Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Monday 11 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. A curve has equation

$$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

The point P lies on the curve and has x coordinate $\frac{1}{2}$

(b) Find an equation of the normal to the curve at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(a) $y = 2x^3 - 5x^2 - \frac{3}{2}x^{-1} + 7$

$\frac{dy}{dx} = 6x^2 - 10x + \frac{3}{2}x^{-2} = 6x^2 - 10x + \frac{3}{2x^2}$

(b) $x = \frac{1}{2}$: $y = 2(\frac{1}{8}) - 5(\frac{1}{4}) - \frac{3}{2 \times 0.5} + 7$

$y = 0.25 - 1.25 - 3 + 7 = 3$ $\quad P(0.5, 3)$

$\frac{dy}{dx} = 6(\frac{1}{4}) - 10(\frac{1}{2}) + \frac{3}{2 \times 0.25} = 1.5 - 5 + 6 = \frac{5}{2}$

Gradient of normal $= -1 \div \frac{5}{2} = -\frac{2}{5}$

$y - 3 = -\frac{2}{5}(x - \frac{1}{2})$

$5y - 15 = -2x + 1$

$2x + 5y - 16 = 0$

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Question 1 continued

Q1

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2. A tree was planted.

Exactly 3 years after it was planted, the height of the tree was 2 m.

Exactly 5 years after it was planted, the height of the tree was 2.4 m.

Given that the height, H metres, of the tree, t years after it was planted, can be modelled by the equation

$$H^3 = pt^2 + q$$

where p and q are constants,

(a) find, to 3 significant figures where necessary, the value of p and the value of q.

(4)

Exactly T years after the tree was planted, its height was 5 m.

(b) Find the value of T according to the model, giving your answer to one decimal place.

(2)

(a) $t = 3, H = 2$: $2^3 = p \times 3^2 + q \Rightarrow 9p + q = 8$

$t = 5, H = 2.4$: $2.4^3 = p \times 5^2 + q \Rightarrow 25p + q = 13.824$

Subtract: $16p = 13.824 - 8 = 5.824$

$p = 0.364$

$q = 8 - 9 \times 0.364 = 8 - 3.276 = 4.724$

$p = 0.364$, $q = 4.72$ (3 s.f.)

(b) $H = 5$: $5^3 = 0.364T^2 + 4.724$

$125 - 4.724 = 0.364T^2 \Rightarrow T^2 = 330.42...$

$T = \sqrt{\left(\frac{125 - 4.724}{0.364}\right)} = 18.177...$

$T = 18.2$ (years)

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Question 2 continued

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Question 2 continued

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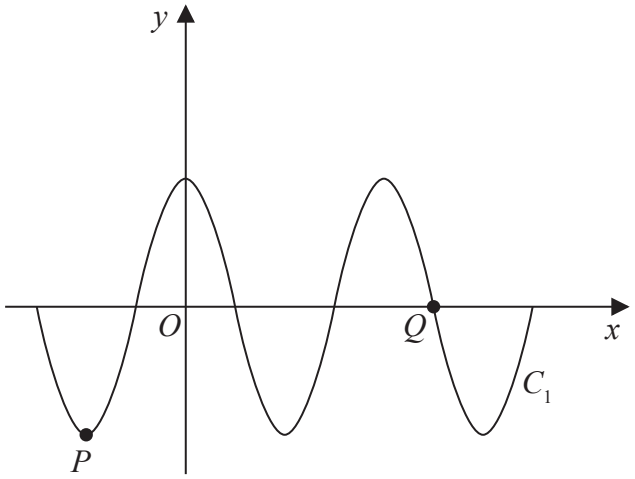


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation $y = 4\cos x°$

The point P and the point Q lie on C_1 and are shown in Figure 1.

(a) State

(i) the coordinates of P,

(ii) the coordinates of Q.

(3)

The curve C_2 has equation $y = 4\cos x° + k$, where k is a constant.

Curve C_2 has a minimum y value of -1

The point R is the maximum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

(2)

(a) (i) P is the minimum to the left of O: P (−180, −4)

(ii) Q is where the curve crosses after the max at 360:

Q (450, 0)

(b) Minimum of $C_2 = -4 + k = -1 \Rightarrow k = 3$

Maximum value = 4 + 3 = 7, first positive max at x = 360

R (360, 7)

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Question 3 continued

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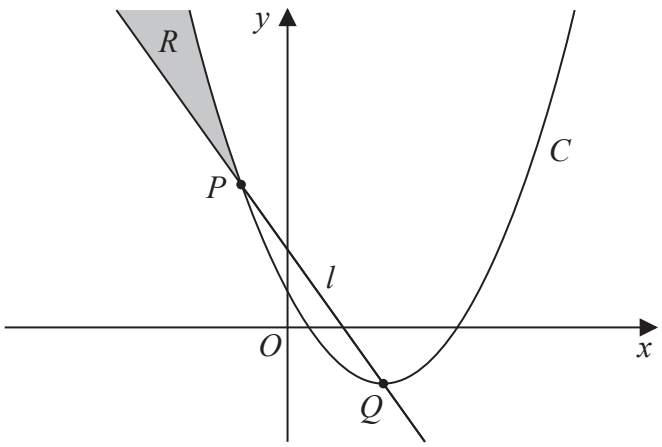


Figure 2

The points P and Q, as shown in Figure 2, have coordinates $(-2, 13)$ and $(4, -5)$ respectively.

The straight line l passes through P and Q.

(a) Find an equation for l, writing your answer in the form $y = mx + c$, where m and c are integers to be found.

(3)

The quadratic curve C passes through P and has a minimum point at Q.

(b) Find an equation for C.

(3)

The region R, shown shaded in Figure 2, lies in the second quadrant and is bounded by C and l only.

(c) Use inequalities to define region R.

(2)

(a) Gradient of $l = \frac{-5 - 13}{4 - (-2)} = \frac{-18}{6} = -3$

$y - 13 = -3(x + 2) \Rightarrow y - 13 = -3x - 6$

$y = -3x + 7$

(b) Minimum at Q $(4, -5)$: $y = a(x - 4)^2 - 5$

P $(-2, 13)$: $13 = a(-2 - 4)^2 - 5 \Rightarrow 36a = 18$

$a = \frac{1}{2}$

$y = \frac{1}{2}(x - 4)^2 - 5 \quad \left(y = \frac{1}{2}x^2 - 4x + 3\right)$

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Question 4 continued

(c) R is above l, below C and to the left of P:

$y > -3x + 7, \quad y < \frac{1}{2}(x - 4)^2 - 5, \quad x < -2$

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Question 4 continued

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Question 4 continued

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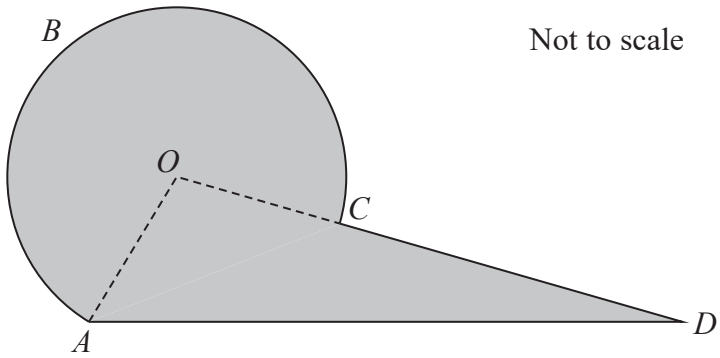


Figure 3

Figure 3 shows the plan view of a viewing platform at a tourist site.

The shape of the viewing platform consists of a sector $ABCOA$ of a circle, centre O, joined to a triangle AOD.

Given that

- $OA = OC = 6\,\text{m}$
- $AD = 14\,\text{m}$
- angle $ADC = 0.43$ radians
- angle AOD is an obtuse angle
- OCD is a straight line

find

(a) the size of angle AOD, in radians, to 3 decimal places,

(3)

(b) the length of arc ABC, in metres, to one decimal place,

(2)

(c) the total area of the viewing platform, in m^2, to one decimal place.

(4)

(a) $\frac{\sin \alpha}{14} = \frac{\sin 0.43}{6}$ where α = angle AOD

$\sin \alpha = \frac{14 \sin 0.43}{6} = 0.97269...$

$\alpha = 1.3366...$ (acute) but AOD is obtuse:

angle AOD $= \pi - 1.3366... = 1.8050...$

angle AOD = 1.805 radians

(b) Reflex angle AOC $= 2\pi - 1.8050... = 4.4782...$

Arc ABC $= r\theta = 6 \times 4.4782... = 26.869...$

Arc ABC = 26.9 m

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Question 5 continued

(c) Sector area $= \frac{1}{2} r^2\theta = \frac{1}{2} \times 6^2 \times 4.4782... = 80.607...$

angle OAD $= \pi - 0.43 - 1.8050... = 0.90659...$

Area AOD $= \frac{1}{2} \times 6 \times 14 \times \sin 0.90659... = 33.071...$

Total area $= 80.607... + 33.071... = 113.678...$

Total area $= 113.7\ m^2$

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Question 5 continued

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Question 5 continued

Q5

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6. (a) Sketch the curve with equation

$$y = -\frac{k}{x} \qquad k > 0 \qquad x \neq 0$$

(2)

(b) On a separate diagram, sketch the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

stating the coordinates of the point of intersection with the x-axis and, in terms of k, the equation of the horizontal asymptote.

(3)

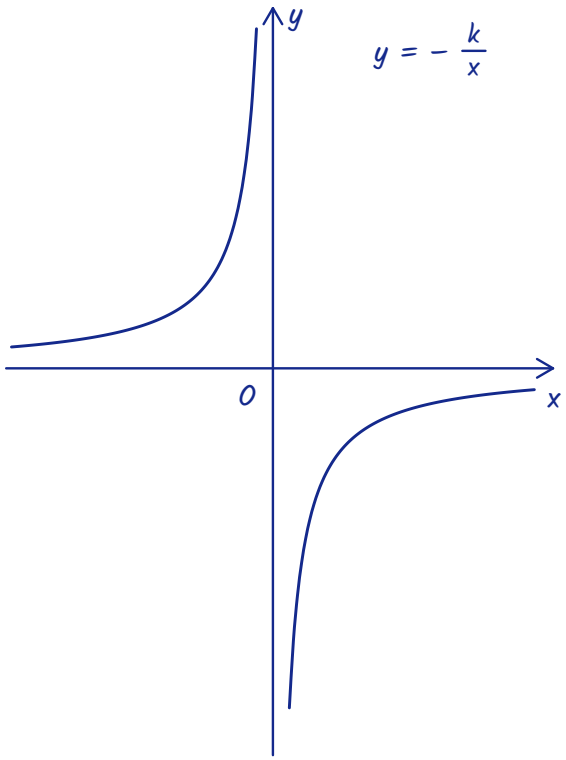
(c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

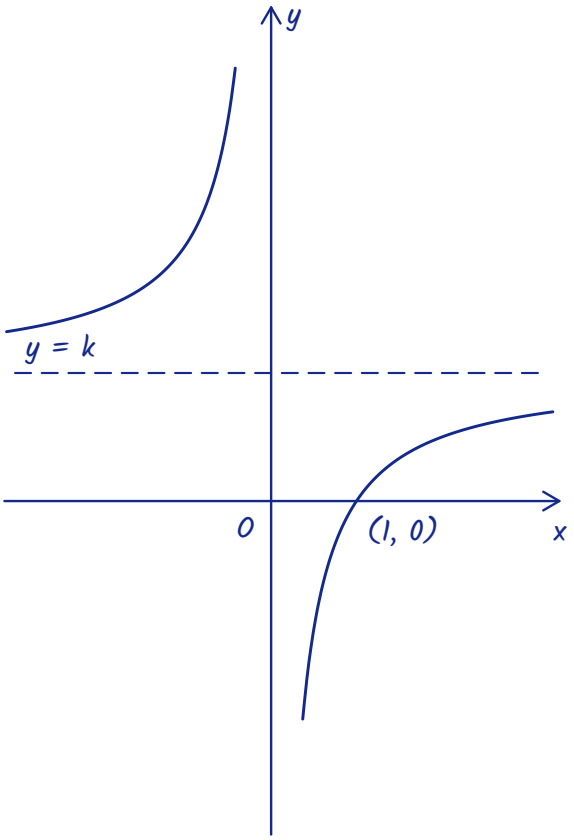
does not touch or intersect the line with equation $y = 3x + 4$

(5)

(a)



(b)



Asymptotes: $y = k$ and $x = 0$

Crosses x-axis at (1, 0)

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Question 6 continued

(c) $3x + 4 = -\frac{k}{x} + k \qquad (\times x)$

$3x^2 + 4x = -k + kx$

$3x^2 + (4 - k)x + k = 0$

Does not touch or intersect: $b^2 - 4ac < 0$

$(4 - k)^2 - 4 \times 3 \times k < 0$

$16 - 8k + k^2 - 12k < 0 \quad \Rightarrow \quad k^2 - 20k + 16 < 0$

Critical values: $k = \frac{20 \pm \sqrt{(400 - 64)}}{2} = \frac{20 \pm \sqrt{336}}{2} = 10 \pm 2\sqrt{21}$

Quadratic in k is negative between the roots:

$10 - 2\sqrt{21} < k < 10 + 2\sqrt{21}$

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Question 6 continued

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Question 6 continued

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7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x - 3\sqrt{x} - 5 \qquad x > 0$$

(a) Solve the equation

$$f(x) = 9$$

(4)

(b) Solve the equation

$$f''(x) = 6$$

(5)

(a) $2x - 3\sqrt{x} - 5 = 9 \Rightarrow 2x - 3\sqrt{x} - 14 = 0$

$(2\sqrt{x} - 7)(\sqrt{x} + 2) = 0$

$\sqrt{x} = \frac{7}{2}$ ($\sqrt{x} = -2$ is impossible)

$x = (\frac{7}{2})^2 = \frac{49}{4}$

(b) $f(x) = 2x - 3x^{1/2} - 5 \Rightarrow f'(x) = 2 - \frac{3}{2}x^{-1/2}$

$f''(x) = \frac{3}{4}x^{-3/2}$

$\frac{3}{4}x^{-3/2} = 6 \Rightarrow x^{-3/2} = 8$

$x^{3/2} = \frac{1}{8} \Rightarrow x = (\frac{1}{8})^{2/3}$

$x = \frac{1}{4}$

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Question 7 continued

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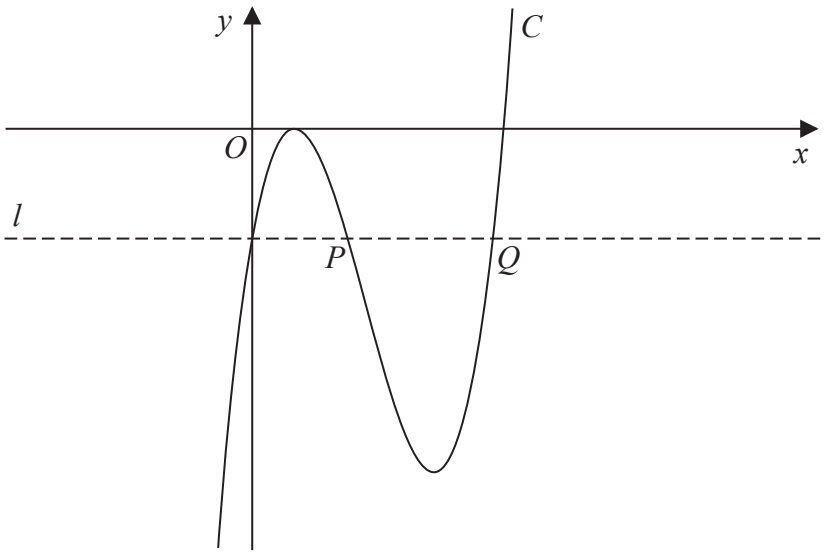


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = f(x)$, where

$$f(x) = (3x - 2)^2 (x - 4)$$

(a) Deduce the values of x for which $f(x) > 0$

(1)

(b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

The line l, also shown in Figure 4, passes through the y intercept of C and is parallel to the x-axis.

The line l cuts C again at points P and Q, also shown in Figure 4.

(c) Using algebra and showing your working, find the length of line PQ. Write your answer in the form $k\sqrt{3}$, where k is a constant to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

(a) $x > 4$

(b) $(3x - 2)^2 = 9x^2 - 12x + 4$

$(9x^2 - 12x + 4)(x - 4) = 9x^3 - 36x^2 - 12x^2 + 48x + 4x - 16$

$f(x) = 9x^3 - 48x^2 + 52x - 16$

(c) y intercept: $f(0) = -16$, so l is $y = -16$

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Question 8 continued

$9x^3 - 48x^2 + 52x - 16 = -16 \Rightarrow 9x^3 - 48x^2 + 52x = 0$

$x(9x^2 - 48x + 52) = 0$, $x = 0$ is the y intercept

$$x = \frac{48 \pm \sqrt{(48^2 - 4 \times 9 \times 52)}}{2 \times 9} = \frac{48 \pm \sqrt{432}}{18} = \frac{16 \pm 4\sqrt{3}}{6}$$

$$PQ = \frac{16 + 4\sqrt{3}}{6} - \frac{16 - 4\sqrt{3}}{6} = \frac{8\sqrt{3}}{6}$$

$$PQ = \frac{4}{3}\sqrt{3}$$

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Question 8 continued

Q8

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9. (i) Find

$$\int \frac{(3x+2)^2}{4\sqrt{x}}\,dx \qquad x > 0$$

giving your answer in simplest form.

(5)

(ii) A curve C has equation $y = f(x)$.

Given

- $f'(x) = x^2 + ax + b$ where a and b are constants
- the y intercept of C is -8
- the point $P(3,-2)$ lies on C
- the gradient of C at P is 2

find, in simplest form, $f(x)$.

(6)

(i) $\frac{(3x+2)^2}{4\sqrt{x}} = \frac{9x^2+12x+4}{4x^{1/2}} = \frac{9}{4}x^{3/2} + 3x^{1/2} + x^{-1/2}$

$\int \frac{(3x+2)^2}{4\sqrt{x}}\,dx = \frac{9}{4}\times\frac{2}{5}x^{5/2} + 3\times\frac{2}{3}x^{3/2} + 2x^{1/2} + c$

$= \frac{9}{10}x^{5/2} + 2x^{3/2} + 2x^{1/2} + c$

(ii) $f'(3) = 2$: $9 + 3a + b = 2 \Rightarrow 3a + b = -7$

$f(x) = \frac{1}{3}x^3 + \frac{1}{2}ax^2 + bx + c$, y intercept $-8 \Rightarrow c = -8$

$f(3) = -2$: $-2 = 9 + \frac{9}{2}a + 3b - 8 \Rightarrow 9a + 6b = -6$

$3a + 2b = -2$ and $3a + b = -7$

Subtract: $b = 5$, $3a = -7 - 5 \Rightarrow a = -4$

$f(x) = \frac{1}{3}x^3 - 2x^2 + 5x - 8$

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Question 9 continued

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Question 9 continued

Q9

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