Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Monday 18 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. $f(x) = x^4 + ax^3 - 3x^2 + bx + 5$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b.

(4)

(a) $f(-1) = (-1)^4 + a(-1)^3 - 3(-1)^2 + b(-1) + 5 = 4$

$1 - a - 3 - b + 5 = 4$

$3 - a - b = 4 \Rightarrow a + b = -1$ (as required)

(b) $f(2) = (2)^4 + a(2)^3 - 3(2)^2 + b(2) + 5 = -23$

$16 + 8a - 12 + 2b + 5 = -23$

$8a + 2b = -32 \Rightarrow 4a + b = -16$

$b = -1 - a \Rightarrow 4a - 1 - a = -16$

$3a = -15 \Rightarrow a = -5$

$b = -1 - (-5) = 4$

$a = -5, \quad b = 4$

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Question 1 continued

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Question 1 continued

Q1

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2. A curve has equation

$$y = x^3 - x^2 - 16x + 2$$

(a) Using calculus, find the x coordinates of the stationary points of the curve.

(4)

(b) Justify, by further calculus, the nature of all of the stationary points of the curve.

(3)

(a) $\frac{dy}{dx} = 3x^2 - 2x - 16$

Stationary points: $3x^2 - 2x - 16 = 0$

$(3x - 8)(x + 2) = 0$

$x = \frac{8}{3}$, $x = -2$

(b) $\frac{d^2y}{dx^2} = 6x - 2$

When $x = \frac{8}{3}$: $\frac{d^2y}{dx^2} = 6 \times \frac{8}{3} - 2 = 14 > 0 \Rightarrow$ minimum

When $x = -2$: $\frac{d^2y}{dx^2} = 6(-2) - 2 = -14 < 0 \Rightarrow$ maximum

So $x = \frac{8}{3}$ is a minimum and $x = -2$ is a maximum.

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Question 2 continued

Q2

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3. (i) Solve

$$7^{x+2} = 3$$

giving your answer in the form $x = \log_7 a$ where a is a rational number in its simplest form.

(3)

(ii) Using the laws of logarithms, solve

$$1 + \log_2 y + \log_2 (y + 4) = \log_2 (5 - y)$$

(5)

(i) $7^{x+2} = 3 \Rightarrow 7^2 \times 7^x = 3$

$7^x = \frac{3}{7^2} \Rightarrow x = \log_7 \frac{3}{7^2}$

$x = \log_7 \frac{3}{49}$

(ii) $1 + \log_2 y + \log_2(y + 4) = \log_2(5 - y)$

$\log_2 2 + \log_2 y + \log_2(y + 4) = \log_2(5 - y)$

$\log_2 (2y(y + 4)) = \log_2(5 - y)$

$2y(y + 4) = 5 - y$

$2y^2 + 8y = 5 - y \Rightarrow 2y^2 + 9y - 5 = 0$

$(2y - 1)(y + 5) = 0 \Rightarrow y = \frac{1}{2}$ or $y = -5$

$y = -5$ rejected as $\log_2 y$ is undefined for $y < 0$

$y = \frac{1}{2}$

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Question 3 continued

Q3

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4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(2 + px)^6$$

where p is a constant. Give each term in simplest form.

(4)

Given that in the expansion of

$$\left(3 - \frac{1}{2}x\right)(2 + px)^6$$

the coefficient of x^2 is $-\frac{3}{4}$

(b) find the possible values of p.

(4)

(a) $(2 + px)^6 = 2^6 + 6 \times 2^5(px) + \frac{6 \times 5}{2} \times 2^4(px)^2 + \ldots$

$= 64 + 6 \times 32px + 15 \times 16p^2x^2 + \ldots$

$= 64 + 192px + 240p^2x^2 + \ldots$

(b) $\left(3 - \frac{1}{2}x\right)(64 + 192px + 240p^2x^2 + \ldots)$

Coefficient of x^2: $3 \times 240p^2 + \left(-\frac{1}{2}\right) \times 192p = -\frac{3}{4}$

$720p^2 - 96p = -\frac{3}{4}$

$2880p^2 - 384p + 3 = 0$

$p = \frac{384 \pm \sqrt{(384^2 - 4 \times 2880 \times 3)}}{2 \times 2880} = \frac{384 \pm 336}{5760}$

$p = \frac{1}{8}$, $p = \frac{1}{120}$

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Question 4 continued

Q4

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5. (i) Use algebra to prove that for all $x \geqslant 0$

$$3x + 1 \geqslant 2\sqrt{3x}$$

(3)

(ii) Show that the following statement is not true.

"The sum of three consecutive prime numbers is always a multiple of 5"

(1)

(i) As $x \geq 0$, $\sqrt{(3x)}$ exists and is real, so

$(\sqrt{(3x)} - 1)^2 \geq 0$ (a square is never negative)

$(\sqrt{(3x)})^2 - 2\sqrt{(3x)} + 1 \geq 0$

$3x - 2\sqrt{(3x)} + 1 \geq 0$

$\therefore$ $3x + 1 \geq 2\sqrt{(3x)}$ for all $x \geq 0$ (as required)

(ii) 5, 7 and 11 are three consecutive prime numbers.

$5 + 7 + 11 = 23$, which is not a multiple of 5,

so the statement is not true.

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Question 5 continued

Q5

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6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta-1}=5\tan\theta \qquad \sin\theta\neq\frac{1}{2}$$

can be written in the form

$$3\sin^3\theta+10\sin^2\theta-8\sin\theta=0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4}<x<\frac{\pi}{4}$

$$\frac{3\sin 2x\cos 2x}{2\sin 2x-1}=5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

(a) $\frac{3\sin\theta\cos\theta}{2\sin\theta-1}=5\tan\theta \Rightarrow \frac{3\sin\theta\cos\theta}{2\sin\theta-1}=\frac{5\sin\theta}{\cos\theta}$

$3\sin\theta\cos^2\theta=5\sin\theta(2\sin\theta-1)$

$3\sin\theta\cos^2\theta=10\sin^2\theta-5\sin\theta$

$3\sin\theta(1-\sin^2\theta)=10\sin^2\theta-5\sin\theta$

$3\sin\theta-3\sin^3\theta=10\sin^2\theta-5\sin\theta$

$3\sin^3\theta+10\sin^2\theta-8\sin\theta=0$ (as required)

(b) With $\theta=2x$: $3\sin^3 2x+10\sin^2 2x-8\sin 2x=0$

$\sin 2x(3\sin^2 2x+10\sin 2x-8)=0$

$\sin 2x(3\sin 2x-2)(\sin 2x+4)=0$

$\sin 2x=0$, $\sin 2x=\frac{2}{3}$, $\sin 2x=-4$ (no solutions)

$\frac{-\pi}{4}<x<\frac{\pi}{4} \Rightarrow \frac{-\pi}{2}<2x<\frac{\pi}{2}$

$\sin 2x=0 \Rightarrow 2x=0 \Rightarrow x=0$

$\sin 2x=\frac{2}{3} \Rightarrow 2x=0.72972\ldots \Rightarrow x=0.36486\ldots$

$x=0$, $x=0.365$ (3 d.p.)

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Question 6 continued

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7.

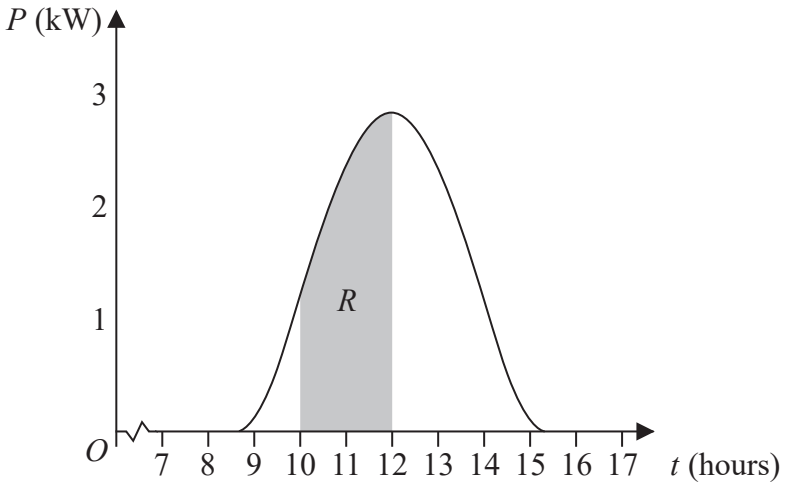


Figure 1

Solar panels are installed on the roof of a building.

The power, P, produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \qquad 8.5 \leqslant t \leqslant 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)	1.2	1.882	2.45	2.821	2.95

(a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R, shown shaded in Figure 1.

(b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

(a) $t = 10$: $P = 0.95 + 2^{-2} + 2^{2} - (-2)^2 = 1.2$

$t = 11.5$: $P = 0.95 + 2^{-0.5} + 2^{0.5} - (-0.5)^2 = 2.82132\ldots = 2.821$

(b) $h = 0.5$

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Question 7 continued

$E \approx \frac{1}{2} \times 0.5 \times [1.2 + 2.95 + 2(1.882 + 2.45 + 2.821)]$

$= 0.25 \times [4.15 + 2(7.153)]$

$= 0.25 \times 18.456 = 4.614$

$E \approx 4.61$ kWh

Q7

(Total 6 marks)

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8. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$
$$a_1 = p - 3$$

where p is a constant.

(a) Find an expression for a_2 in terms of p, giving your answer in simplest form.

(1)

Given that $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of a_2

(6)

(a) $a_2 = 2(p - 3 + 3)^2 - 7 = 2p^2 - 7$

(b) $a_3 = 2(2p^2 - 7 + 3)^2 - 7 = 2(2p^2 - 4)^2 - 7$

$= 2(4p^4 - 16p^2 + 16) - 7 = 8p^4 - 32p^2 + 25$

$a_1 + a_2 + a_3 = p + 15$

$p - 3 + 2p^2 - 7 + 8p^4 - 32p^2 + 25 = p + 15$

$8p^4 - 30p^2 + p + 15 = p + 15$

$8p^4 - 30p^2 = 0 \Rightarrow 2p^2(4p^2 - 15) = 0$

$p^2 = 0$ or $p^2 = \frac{15}{4}$

$p^2 = 0$: $a_2 = 2(0) - 7 = -7$

$p^2 = \frac{15}{4}$: $a_2 = 2 \times \frac{15}{4} - 7 = \frac{1}{2}$

Possible values: $a_2 = -7$, $a_2 = \frac{1}{2}$

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Question 8 continued

Q8

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9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k < 2$

(c) calculate the exact area of triangle OPT.

(5)

(a) (i) Centre $(k, 2k)$

(ii) Radius $= \sqrt{(k + 7)}$

(b) (i) $(2 - k)^2 + (3 - 2k)^2 = k + 7$

$4 - 4k + k^2 + 9 - 12k + 4k^2 = k + 7$

$5k^2 - 16k + 13 = k + 7$

$5k^2 - 17k + 6 = 0$ (as required)

(ii) $(5k - 2)(k - 3) = 0 \Rightarrow k = \frac{2}{5}, \quad k = 3$

(c) $k < 2 \Rightarrow k = \frac{2}{5}$, centre $\left(\frac{2}{5}, \frac{4}{5}\right)$

Gradient of radius to P $= \frac{3 - 0.8}{2 - 0.4} = \frac{2.2}{1.6} = \frac{11}{8}$

Gradient of tangent $= -\frac{8}{11}$

Tangent: $y - 3 = -\frac{8}{11}(x - 2)$

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Question 9 continued

$y = 0$: $-3 = -\frac{8}{11}(x - 2) \Rightarrow x - 2 = \frac{33}{8} \Rightarrow x = \frac{49}{8}$

$T\left(\frac{49}{8}, 0\right)$, height of triangle OPT = 3 (y-coordinate of P)

Area OPT $= \frac{1}{2} \times \frac{49}{8} \times 3 = \frac{147}{16}$

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Question 9 continued

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Question 9 continued

Q9

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10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan A.

(3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan B.
Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(a) Arithmetic: $a = 15$, $u_{12} = a + 11d = 37$

$d = \frac{37 - 15}{11} = 2$

$u_5 = a + 4d = 15 + 4 \times 2 = 23$

Week 5 Sunday: 23 km

(b) Geometric: $a = 15$, $ar^{11} = 37 \Rightarrow r^{11} = \frac{37}{15}$

$r = \left(\frac{37}{15}\right)^{1/11} = 1.085541...$

$u_5 = ar^4 = 15 \times 1.085541...^4 = 20.829...$

Week 5 Sunday: 20.8 km (1 d.p.)

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Question 10 continued

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(c) (i) Plan A: $S_{12} = \frac{12}{2}[2 \times 15 + (12 - 1) \times 2] = 6 \times 52 = 312$

Wednesdays: $12x$, so $312 + 12x \leq 360$

$12x \leq 48 \Rightarrow x \leq \frac{360 - 312}{12} = 4$

Maximum value of $x = 4$

(ii) Plan B: $S_{12} = \frac{15(1.085541...^{12} - 1)}{1.085541... - 1} = 294.185...$

$294.185... + 12x \leq 360$

$x \leq \frac{360 - 294.185...}{12} = 5.484...$

x is an integer, so maximum value of $x = 5$

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Question 10 continued

Q10

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

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