Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. The table below shows corresponding values of x and y for

$$y = 2^{5-\sqrt{x}}$$

The values of y are given to 3 decimal places.

x	5	5.5	6	6.5	7
y	6.792	6.298	5.858	5.466	5.113

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_5^7 2^{5-\sqrt{x}}\,dx$$

giving your answer to 2 decimal places.

(3)

(b) Using your answer to part (a) and making your method clear, estimate

(i) $\int_5^7 2^{6-\sqrt{x}}\,dx$

(ii) $\int_5^7 \left(3 + 2^{5-\sqrt{x}}\right)dx$

(4)

(a) $h = 0.5$

$\int_5^7 2^{5-\sqrt{x}}\,dx \approx \frac{1}{2} \times 0.5 \times \{6.792 + 5.113 + 2(6.298 + 5.858 + 5.466)\}$

$= 0.25 \times \{11.905 + 35.294\}$

$= 0.25 \times 47.199 = 11.78725 = 11.79$ (2 d.p.)

(b) (i) $2^{6-\sqrt{x}} = 2 \times 2^{5-\sqrt{x}} \Rightarrow \int_5^7 2^{6-\sqrt{x}}\,dx \approx 2 \times 11.79$

$= 23.58$

(ii) $\int_5^7 (3 + 2^{5-\sqrt{x}})\,dx = \int_5^7 3\,dx + \int_5^7 2^{5-\sqrt{x}}\,dx$

$= [3x]_5^7 + 11.79 = (21 - 15) + 11.79$

$= 6 + 11.79 = 17.79$

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Question 1 continued

Q1

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2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 27x^{\frac{1}{2}} - x^{\frac{3}{2}} - 20 \qquad x > 0$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, giving each term in simplest form.

(2)

(b) Hence find the coordinates of the stationary point of C.

(4)

(c) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and hence determine the nature of the stationary point of C.

(2)

(a) $y = 27x^{1/2} - x^{3/2} - 20 \Rightarrow \frac{dy}{dx} = \frac{27}{2}x^{-1/2} - \frac{3}{2}x^{1/2}$

(b) $\frac{27}{2}x^{-1/2} - \frac{3}{2}x^{1/2} = 0$

Multiply by $2x^{1/2}$: $27 - 3x = 0$

$x = 9$

$y = 27(9)^{1/2} - (9)^{3/2} - 20 = 81 - 27 - 20 = 34$

Stationary point $(9, 34)$

(c) $\frac{d^2y}{dx^2} = -\frac{27}{4}x^{-3/2} - \frac{3}{4}x^{-1/2}$

At $x = 9$: $\frac{d^2y}{dx^2} = -\frac{27}{4} \times \frac{1}{27} - \frac{3}{4} \times \frac{1}{3} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

$\frac{d^2y}{dx^2} = -\frac{1}{2} < 0$, so $(9, 34)$ is a maximum

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Question 2 continued

Q2

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3. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{kx}{4}\right)^8$$

where k is a non-zero constant. Give each term in simplest form.

(4)

$$f(x) = (5 - 3x)\left(2 - \frac{kx}{4}\right)^8$$

In the expansion of $f(x)$, the constant term is 3 times the coefficient of x.

(b) Find the value of k.

(3)

(a) $\left(2 - \frac{kx}{4}\right)^8 = 2^8 + \binom{8}{1}2^7\left(-\frac{kx}{4}\right) + \binom{8}{2}2^6\left(-\frac{kx}{4}\right)^2$

$+ \binom{8}{3}2^5\left(-\frac{kx}{4}\right)^3 + \ldots$

$= 256 + 1024\left(-\frac{kx}{4}\right) + 1792\left(\frac{k^2x^2}{16}\right) + 1792\left(-\frac{k^3x^3}{64}\right) + \ldots$

$= 256 - 256kx + 112k^2x^2 - 28k^3x^3 + \ldots$

(b) $f(x) = (5 - 3x)(256 - 256kx + 112k^2x^2 - \ldots)$

Constant term: $5 \times 256 = 1280$

Coefficient of x: $5 \times (-256k) - 3 \times 256 = -1280k - 768$

$1280 = 3(-1280k - 768)$

$1280 = -3840k - 2304$

$3840k = -3584$

$k = -\frac{3584}{3840} = -\frac{14}{15}$

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Question 3 continued

Q3

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4. Using the laws of logarithms, solve

$$\log_3(32 - 12x) = 2\log_3(1 - x) + 3$$

(5)

$\log_3(32 - 12x) = \log_3(1 - x)^2 + 3$

$\log_3(32 - 12x) - \log_3(1 - x)^2 = 3$

$\log_3\left(\frac{32 - 12x}{(1 - x)^2}\right) = 3$

$\frac{32 - 12x}{(1 - x)^2} = 3^3 = 27$

$32 - 12x = 27(1 - 2x + x^2) = 27 - 54x + 27x^2$

$27x^2 - 42x - 5 = 0$

$(9x + 1)(3x - 5) = 0$

$x = -\frac{1}{9}$ or $x = \frac{5}{3}$

$x = \frac{5}{3}$ is rejected since $1 - x < 0$ and $\log_3(1 - x)$ is undefined

$x = -\frac{1}{9}$

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Question 4 continued

Q4

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5. $f(x) = 3x^3 + Ax^2 + Bx - 10$

where A and B are integers.

Given that

- when $f(x)$ is divided by $(x - 1)$ the remainder is k
- when $f(x)$ is divided by $(x + 1)$ the remainder is $-10k$
- k is a constant

(a) show that

$$11A + 9B = 83$$

(3)

Given also that $(3x - 2)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(3)

(c) Hence find the quadratic expression $g(x)$ such that

$$f(x) = (3x - 2)g(x)$$

(2)

(a) $f(1) = 3(1)^3 + A(1)^2 + B(1) - 10 = k \Rightarrow A + B - 7 = k$

$f(-1) = 3(-1)^3 + A(-1)^2 + B(-1) - 10 = -10k \Rightarrow A - B - 13 = -10k$

$A - B - 13 = -10(A + B - 7)$

$A - B - 13 = -10A - 10B + 70$

$11A + 9B = 83$ (as required)

(b) $f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 + A\left(\frac{2}{3}\right)^2 + B\left(\frac{2}{3}\right) - 10 = 0$

$\frac{8}{9} + \frac{4A}{9} + \frac{2B}{3} - 10 = 0 \Rightarrow 4A + 6B = 82 \Rightarrow 12A + 18B = 246$

$2 \times (11A + 9B = 83)$: $22A + 18B = 166$

Subtract: $10A = -80 \Rightarrow A = -8$

$11(-8) + 9B = 83 \Rightarrow 9B = 171 \Rightarrow B = 19$

$A = -8, \quad B = 19$

(c) $f(x) = 3x^3 - 8x^2 + 19x - 10 = (3x - 2)(x^2 + px + 5)$

x^2 terms: $3p - 2 = -8 \Rightarrow p = -2$

Check x terms: $-2p + 15 = 4 + 15 = 19$ ✓

$g(x) = x^2 - 2x + 5$

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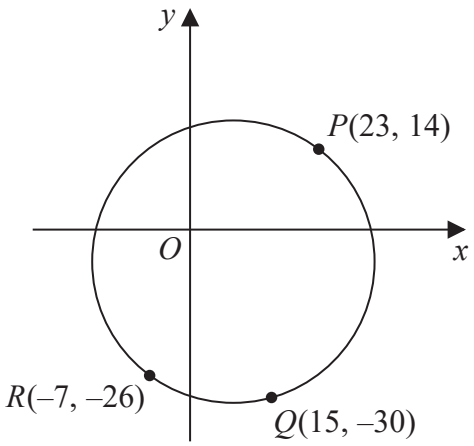


Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(a) $m_{PQ} = \frac{14 - (-30)}{23 - 15} = \frac{44}{8} = \frac{11}{2}$, $m_{QR} = \frac{-26 - (-30)}{-7 - 15} = \frac{4}{-22} = -\frac{2}{11}$

$m_{PQ} \times m_{QR} = \frac{11}{2} \times \left(-\frac{2}{11}\right) = -1$, so PQ ⊥ QR, ∴ angle PQR = 90°

(b) (i) Angle PQR = 90°, so PR is a diameter of C

Centre = midpoint of PR = $\left(\frac{23 + (-7)}{2}, \frac{14 + (-26)}{2}\right) = (8, -6)$

(ii) $r = \sqrt{((23 - 8)^2 + (14 + 6)^2)} = \sqrt{(225 + 400)} = \sqrt{625}$

$r = 25$

(c) QS greatest ⇒ QS is a diameter, so centre is the midpoint of QS

$S = (2 \times 8 - 15, 2 \times (-6) + 30) = (1, 18)$

Gradient of radius from (8, –6) to S = $\frac{18 - (-6)}{1 - 8} = -\frac{24}{7}$

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Question 6 continued

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Tangent is perpendicular to radius, so gradient $= \frac{7}{24}$

$y - 18 = \frac{7}{24}(x - 1)$

$24y - 432 = 7x - 7$

$7x - 24y + 425 = 0$

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Question 6 continued

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Question 6 continued

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7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-90° < x < 90°$, the equation

$$3\sin(2x - 15°) = \cos(2x - 15°)$$

giving your answers to one decimal place.

(4)

(ii) Solve, for $0 < \theta < 2\pi$, the equation

$$4\sin^2\theta + 8\cos\theta = 3$$

giving your answers to 3 significant figures.

(4)

(i) $3\sin(2x - 15°) = \cos(2x - 15°) \Rightarrow \frac{\sin(2x - 15°)}{\cos(2x - 15°)} = \frac{1}{3}$

$\tan(2x - 15°) = \frac{1}{3}$

$-90° < x < 90° \Rightarrow -195° < 2x - 15° < 165°$

$2x - 15° = \tan^{-1}\left(\frac{1}{3}\right) = 18.434...°$ or $18.434...° - 180° = -161.565...°$

$2x = 33.434...°$ or $-146.565...°$

$x = 16.717...°$ or $-73.282...°$

$x = 16.7°, \quad x = -73.3°$

(ii) $4\sin^2\theta + 8\cos\theta = 3$

$4(1 - \cos^2\theta) + 8\cos\theta = 3$

$4\cos^2\theta - 8\cos\theta - 1 = 0$

$\cos\theta = \frac{8 \pm \sqrt{(64 + 4 \times 4 \times 1)}}{2 \times 4} = \frac{8 \pm \sqrt{80}}{8} = 1 \pm \frac{\sqrt{5}}{2}$

$\cos\theta = 1 + \frac{\sqrt{5}}{2} = 2.118... > 1$, no solutions

$\cos\theta = 1 - \frac{\sqrt{5}}{2} = -0.11803...$

$\theta = \cos^{-1}(-0.11803...) = 1.68910...$

or $\theta = 2\pi - 1.68910... = 4.59407...$

$\theta = 1.69, \quad \theta = 4.59$ (3 s.f.)

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Question 7 continued

Q7

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8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.

(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.

(4)

(a) AP with $a = 100$, $d = -2$: $u_{20} = a + 19d = 100 + 19(-2) = 62$ mm

(b) $S_{20} = \frac{20}{2}\{2 \times 100 + 19 \times (-2)\} = 10 \times 162$

$= 1620$ mm

(c) 21st hit: $62r$, 22nd hit: $62r^2$

$62r^2 = 60 \Rightarrow r^2 = \frac{60}{62} \Rightarrow r = \frac{\sqrt{60}}{\sqrt{62}} = 0.983738...$

$r = 0.984$ (3 d.p.)

(d) Hits after the 20th form a GP with first term $62r$ and ratio r

Distance after N hits $= 1620 + \frac{62r(1 - r^n)}{1 - r}$ where $n = N - 20$

$1620 + \frac{62 \times 0.98373...(1 - 0.98373...^n)}{1 - 0.98373...} > 3000$

$3750.745...(1 - 0.98373...^n) > 1380$

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Question 8 continued

$1 - 0.98373...^n > \frac{1380}{3750.745...} = 0.36792...$

$0.98373...^n < 0.63207...$

$n > \frac{\log(0.63207...)}{\log(0.98373...)} = 27.981...$

(inequality reverses as $\log(0.98373...) < 0$)

$n = 28$, so $N = 20 + 28 = 48$

Smallest $N = 48$

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Question 8 continued

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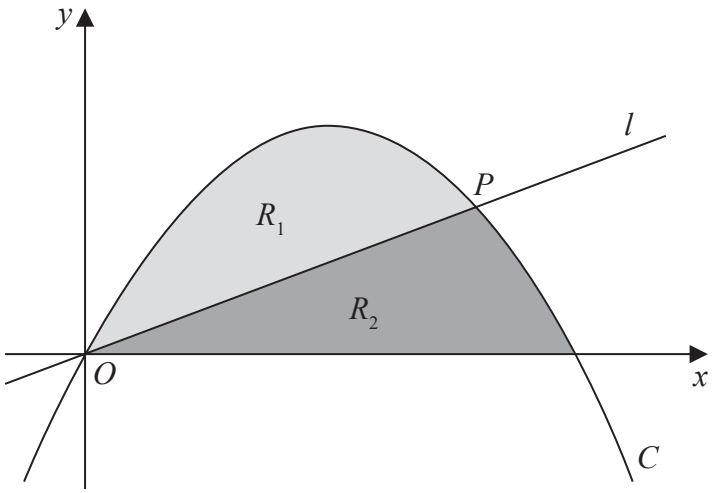


Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1-m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

(a) $mx = x - x^2 \Rightarrow x^2 - x + mx = 0 \Rightarrow x(x - (1 - m)) = 0$

$x = 1 - m \Rightarrow y = m(1 - m)$

$P(1 - m, m(1 - m))$

(b) Area of $R_1 = \int_0^{1-m} \{(x - x^2) - mx\}\, dx = \int_0^{1-m} \{(1 - m)x - x^2\}\, dx$

$= \left[\frac{(1-m)x^2}{2} - \frac{x^3}{3}\right]_0^{1-m}$

$= \frac{(1-m)(1-m)^2}{2} - \frac{(1-m)^3}{3} - 0$

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Question 9 continued

$= \frac{(1-m)^3}{2} - \frac{(1-m)^3}{3} = \frac{(1-m)^3}{6}$ (as required)

(c) Area of $R_1 + R_2 = \int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

$R_1 = R_2 \Rightarrow R_1 = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

$\frac{(1-m)^3}{6} = \frac{1}{12} \Rightarrow (1-m)^3 = \frac{1}{2}$

$1 - m = \frac{1}{\sqrt[3]{2}} \Rightarrow m = 1 - \frac{1}{\sqrt[3]{2}}$

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Question 9 continued

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Question 9 continued

Q9

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10. (i) Prove by counter example that the statement

"if p is a prime number then $2p + 1$ is also a prime number"

is not true.

(1)

(ii) Use proof by exhaustion to prove that if n is an integer then

$$5n^2 + n + 12$$

is always even.

(4)

(i) $p = 7$ is prime, but $2p + 1 = 2 \times 7 + 1 = 15 = 3 \times 5$,
which is not prime, so the statement is not true.

(ii) Case 1: n even, $n = 2k$ for some integer k
$5n^2 + n + 12 = 5(2k)^2 + 2k + 12 = 20k^2 + 2k + 12$
$= 2(10k^2 + k + 6)$, which is even

Case 2: n odd, $n = 2k + 1$ for some integer k
$5n^2 + n + 12 = 5(2k + 1)^2 + (2k + 1) + 12$
$= 20k^2 + 20k + 5 + 2k + 1 + 12 = 20k^2 + 22k + 18$
$= 2(10k^2 + 11k + 9)$, which is even

Every integer is either even or odd, and in both cases
$5n^2 + n + 12$ is even, so $5n^2 + n + 12$ is always even.

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Question 10 continued

Q10

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