Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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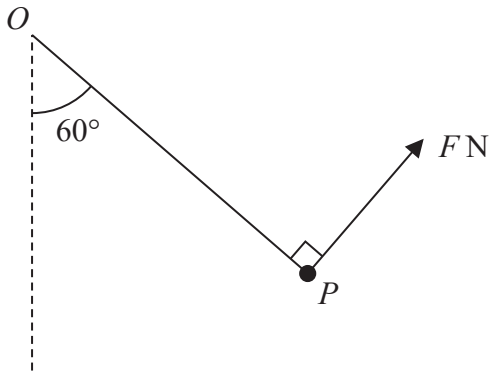


Figure 1

A particle P of weight 5 N is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle P is held in equilibrium by a force of magnitude F newtons. The direction of this force is perpendicular to the string and OP makes an angle of 60° with the vertical, as shown in Figure 1.

Find

(a) the value of F

(3)

(b) the tension in the string.

(3)

(a) Resolving perpendicular to the string:

$F = 5 \cos 30°$

$F = \frac{5\sqrt{3}}{2} = 4.3301...$

$F = 4.33$ (N)

(b) Resolving along the string:

$T = 5 \sin 30°$

$T = \frac{5}{2} = 2.5$

Tension = 2.5 N

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Question 1 continued

Q1

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2. A particle P has mass km and a particle Q has mass m. The particles are moving towards each other in opposite directions along the same straight line when they collide directly.

Immediately before the collision, P has speed $3u$ and Q has speed u.

As a result of the collision, the direction of motion of each particle is reversed and the speed of each particle is halved.

(a) Find the value of k.

(4)

(b) Find, in terms of m and u, the magnitude of the impulse exerted on Q in the collision.

(3)

(a) Take the direction of motion of P before impact as positive.

After impact: P moves at $1.5u$ (backwards), Q moves at $0.5u$ (forwards)

CLM: $km(3u) - m(u) = km(-1.5u) + m(0.5u)$

$3k - 1 = -1.5k + 0.5$

$4.5k = 1.5$

$k = \frac{1}{3}$

(b) Impulse on Q = change in momentum of Q

$I = m(0.5u - (-u))$

$I = m(\frac{1}{2}u + u) = \frac{3}{2}mu$

Magnitude of impulse on Q $= \frac{3}{2}mu$

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Question 2 continued

Q2

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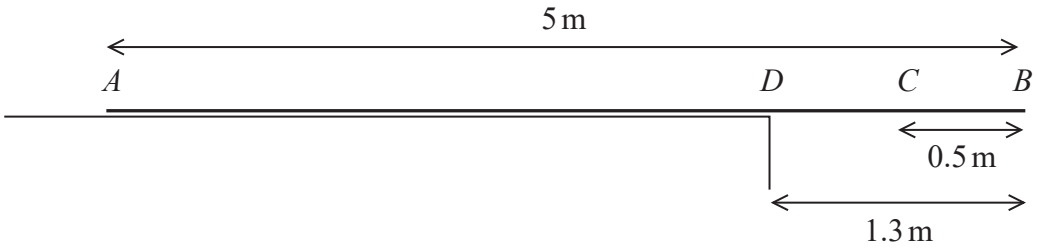


Figure 2

A beam $ADCB$ has length 5 m. The beam lies on a horizontal step with the end A on the step and the end B projecting over the edge of the step. The edge of the step is at the point D where $DB = 1.3$ m, as shown in Figure 2.

When a small boy of mass 30 kg stands on the beam at C, where $CB = 0.5$ m, the beam is on the point of tilting.

The boy is modelled as a particle and the beam is modelled as a uniform rod.

(a) Find the mass of the beam.

(3)

A block of mass X kg is now placed on the beam at A.

The block is modelled as a particle.

(b) Find the smallest value of X that will enable the boy to stand on the beam at B without the beam tilting.

(3)

(c) State how you have used the modelling assumption that the block is a particle in your calculations.

(1)

(a) Beam is uniform so its weight acts at G, 2.5 m from A.

AD = 5 − 1.3 = 3.7 m, so GD = 1.2 m; CD = 1.3 − 0.5 = 0.8 m

On the point of tilting about D, so reaction at A = 0.

M(D): $mg \times 1.2 = 30g \times 0.8$

$1.2m = 24$

Mass of beam = 20 kg

(b) Boy at B, block at A, beam about to tilt about D (reaction at A = 0):

M(D): $Xg \times 3.7 + 20g \times 1.2 = 30g \times 1.3$

$3.7X + 24 = 39$

$3.7X = 15$

$X = \frac{150}{37} = 4.05405...$

Smallest value of X = 4.05

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Question 3 continued

(c) The mass of the block is concentrated at the single point A, so its weight acts exactly at A, at a distance of 3.7 m from D.

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Question 3 continued

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4. At time $t = 0$, a small ball is projected vertically upwards from a point A which is 24.5 m above the ground. The ball first comes to instantaneous rest at the point B, where $AB = 19.6$ m and first hits the ground at time $t = T$ seconds.

The ball is modelled as a particle moving freely under gravity.

(a) Find the value of T.

(6)

(b) Sketch a speed-time graph for the motion of the ball from $t = 0$ to $t = T$ seconds.

(No further calculations are needed in order to draw this sketch.)

(2)

(a) Take upwards as positive, $g = 9.8\ m\ s^{-2}$

A to B: $v^2 = u^2 + 2as \Rightarrow 0 = u^2 - 2 \times 9.8 \times 19.6$

$u^2 = 384.16 \Rightarrow u = 19.6\ m\ s^{-1}$

A to ground: $s = -24.5,\ u = 19.6,\ a = -9.8,\ t = T$

$s = ut + \frac{1}{2}at^2 \Rightarrow -24.5 = 19.6T - \frac{1}{2} \times 9.8T^2$

$4.9T^2 - 19.6T - 24.5 = 0$

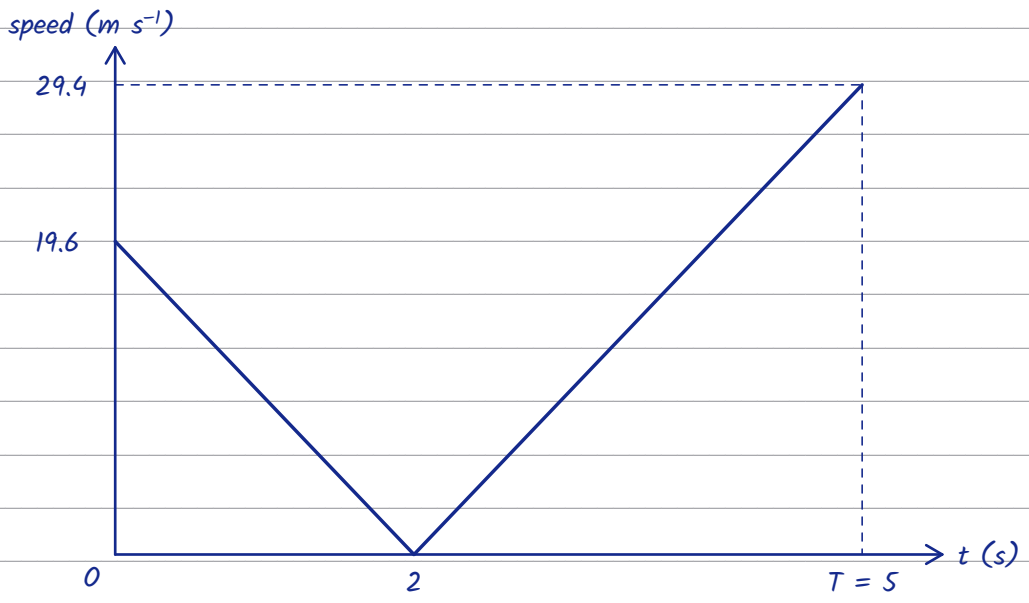
$T^2 - 4T - 5 = 0$

$(T - 5)(T + 1) = 0$

$T = 5$ ($T = -1$ rejected as $T > 0$)

$T = 5$

(b)



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Question 4 continued

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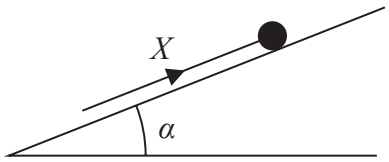


Figure 3

A particle of mass m rests in equilibrium on a fixed rough plane under the action of a force of magnitude X. The force acts up a line of greatest slope of the plane, as shown in Figure 3.

The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

The coefficient of friction between the particle and the plane is μ.

- When $X = 2P$, the particle is on the point of sliding up the plane.
- When $X = P$, the particle is on the point of sliding down the plane.

Find the value of μ.

(8)

$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \quad \cos\alpha = \frac{4}{5}$

Resolve perpendicular to the plane: $R = mg\cos\alpha = 0.8mg$

Limiting friction in both cases: $F = \mu R$

$X = 2P$, about to slide up (friction acts down the plane):

Resolve parallel to the plane: $2P = mg\sin\alpha + F$ (1)

$X = P$, about to slide down (friction acts up the plane):

Resolve parallel to the plane: $P = mg\sin\alpha - F$ (2)

(1) $- 2 \times$ (2): $0 = mg\sin\alpha + F - 2mg\sin\alpha + 2F$

$3F = mg\sin\alpha = 0.6mg \Rightarrow F = 0.2mg$

$\mu = \frac{F}{R} = \frac{0.2mg}{0.8mg}$

$\mu = 0.25$

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Question 5 continued

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6. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 2 kg moves under the action of two forces, $(p\mathbf{i} + q\mathbf{j})$N and $(2q\mathbf{i} + p\mathbf{j})$N, where p and q are constants.

Given that the acceleration of P is $(\mathbf{i} - \mathbf{j})\,\text{m s}^{-2}$

(a) find the value of p and the value of q.

(5)

(b) Find the size of the angle between the direction of the acceleration and the vector **j**.

(2)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$

At $t = T$ seconds, P is moving in the direction of the vector $(11\mathbf{i} - 13\mathbf{j})$.

(c) Find the value of T.

(5)

(a) $F = ma$: $(pi + qj) + (2qi + pj) = 2(i - j)$

i: $p + 2q = 2$

j: $q + p = -2$

Subtract: $q = 4$

$p = -2 - 4 = -6$

$p = -6, \quad q = 4$

(b) $a = i - j$ makes an angle θ with $-j$ where $\tan\theta = 1$, so $\theta = 45°$

Angle between a and $j = 180° - 45° = 135°$

(c) $v = u + at = (3i - 4j) + T(i - j)$

$v = (3 + T)i + (-4 - T)j$

v is parallel to $(11i - 13j)$:

$$\frac{3 + T}{-4 - T} = \frac{11}{-13}$$

$-13(3 + T) = 11(-4 - T)$

$-39 - 13T = -44 - 11T$

$2T = 5$

$T = 2.5$

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Question 6 continued

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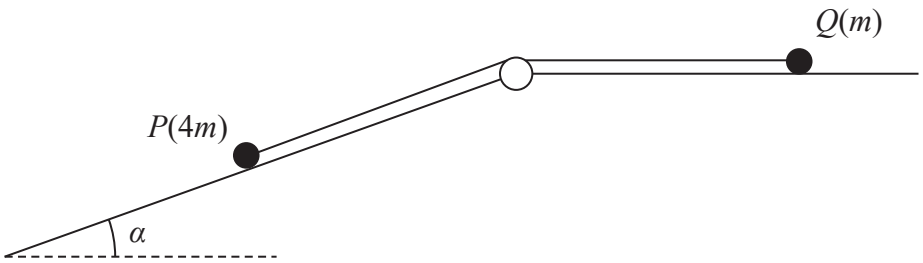


Figure 4

A particle P of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The particle P is attached to one end of a light inextensible string.

The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.

The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$

For the motion before Q reaches the pulley

(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string,

(7)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State where in your working you have used the information that the string is light.

(1)

(a) N2L for Q: $T = ma$

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Question 7 continued

(b) $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \quad \cos\alpha = \frac{4}{5}$

For P, perpendicular to the plane: $R = 4mg\cos\alpha = 3.2mg$

Friction (P moving): $F = \frac{1}{4}R = 0.8mg$

N2L for P down the plane: $4mg\sin\alpha - T - F = 4ma$

$2.4mg - T - 0.8mg = 4ma \Rightarrow 1.6mg - T = 4ma$

Using $a = T/m$ from (a): $1.6mg - T = 4T$

$5T = 1.6mg$

$T = \frac{8mg}{25} \quad (= 0.32mg)$

(c) The string pulls on the pulley with tension T horizontally towards Q and with tension T down the line of greatest slope towards P.

Horizontal component: $T - T\cos\alpha = T - 0.8T = 0.2T$

Vertical component: $T\sin\alpha = 0.6T$

Force on pulley $= \sqrt{((T - T\cos\alpha)^2 + (T\sin\alpha)^2)}$

$= T\sqrt{(0.04 + 0.36)} = T\sqrt{0.4} = \frac{8mg}{25} \times \frac{\sqrt{10}}{5}$

Magnitude of force $= \frac{8\sqrt{10}\,mg}{125} = 0.202mg \quad (= 1.98m\ N)$

(d) The tension is the same throughout the section of string between P and Q, so I used the same T in the equations of motion for P and Q.

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Question 7 continued

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Question 7 continued

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8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

A ship A moves with constant velocity $(3\mathbf{i} - 10\mathbf{j})\,\text{km h}^{-1}$

At time t hours, the position vector of A is $\mathbf{r}$ km.

At time $t = 0$, A is at the point with position vector $(13\mathbf{i} + 5\mathbf{j})$ km.

(a) Find $\mathbf{r}$ in terms of t.

(2)

Another ship B moves with constant velocity $(15\mathbf{i} + 14\mathbf{j})\,\text{km h}^{-1}$

At time $t = 0$, B is at the point with position vector $(3\mathbf{i} - 5\mathbf{j})$ km.

(b) Show that, at time t hours,

$$\overrightarrow{AB} = \left[(12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}\right]\text{km}$$

(4)

Given that the two ships do not change course,

(c) find the shortest distance between the two ships,

(6)

(d) find the bearing of ship B from ship A when the ships are closest.

(2)

(a) $r = (13i + 5j) + t(3i - 10j)$

$r = (13 + 3t)i + (5 - 10t)j$

(b) Position of B: $s = (3i - 5j) + t(15i + 14j)$

$AB = s - r$

$AB = (3 + 15t - 13 - 3t)i + (-5 + 14t - 5 + 10t)j$

$AB = [(12t - 10)i + (24t - 10)j]$ km as required

(c) $AB^2 = (12t - 10)^2 + (24t - 10)^2$

$= 144t^2 - 240t + 100 + 576t^2 - 480t + 100$

$= 720t^2 - 720t + 200$

$= 720(t^2 - t) + 200 = 720(t - 0.5)^2 - 180 + 200$

$= 720(t - 0.5)^2 + 20$

Minimum when $t = 0.5$: $AB^2 = 20$

Shortest distance $= \sqrt{20} = 2\sqrt{5} = 4.4721\ldots = 4.47$ km

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Question 8 continued

(d) At $t = 0.5$: $\overrightarrow{AB} = (6 - 10)\mathbf{i} + (12 - 10)\mathbf{j} = -4\mathbf{i} + 2\mathbf{j}$

B is 4 km west and 2 km north of A.

Angle west of north: $\tan\theta = \frac{4}{2} \Rightarrow \theta = 63.43...°$

Bearing = $360° - 63.43...° = 296.565...°$

Bearing of B from A = 297°

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Question 8 continued

Q8

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TOTAL FOR PAPER: 75 MARKS

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