Please check the examination details below before entering your candidate information

Candidate surname	Other names
StemUp Model Solution	

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P70482A

©2022 Pearson Education Ltd.

L:1/1/1/1/

P 7 0 4 8 2 A 0 1 3 2

Pearson

Leave blank

1. Find

$$\int\left(\frac{8x^3}{5}-\frac{2}{3x^4}-1\right)\mathrm{d}x$$

giving each term in simplest form.

(4)

$\int\left(\frac{8x^3}{5}-\frac{2}{3x^4}-1\right)dx = \int\left(\frac{8}{5}x^3-\frac{2}{3}x^{-4}-1\right)dx$

$=\frac{8}{5}\times\frac{x^4}{4}-\frac{2}{3}\times\frac{x^{-3}}{-3}-x+c$

$=\frac{2}{5}x^4+\frac{2}{9}x^{-3}-x+c$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 1 continued

Q1

(Total 4 marks)

Leave blank

2. $f(x) = 11 - 4x - 2x^2$

(a) Express $f(x)$ in the form

$$a + b(x + c)^2$$

where a, b and c are integers to be found.

(3)

(b) Sketch the graph of the curve C with equation $y = f(x)$, showing clearly the coordinates of the point where the curve crosses the y-axis.

(2)

(c) Write down the equation of the line of symmetry of C.

(1)

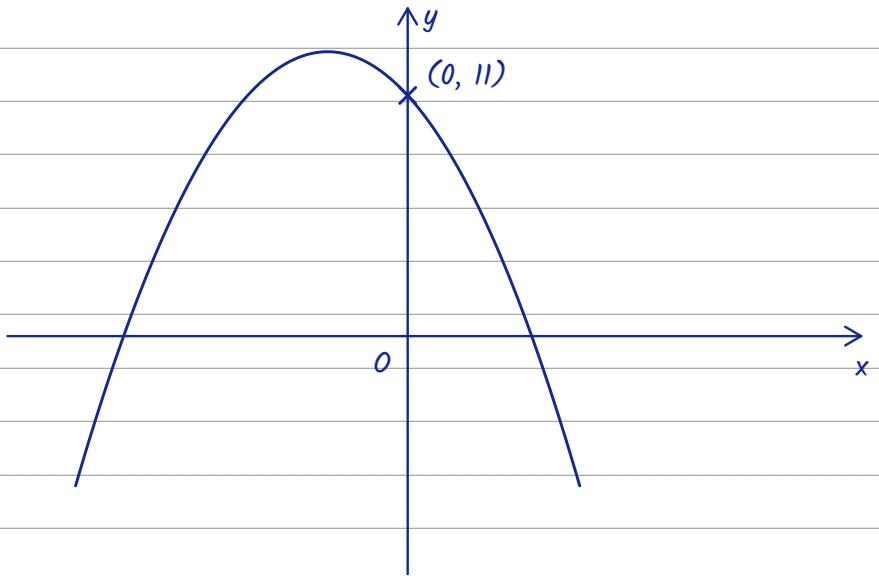
(a) $f(x) = 11 - 4x - 2x^2 = 11 - 2(x^2 + 2x)$

$= 11 - 2[(x + 1)^2 - 1]$

$= 11 - 2(x + 1)^2 + 2$

$f(x) = 13 - 2(x + 1)^2$ $\quad (a = 13, b = -2, c = 1)$

(b)



(c) Line of symmetry: $x = -1$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i)

$$f(x) = (x + \sqrt{2})^2 + (3x - 5\sqrt{8})^2$$

Express f(x) in the form $ax^2 + bx\sqrt{2} + c$ where a, b and c are integers to be found.

(3)

(ii) Solve the equation

$$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are simplified fractions to be found.

(4)

(i) $\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$(x + \sqrt{2})^2 = x^2 + 2x\sqrt{2} + 2$

$(3x - 5\sqrt{8})^2 = (3x - 10\sqrt{2})^2 = 9x^2 - 60x\sqrt{2} + 200$

$f(x) = x^2 + 2x\sqrt{2} + 2 + 9x^2 - 60x\sqrt{2} + 200$

$f(x) = 10x^2 - 58x\sqrt{2} + 202$

(ii) $\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$

$4\sqrt{3}\,y - 9 = 5y + \sqrt{3}$

$4\sqrt{3}\,y - 5y = 9 + \sqrt{3}$

$y(4\sqrt{3} - 5) = 9 + \sqrt{3}$

$$y = \frac{9 + \sqrt{3}}{4\sqrt{3} - 5} \times \frac{4\sqrt{3} + 5}{4\sqrt{3} + 5}$$

$$y = \frac{36\sqrt{3} + 45 + 12 + 5\sqrt{3}}{48 - 25}$$

$$y = \frac{57 + 41\sqrt{3}}{23}$$

$$y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Leave blank

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4.

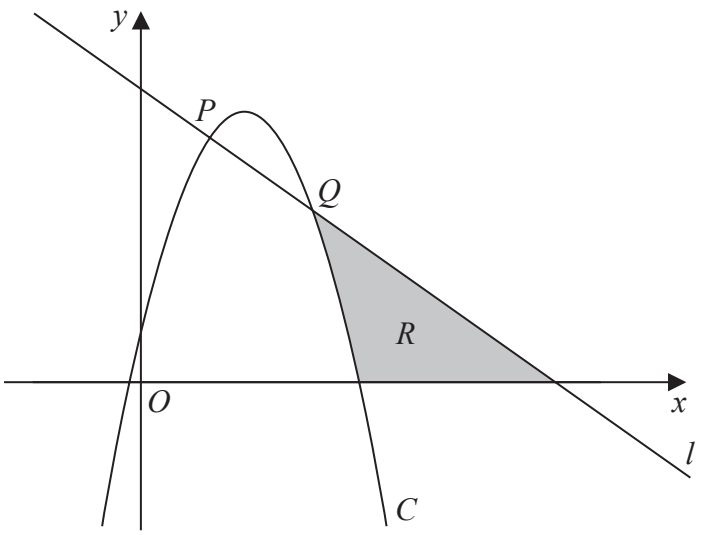


Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 1 shows a line l with equation $x + y = 6$ and a curve C with equation $y = 6x - 2x^2 + 1$

The line l intersects the curve C at the points P and Q as shown in Figure 1.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 1, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

(a) $x + y = 6 \Rightarrow y = 6 - x$

$6 - x = 6x - 2x^2 + 1$

$2x^2 - 7x + 5 = 0$

$(2x - 5)(x - 1) = 0$

$x = \frac{5}{2}$ or $x = 1$

$x = \frac{5}{2} \Rightarrow y = 6 - \frac{5}{2} = \frac{7}{2}$; $x = 1 \Rightarrow y = 6 - 1 = 5$

P (1, 5) and Q (2.5, 3.5)

(b) $y \geq 6x - 2x^2 + 1$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

$x + y \leq 6$

$y \geq 0$

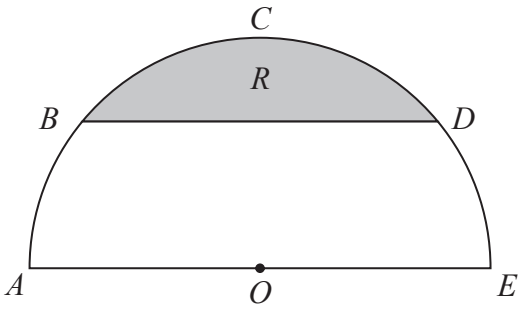
$x \geq 2.5$

Q4

(Total 7 marks)

Leave blank

5.



Not to scale

Figure 2

Figure 2 shows a plan view of a semicircular garden *ABCDEOA*

The semicircle has

- centre *O*
- diameter *AOE*
- radius 3 m

The straight line *BD* is parallel to *AE* and angle *BOA* is 0.7 radians.

(a) Show that, to 4 significant figures, angle *BOD* is 1.742 radians.

(1)

The flowerbed *R*, shown shaded in Figure 2, is bounded by *BD* and the arc *BCD*.

(b) Find the area of the flowerbed, giving your answer in square metres to one decimal place.

(3)

(c) Find the perimeter of the flowerbed, giving your answer in metres to one decimal place.

(3)

(a) $\angle BOD = \pi - 2 \times 0.7 = 1.74159...$

$\angle BOD = 1.742$ radians (4 s.f.)

(b) Area of sector $BOD = \frac{1}{2} \times 3^2 \times 1.742 = 7.839$

Area of triangle $BOD = \frac{1}{2} \times 3^2 \times \sin 1.742 = 4.4342...$

Area of $R = 7.839 - 4.4342... = 3.4047...$

Area of flowerbed $= 3.4\ m^2$

(c) $BD = 2 \times 3 \cos 0.7 = 4.5890...$

Arc $BCD = r\theta = 3 \times 1.742 = 5.226$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

Perimeter = 5.226 + 4.5890... = 9.8150...

Perimeter of flowerbed = 9.8 m

Leave blank

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = \dfrac{(x+3)^2}{x\sqrt{x}}$
- the point $P(4, 20)$ lies on C

(a) (i) find the value of the gradient at P

(ii) Hence find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $f(x)$, simplifying your answer.

(7)

(a)(i) $f'(4) = \dfrac{(4+3)^2}{4\sqrt{4}} = \dfrac{49}{8}$ $(= 6.125)$

(ii) $y - 20 = \dfrac{49}{8}(x - 4)$

$8y - 160 = 49x - 196$

$49x - 8y - 36 = 0$

(b) $f'(x) = \dfrac{(x+3)^2}{x\sqrt{x}} = \dfrac{x^2 + 6x + 9}{x^{3/2}}$

$= \dfrac{x^2}{x^{3/2}} + \dfrac{6x}{x^{3/2}} + \dfrac{9}{x^{3/2}}$

$= x^{1/2} + 6x^{-1/2} + 9x^{-3/2}$

$f(x) = \dfrac{x^{3/2}}{3/2} + \dfrac{6x^{1/2}}{1/2} + \dfrac{9x^{-1/2}}{-1/2} + c$

$f(x) = \dfrac{2}{3}x^{3/2} + 12x^{1/2} - 18x^{-1/2} + c$

$P(4, 20)$: $20 = \dfrac{2}{3}(4)^{3/2} + 12(4)^{1/2} - 18(4)^{-1/2} + c$

$20 = \dfrac{16}{3} + 24 - 9 + c \Rightarrow c = -\dfrac{1}{3}$

$f(x) = \dfrac{2}{3}x^{3/2} + 12x^{1/2} - 18x^{-1/2} - \dfrac{1}{3}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

Leave blank

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

(Total 11 marks)

Q6

Leave blank

7.

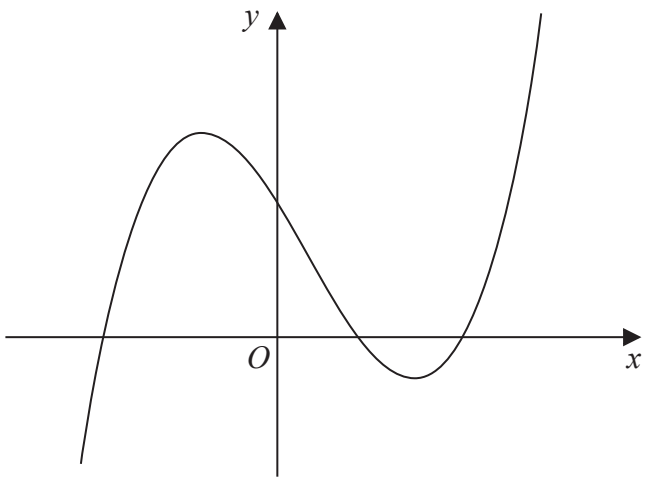


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x + 4)(x - 2)(2x - 9)$$

Given that the curve with equation $y = f(x) - p$ passes through the point with coordinates (0, 50)

(a) find the value of the constant p.

(2)

Given that the curve with equation $y = f(x + q)$ passes through the origin,

(b) write down the possible values of the constant q.

(2)

(c) Find $f'(x)$.

(4)

(d) Hence find the range of values of x for which the gradient of the curve with equation $y = f(x)$ is less than -18

(3)

(a) $f(0) = (4)(-2)(-9) = 72$

$f(0) - p = 50 \Rightarrow 72 - p = 50 \Rightarrow p = 22$

(b) $f(x) = 0$ when $x = -4, 2, 4.5$

$f(0 + q) = 0 \Rightarrow q = -4, 2, 4.5$

(c) $f(x) = (x^2 + 2x - 8)(2x - 9)$

$= 2x^3 - 9x^2 + 4x^2 - 18x - 16x + 72$

$= 2x^3 - 5x^2 - 34x + 72$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

$f'(x) = 6x^2 - 10x - 34$

(d) $6x^2 - 10x - 34 < -18$

$6x^2 - 10x - 16 < 0 \Rightarrow 3x^2 - 5x - 8 < 0$

$(3x - 8)(x + 1) < 0$

Critical values: $x = -1$, $x = \frac{8}{3}$

$-1 < x < \frac{8}{3}$

Leave blank

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8. The line l_1 has equation

$$2x - 5y + 7 = 0$$

(a) Find the gradient of l_1

(1)

Given that

- the point A has coordinates $(6, -2)$
- the line l_2 passes through A and is perpendicular to l_1

(b) find the equation of l_2 giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The lines l_1 and l_2 intersect at the point M.

(c) Using algebra and showing all your working, find the coordinates of M.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

Given that the diagonals of a square $ABCD$ meet at M,

(d) find the coordinates of the point C.

(2)

(a) $5y = 2x + 7 \Rightarrow y = \frac{2}{5}x + \frac{7}{5}$

Gradient of $l_1 = \frac{2}{5}$

(b) Gradient of $l_2 = -1 \div \frac{2}{5} = -\frac{5}{2}$

$y - (-2) = -\frac{5}{2}(x - 6)$

$y = -\frac{5}{2}x + 13$

(c) $2x - 5\left(-\frac{5}{2}x + 13\right) + 7 = 0$

$2x + \frac{25}{2}x - 65 + 7 = 0 \Rightarrow \frac{29}{2}x = 58$

$x = 4$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

$y = -\frac{5}{2}(4) + 13 = -10 + 13 = 3$

$M\ (4, 3)$

(d) M is the midpoint of the diagonal AC

$C = (\ 2 \times 4 - 6\ ,\ \ 2 \times 3 - (-2)\ )$

$C\ (2, 8)$

Leave blank

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

Q8

(Total 9 marks)

Leave blank

9.

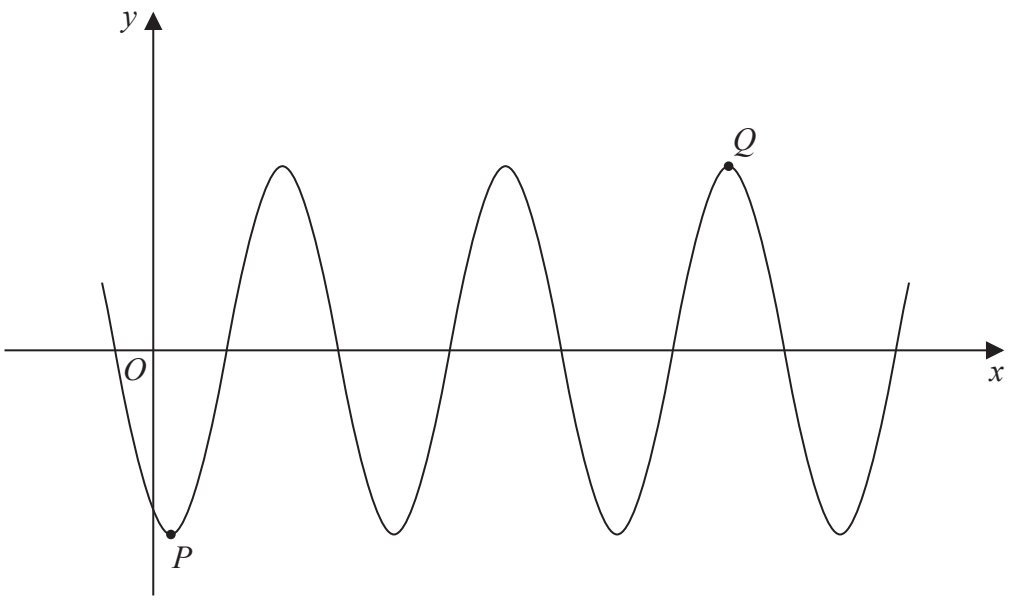


Figure 4

Figure 4 shows part of the curve with equation

$$y = A\cos(x - 30)°$$

where A is a constant.

The point P is a minimum point on the curve and has coordinates (30, –3) as shown in Figure 4.

(a) Write down the value of A.

(1)

The point Q is shown in Figure 4 and is a maximum point.

(b) Find the coordinates of Q.

(3)

(a) At P, x = 30: y = A cos 0° = A ⇒ A = −3

(b) y = −3 cos(x − 30)°, maximum when cos(x − 30)° = −1

so y = 3 at a maximum

x − 30 = 180, 540, 900, ... ⇒ x = 210, 570, 930, ...

Q is the third maximum, so x = 210 + 2 × 360 = 930

Q (930, 3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 9 continued

Q9

(Total 4 marks)

Leave blank

10. The curve C has equation

$$y = \frac{1}{x^2} - 9$$

(a) Sketch the graph of C.

On your sketch

- show the coordinates of any points of intersection with the coordinate axes
- state clearly the equations of any asymptotes

(4)

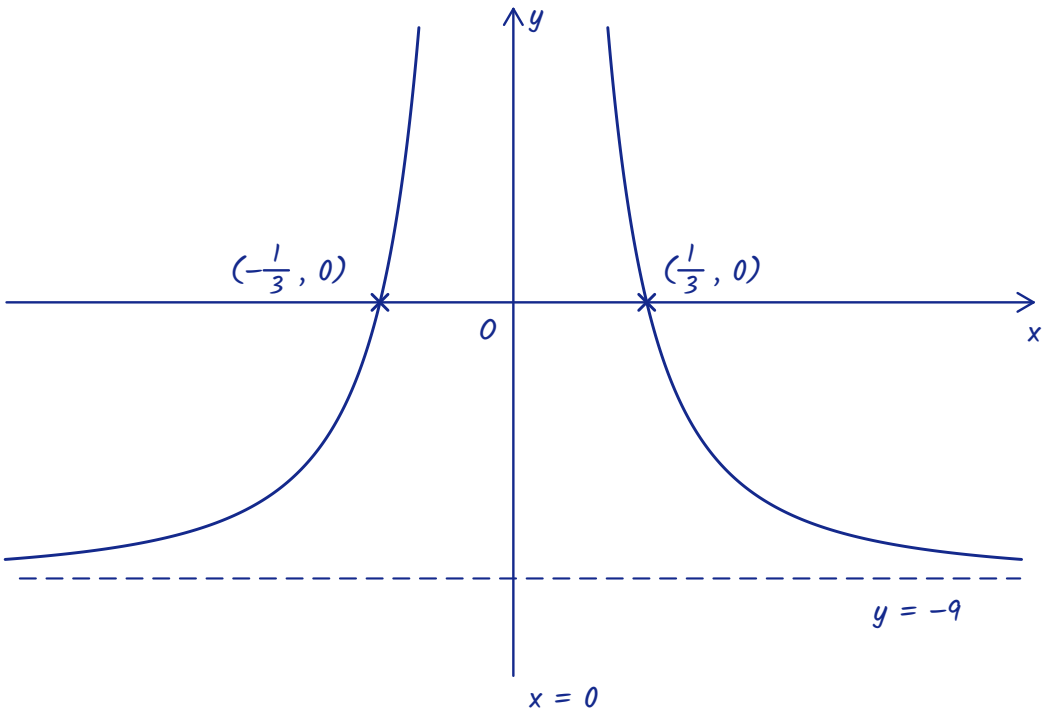
The curve D has equation $y = kx^2$ where k is a constant.

Given that C meets D at 4 distinct points,

(b) find the range of possible values for k.

(5)

(a)



Asymptotes: $x = 0$ and $y = -9$

Crosses x-axis at $x = \pm\frac{1}{3}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 10 continued

(b) $\frac{1}{x^2} - 9 = kx^2$

$1 - 9x^2 = kx^4$

$kx^4 + 9x^2 - 1 = 0$ (a quadratic in x^2)

4 distinct points need 2 distinct positive roots for x^2

Distinct roots: $b^2 - 4ac > 0$

$81 - 4(k)(-1) > 0 \Rightarrow 81 + 4k > 0$

$k > -\frac{81}{4}$

Both roots for x^2 positive: product $= \frac{-1}{k} > 0 \Rightarrow k < 0$

$-\frac{81}{4} < k < 0$

Leave blank

Question 10 continued

Q10

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA