Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. A curve C has equation

$$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}} \qquad x > 0$$

(a) Find $\dfrac{dy}{dx}$ giving your answer in simplest form.

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $x = 2$ giving your answer in simplest form.

(*Solutions relying on calculator technology are not acceptable.*)

(2)

(a) $y = 2 + 10x^{1/2} - 2x^{3/2}$

$\frac{dy}{dx} = 10 \times \frac{1}{2} x^{-1/2} - 2 \times \frac{3}{2} x^{1/2}$

$\frac{dy}{dx} = 5x^{-1/2} - 3x^{1/2}$

(b) $x = 2$: $\frac{dy}{dx} = 5(2)^{-1/2} - 3(2)^{1/2} = \frac{5}{\sqrt{2}} - 3\sqrt{2}$

$= \frac{5\sqrt{2}}{2} - \frac{6\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$

Gradient of tangent $= -\frac{1}{2}\sqrt{2}$

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Question 1 continued

(Total for Question 1 is 5 marks)

2. The points P, Q and R have coordinates $(-3, 7)$, $(9, 11)$ and $(12, 2)$ respectively.

(a) Prove that angle $PQR = 90°$

(3)

Given that the point S is such that $PQRS$ forms a rectangle,

(b) find the coordinates of S.

(2)

(a) Gradient $PQ = \frac{11 - 7}{9 - (-3)} = \frac{4}{12} = \frac{1}{3}$

Gradient $QR = \frac{2 - 11}{12 - 9} = \frac{-9}{3} = -3$

$\frac{1}{3} \times (-3) = -1$

so PQ is perpendicular to QR, $\therefore$ angle $PQR = 90°$

(b) $Q \rightarrow R$: $(9, 11) \rightarrow (12, 2)$ is $+3$ in x, -9 in y

$S = P + (3, -9) = (-3 + 3, 7 - 9)$

$S\ (0, -2)$

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Question 2 continued

(Total for Question 2 is 5 marks)

3. Find

$$\int \frac{4x^5 + 3}{2x^2} \, dx$$

giving your answer in simplest form.

(5)

$\int \frac{4x^5 + 3}{2x^2} dx = \int \left(\frac{4x^5}{2x^2} + \frac{3}{2x^2} \right) dx$

$= \int \left(2x^3 + \frac{3}{2} x^{-2} \right) dx$

$= \frac{2x^4}{4} + \frac{3}{2} \times \frac{x^{-1}}{-1} + c$

$= \frac{1}{2} x^4 - \frac{3}{2} x^{-1} + c$

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Question 3 continued

(Total for Question 3 is 5 marks)

4. Given that the equation

$$kx^2 + 6kx + 5 = 0 \qquad \text{where } k \text{ is a non zero constant}$$

has no real roots, find the range of possible values for k.

(4)

No real roots $\Rightarrow$ $b^2 - 4ac < 0$ $\quad (a = k, b = 6k, c = 5)$

$(6k)^2 - 4 \times k \times 5 < 0$

$36k^2 - 20k < 0$

$4k(9k - 5) < 0$

Critical values: $k = 0$, $k = \frac{5}{9}$

$0 < k < \frac{5}{9}$

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Question 4 continued

(Total for Question 4 is 4 marks)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 3^x$, show that the equation

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

can be rewritten in the form

$$9p^2 + 26p - 3 = 0$$

(3)

(b) Hence solve

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

(3)

(a) $p = 3^x$: $9^x = (3^2)^x = (3^x)^2 = p^2$

$3^{x+2} = 3^2 \times 3^x = 9p$, $3^{x-1} = 3^x \times 3^{-1} = \frac{p}{3}$

$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1} \Rightarrow 3p^2 + 9p = 1 + \frac{p}{3}$

$\times 3$: $9p^2 + 27p = 3 + p$

$9p^2 + 26p - 3 = 0$ *

(b) $(9p - 1)(p + 3) = 0$

$p = \frac{1}{9}$ or $p = -3$

$3^x = \frac{1}{9} = 3^{-2} \Rightarrow x = -2$

$3^x = -3$ has no solutions since $3^x > 0$

$x = -2$

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Question 5 continued

(Total for Question 5 is 6 marks)

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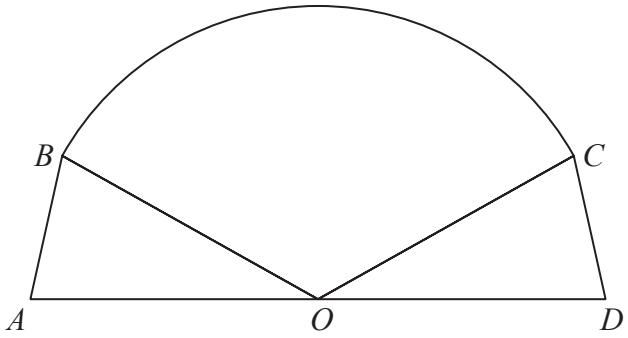


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the plan view for the design of a stage.

The design consists of a sector OBC of a circle, with centre O, joined to two congruent triangles OAB and ODC.

Given that

- angle $BOC = 2.4$ radians
- area of sector $BOC = 40\,m^2$
- AOD is a straight line of length 12.5 m

(a) find the radius of the sector, giving your answer, in m, to 2 decimal places,

(2)

(b) find the size of angle AOB, in radians, to 2 decimal places.

(1)

Hence find

(c) the total area of the stage, giving your answer, in m^2, to one decimal place,

(3)

(d) the total perimeter of the stage, giving your answer, in m, to one decimal place.

(4)

(a) Area $= \frac{1}{2} r^2\theta \Rightarrow 40 = \frac{1}{2} r^2 \times 2.4$

$r^2 = \frac{80}{2.4} = 33.33... \Rightarrow r = 5.7735...$

$r = 5.77$ m (2 d.p.)

(b) Angle AOB $= \frac{\pi - 2.4}{2} = 0.37079...$

Angle AOB $= 0.37$ radians (2 d.p.)

(c) OA = OD = 12.5 ÷ 2 = 6.25 m

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Question 6 continued

Area of triangle $AOB = \frac{1}{2} \times 5.7735... \times 6.25 \times \sin 0.37079... = 6.5377...$

Total area $= 40 + 2 \times 6.5377... = 53.0754...$

Total area of stage $= 53.1\ m^2$

(d) Arc $BC = r\theta = 5.7735... \times 2.4 = 13.856...$

$AB^2 = 6.25^2 + 5.7735...^2 - 2 \times 6.25 \times 5.7735... \times \cos 0.37079...$

$AB^2 = 5.1317... \Rightarrow AB = 2.2653...$

Perimeter $= AD + AB + CD + \text{arc } BC$

$= 12.5 + 2 \times 2.2653... + 13.856... = 30.887...$

Total perimeter of stage $= 30.9\ m$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

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7. (a) On Diagram 1, sketch a graph of the curve C with equation

$$y = \frac{6}{x} \qquad x \neq 0$$

(2)

The curve C is transformed onto the curve with equation $y = \dfrac{6}{x-2} \qquad x \neq 2$

(b) Fully describe this transformation.

(2)

The curve with equation

$$y = \frac{6}{x-2} \qquad x \neq 2$$

and the line with equation

$$y = kx + 7 \qquad \text{where } k \text{ is a constant}$$

intersect at exactly two points, P and Q.

Given that the x coordinate of point P is -4

(c) find the value of k,

(2)

(d) find, using algebra, the coordinates of point Q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(a) sketch on Diagram 1

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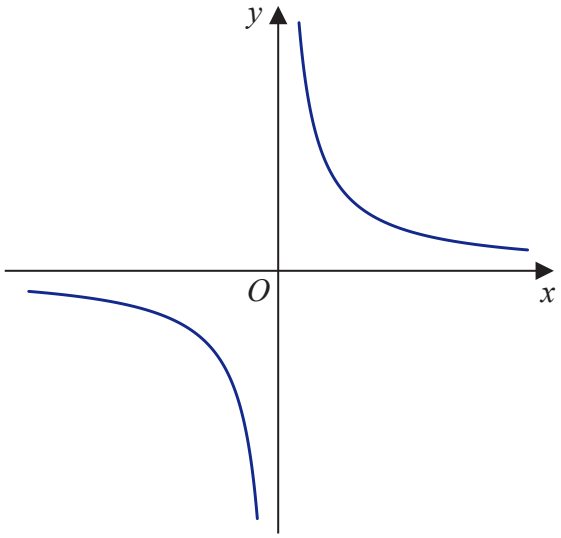


Diagram 1

(b) Translation 2 units to the right (parallel to the x-axis),

i.e. translation by vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(c) $x = -4$: $y = \frac{6}{-4-2} = \frac{6}{-6} = -1$

$-1 = k(-4) + 7 \Rightarrow 4k = 8$

$k = 2$

(d) $\frac{6}{x-2} = 2x + 7$

$6 = (2x + 7)(x - 2) = 2x^2 + 3x - 14$

$2x^2 + 3x - 20 = 0$

$(2x - 5)(x + 4) = 0$

$x = -4$ (point P) or $x = \frac{5}{2}$

$x = \frac{5}{2}$: $y = 2 \times \frac{5}{2} + 7 = 12$

Q $(\frac{5}{2}, 12)$

Turn over for a copy of Diagram 1 if you need to redraw your graph.

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Question 7 continued

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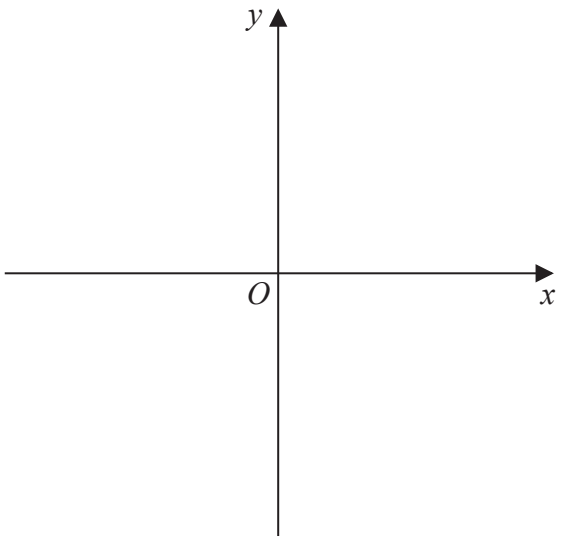
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Question 7 continued

Only use this copy of Diagram 1 if you need to redraw your graph.



Copy of Diagram 1

(Total for Question 7 is 10 marks)

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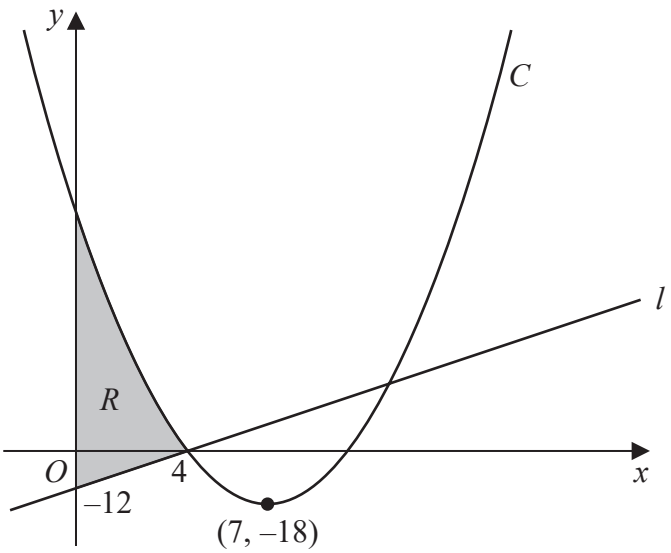


Figure 2

Figure 2 shows a sketch of the straight line l and the curve C.

Given that l cuts the y-axis at –12 and cuts the x-axis at 4, as shown in Figure 2,

(a) find an equation for l, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

Given that C

- has equation $y = f(x)$ where $f(x)$ is a quadratic expression
- has a minimum point at (7, –18)
- cuts the x-axis at 4 and at k, where k is a constant

(b) deduce the value of k,

(1)

(c) find $f(x)$.

(3)

The region R is shown shaded in Figure 2.

(d) Use inequalities to define R.

(2)

(a) Gradient of $l = \frac{0-(-12)}{4-0} = \frac{12}{4} = 3$, $c = -12$

$y = 3x - 12$

(b) Line of symmetry $x = 7$, so the roots are equidistant from 7

$k = 7 + (7 - 4) = 10$

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Question 8 continued

(c) $f(x) = A(x - 4)(x - 10)$

$(7, -18)$: $-18 = A(7 - 4)(7 - 10) = -9A \Rightarrow A = 2$

$f(x) = 2(x - 4)(x - 10)$

$f(x) = 2x^2 - 28x + 80$

(d) $3x - 12 < y < 2x^2 - 28x + 80$

$0 < x < 4$

Question 8 continued

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Question 8 continued

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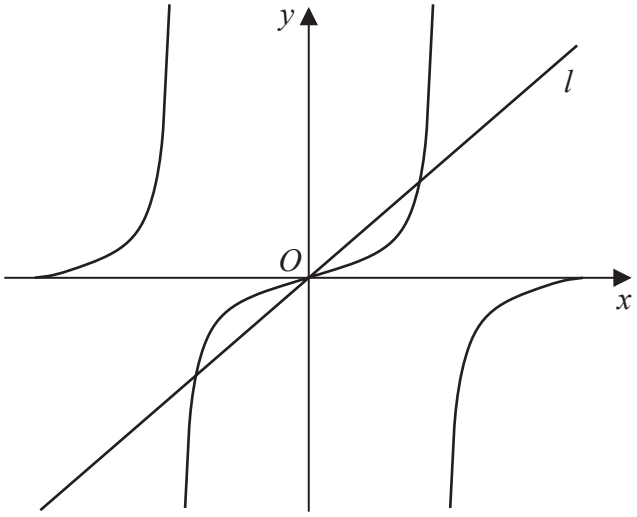


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan x$
- the straight line l with equation $y = \pi x$

in the interval $-\pi < x < \pi$

(a) State the period of $\tan x$

(1)

(b) Write down the number of roots of the equation

(i) $\tan x = (\pi + 2)x$ in the interval $-\pi < x < \pi$

(1)

(ii) $\tan x = \pi x$ in the interval $-2\pi < x < 2\pi$

(1)

(iii) $\tan x = \pi x$ in the interval $-100\pi < x < 100\pi$

(1)

(a) Period of tan x is π (radians)

(b)(i) 3

(ii) 5

(iii) 201

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Question 9 continued

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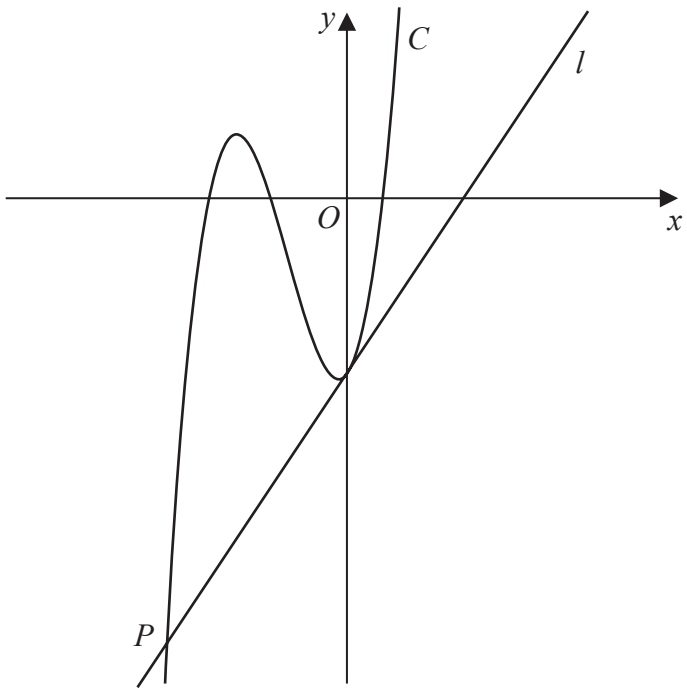


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = f(x)$, where

$$f(x) = (3x + 20)(x + 6)(2x - 3)$$

(a) Use the given information to state the values of x for which

$$f(x) > 0$$

(2)

(b) Expand $(3x + 20)(x + 6)(2x - 3)$, writing your answer as a polynomial in simplest form.

(3)

The straight line l is the tangent to C at the point where C cuts the y-axis.

Given that l cuts C at the point P, as shown in Figure 4,

(c) find, using algebra, the x coordinate of P

(*Solutions based on calculator technology are not acceptable.*)

(5)

(a) $f(x) = 0$ at $x = -\frac{20}{3}$, $x = -6$, $x = \frac{3}{2}$

$f(x) > 0$: $-\frac{20}{3} < x < -6$, $x > \frac{3}{2}$

(b) $(x + 6)(2x - 3) = 2x^2 - 3x + 12x - 18 = 2x^2 + 9x - 18$

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Question 10 continued

$(3x + 20)(2x^2 + 9x - 18)$

$= 6x^3 + 27x^2 - 54x + 40x^2 + 180x - 360$

$= 6x^3 + 67x^2 + 126x - 360$

(c) $\frac{dy}{dx} = 18x^2 + 134x + 126$

At $x = 0$: gradient $= 126$, $y = f(0) = -360$

Equation of l: $y = 126x - 360$

$6x^3 + 67x^2 + 126x - 360 = 126x - 360$

$6x^3 + 67x^2 = 0$

$x^2(6x + 67) = 0$

$x = 0$ (point of contact) or $x = -\frac{67}{6}$

x coordinate of $P = -\frac{67}{6}$

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Question 10 continued

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Question 10 continued

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11. A curve C has equation $y = f(x)$, $x > 0$

Given that

- $f''(x) = 4x + \frac{1}{\sqrt{x}}$
- the point P has x coordinate 4 and lies on C
- the tangent to C at P has equation $y = 3x + 4$

(a) find an equation of the normal to C at P

(2)

(b) find $f(x)$, writing your answer in simplest form.

(6)

(a) At P, $x = 4$: $y = 3(4) + 4 = 16$, so $P(4, 16)$

Gradient of tangent $= 3$, so gradient of normal $= -\frac{1}{3}$

$y - 16 = -\frac{1}{3}(x - 4)$

$3y + x - 52 = 0$

(b) $f''(x) = 4x + x^{-1/2}$

$f'(x) = \frac{4x^2}{2} + \frac{x^{1/2}}{1/2} + c = 2x^2 + 2x^{1/2} + c$

$f'(4) = 3$: $3 = 2(16) + 2(2) + c = 36 + c \Rightarrow c = -33$

$f'(x) = 2x^2 + 2x^{1/2} - 33$

$f(x) = \frac{2x^3}{3} + \frac{2x^{3/2}}{3/2} - 33x + d = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + d$

$f(4) = 16$: $16 = \frac{2}{3}(64) + \frac{4}{3}(8) - 33(4) + d$

$16 = \frac{128}{3} + \frac{32}{3} - 132 + d = -\frac{236}{3} + d \Rightarrow d = \frac{284}{3}$

$f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + \frac{284}{3}$

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Question 11 continued

Question 11 continued

(Total for Question 11 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

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