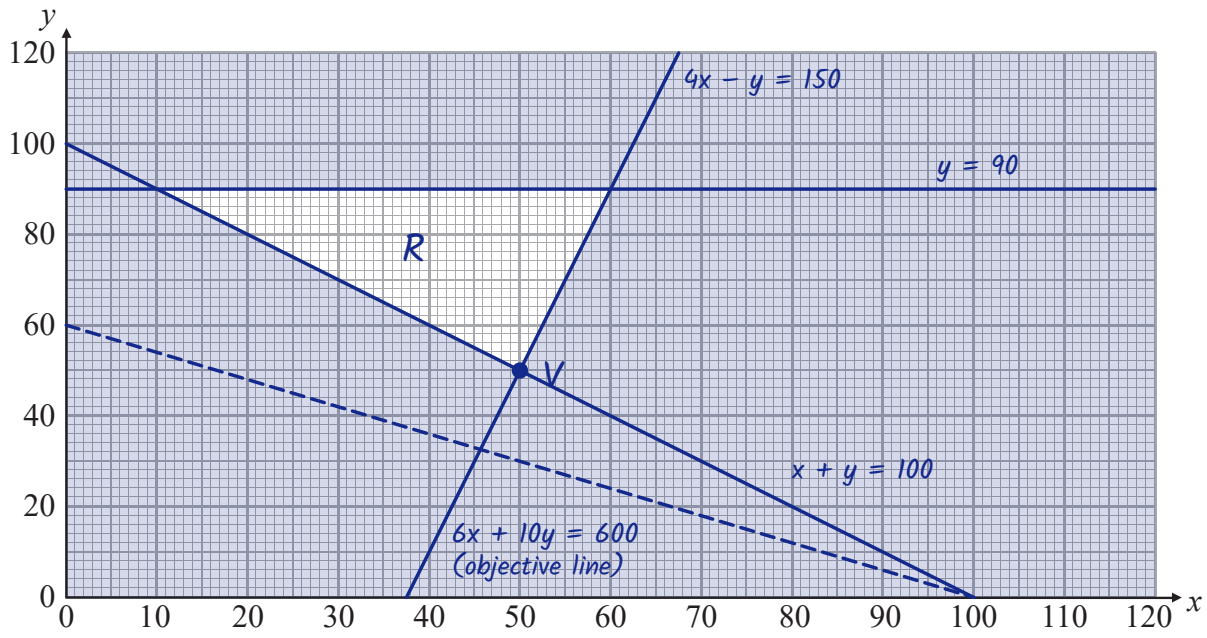


Diagram 1

(e) Objective line $6x + 10y = 600$ (gradient -0.6) drawn dashed.

Moving it towards the origin, the last point of R it touches is V,

where $x + y = 100$ and $4x - y = 150 \Rightarrow 5x = 250$

$x = 50$, $y = 50$, $V = (50, 50)$

(f) Order 50 small, 50 medium and 150 large shirts.

Cost $= 6(50) + 10(50) + 15(150) = 300 + 500 + 2250$

$= £3050$

(g) $x = 50$, $y = 75$:

$5y \leqslant 3z \Rightarrow 375 \leqslant 3z \Rightarrow z \geqslant 125$

$x + y + z \geqslant 250 \Rightarrow z \geqslant 125$

$50 \leqslant 0.2(125 + z) \Rightarrow 250 \leqslant 125 + z \Rightarrow z \geqslant 125$

Minimum number of large shirts = 125

Cost $= 6(50) + 10(75) + 15(125) = 300 + 750 + 1875$

$= £2925$

£2925 < £3050, so this leads to a lower cost.

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Question 5 continued

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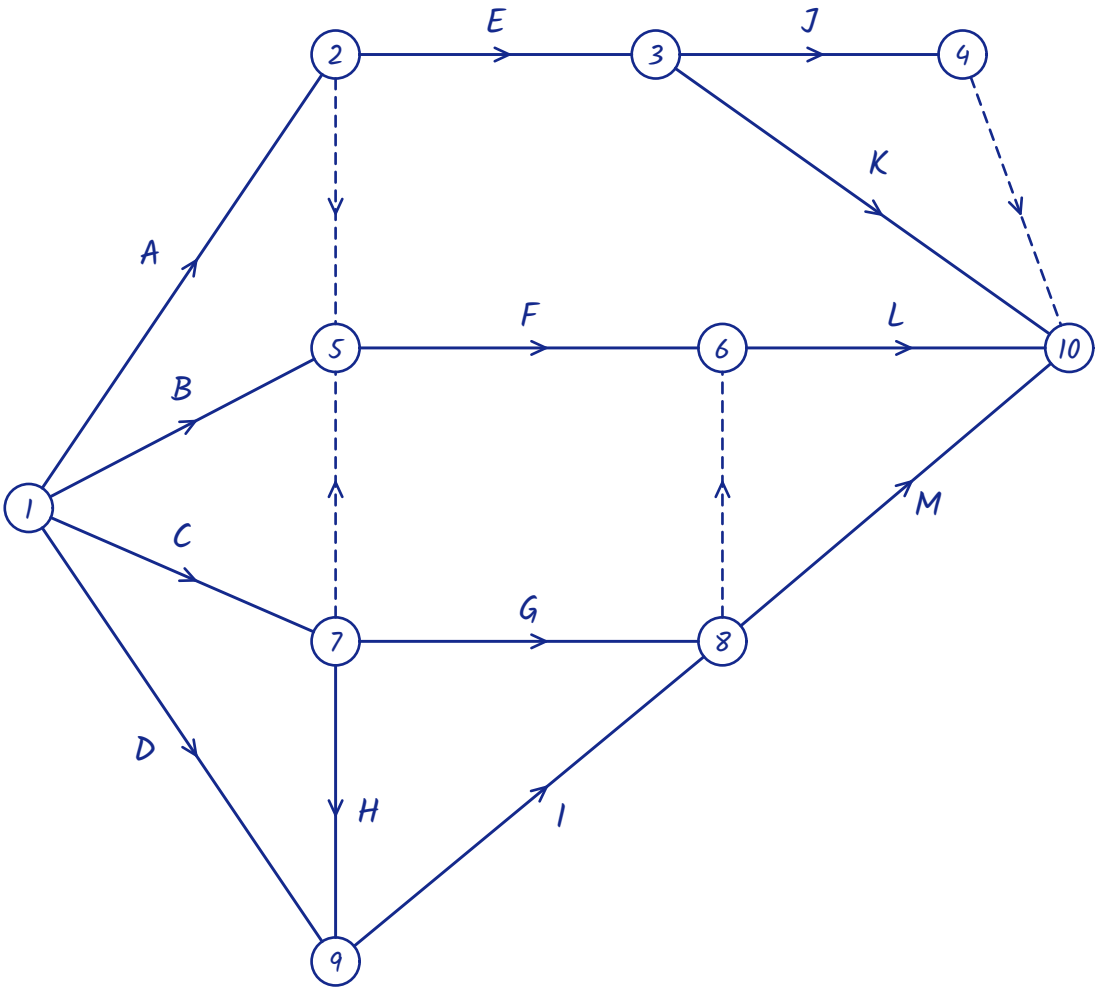
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6.



Activity on arc; dashed arcs are the four dummies.

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Question 6 continued

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(b) Critical path: A – E – K

(c) All activities equal duration, so the critical paths are the paths with the most activities (4 activities):

C – H – I – L

C – H – I – M

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Question 6 continued

Q6

(Total 8 marks)

TOTAL FOR PAPER: 75 MARKS

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