Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Tuesday 19 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. A small stone is projected vertically upwards with speed $20\,\text{m s}^{-1}$ from a point O which is 5 m above horizontal ground. The stone is modelled as a particle moving freely under gravity.

Find

(a) the speed of the stone at the instant when it is 2 m above the ground,

(2)

(b) the total time between the instant when the stone is projected from O and the instant when it first strikes the ground.

(4)

(a) Take upwards as positive. At 2 m above ground, $s = -3$

$v^2 = u^2 + 2as = 20^2 + 2(-9.8)(-3)$

$v^2 = 400 + 58.8 = 458.8$

$v = 21.4196\ldots = 21.4\ \text{m s}^{-1}$

(b) Ground is 5 m below O: $s = -5$, $u = 20$, $a = -9.8$

$s = ut + \frac{1}{2}at^2 \Rightarrow -5 = 20t - 4.9t^2$

$4.9t^2 - 20t - 5 = 0$

$t = \frac{20 \pm \sqrt{(20^2 + 4 \times 4.9 \times 5)}}{2 \times 4.9} = \frac{20 \pm \sqrt{498}}{9.8}$

$t = 4.3179\ldots$ or $t = -0.236\ldots$ (reject, $t > 0$)

Total time = 4.32 s

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Question 1 continued

Q1

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2. Two particles, P and Q, have masses $2m$ and m respectively. The particles are moving towards each other in opposite directions along the same straight line on a smooth horizontal plane. The particles collide directly.

Immediately before the collision, the speed of P is $3u$ and the speed of Q is $2u$.

The magnitude of the impulse exerted on Q by P in the collision is $5mu$.

Find

(a) the speed of P immediately after the collision,

(3)

(b) the speed of Q immediately after the collision.

(3)

(a) Take the direction of motion of P before impact as positive.

Impulse on P = $-5mu$ (equal and opposite to impulse on Q)

For P: $-5mu = 2m(v - 3u)$

$v - 3u = -\frac{5u}{2}$

$v = \frac{1}{2}u$, so the speed of P is $\frac{1}{2}u$

(b) For Q: $5mu = m(w - (-2u))$

$5u = w + 2u$

$w = 3u$, so the speed of Q is $3u$

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Question 2 continued

Q2

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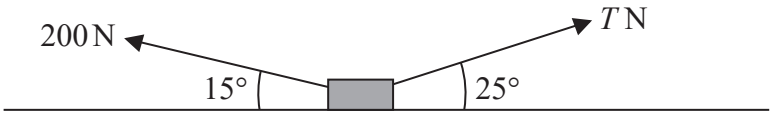


Figure 1

A parcel of mass 20 kg is at rest on a rough horizontal floor. The coefficient of friction between the parcel and the floor is 0.3

Two forces, both acting in the same vertical plane, of magnitudes 200 N and T N are applied to the parcel. The line of action of the 200 N force makes an angle of 15° with the horizontal and the line of action of the T N force makes an angle of 25° with the horizontal, as shown in Figure 1. The parcel is modelled as a particle P.

Find the smallest value of T for which P remains in equilibrium.

(9)

For the smallest T, P is about to slip in the direction of the 200 N force, so friction F acts towards the T force and is limiting.

(↑): $R + 200 \sin 15° + T \sin 25° = 20g$

$R = 196 - 51.7638... - 0.42261...T = 144.2361... - 0.42261...T$

(←): $200 \cos 15° - T \cos 25° - F = 0$

Limiting friction: $F = 0.3R$

$193.1851... - 0.90630...T - 0.3(144.2361... - 0.42261...T) = 0$

$193.1851... - 43.2708... = 0.90630...T - 0.12678...T$

$149.9143... = 0.77952...T$

$T = 192.315...$

Smallest T = 192 (N)

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Question 3 continued

Q3

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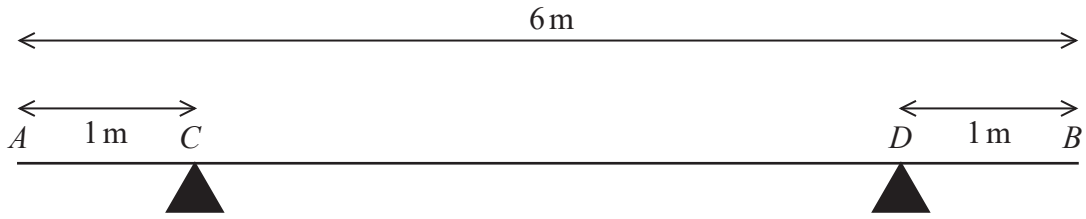


Figure 2

A metal girder AB has weight W newtons and length 6 m. The girder rests in a horizontal position on two supports C and D where $AC = DB = 1$ m, as shown in Figure 2.

When a force of magnitude 900 N is applied vertically upwards to the girder at A, the girder is about to tilt about D.

When a force of magnitude 1500 N is applied vertically upwards to the girder at B, the girder is about to tilt about C.

The girder is modelled as a non-uniform rod whose centre of mass is a distance x metres from A.

Find the value of x.

(6)

900 N at A, about to tilt about D, so reaction at C = 0:

M(D): $900 \times 5 = W(5 - x)$ (1)

1500 N at B, about to tilt about C, so reaction at D = 0:

M(C): $1500 \times 5 = W(x - 1)$ (2)

(1) ÷ (2): $\frac{4500}{7500} = \frac{5 - x}{x - 1} \Rightarrow \frac{3}{5} = \frac{5 - x}{x - 1}$

$3(x - 1) = 5(5 - x)$

$3x - 3 = 25 - 5x$

$8x = 28$

$x = 3.5$

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Question 4 continued

Q4

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5. A particle is acted upon by two forces **F** and **G**. The force **F** has magnitude 8 N and acts in a direction with a bearing of 240°. The force **G** has magnitude 10 N and acts due South.

Given that $\mathbf{R} = \mathbf{F} + \mathbf{G}$, find

(i) the magnitude of **R**,

(ii) the direction of **R**, giving your answer as a bearing to the nearest degree.

(7)

(i) Angle between the directions of F (240°) and G (180°) is 60°.

Components of R: West: $8 \sin 60° = 4\sqrt{3} = 6.928...$

South: $10 + 8 \cos 60° = 14$

$|R|^2 = (8 \sin 60°)^2 + (10 + 8 \cos 60°)^2 = 48 + 196 = 244$

$|R| = \sqrt{244} = 15.6204... = 15.6$ N

(ii) Let α be the angle between R and due South (towards West).

$$\tan \alpha = \frac{8 \sin 60°}{10 + 8 \cos 60°} = \frac{4\sqrt{3}}{14} = 0.49487...$$

$\alpha = 26.3295...°$

Bearing $= 180° + 26.33° = 206.33...°$

Bearing of R = 206°

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Question 5 continued

Q5

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6. Two girls, Agatha and Brionie, are roller skating inside a large empty building. The girls are modelled as particles.

At time $t = 0$, Agatha is at the point with position vector $(11\mathbf{i} + 11\mathbf{j})$ m and Brionie is at the point with position vector $(7\mathbf{i} + 16\mathbf{j})$ m. The position vectors are given relative to the door, O, and $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors.

Agatha skates with constant velocity $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

Brionie skates with constant velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the position vector of Agatha at time t seconds.

(2)

At time $t = 6$ seconds, Agatha passes through the point P.

(b) Show that Brionie also passes through P and find the value of t when this occurs.

(4)

At time t seconds, Agatha is at the point A and Brionie is at the point B.

(c) Show that $\overrightarrow{AB} = [(t - 4)\mathbf{i} + (5 - t)\mathbf{j}]$ m

(2)

(d) Find the distance between the two girls when they are closest together.

(4)

(a) $r_A = (11i + 11j) + t(3i - j)$

$r_A = (11 + 3t)i + (11 - t)j$ m

(b) $t = 6$: $r_A = (11 + 18)i + (11 - 6)j = 29i + 5j$, so P is $(29i + 5j)$

$r_B = (7i + 16j) + t(4i - 2j) = (7 + 4t)i + (16 - 2t)j$

Brionie at P: $(7 + 4t)i + (16 - 2t)j = 29i + 5j$

i: $7 + 4t = 29 \Rightarrow 4t = 22 \Rightarrow t = 5.5$

j: $16 - 2t = 5 \Rightarrow 2t = 11 \Rightarrow t = 5.5$

Both components give $t = 5.5$, so Brionie also passes through P, when $t = 5.5$

(c) $AB = r_B - r_A$

$AB = \{(7i + 16j) + t(4i - 2j)\} - \{(11i + 11j) + t(3i - j)\}$

$AB = (7 + 4t - 11 - 3t)i + (16 - 2t - 11 + t)j$

$AB = [(t - 4)i + (5 - t)j]$ m

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Question 6 continued

(d) $AB^2 = (t - 4)^2 + (5 - t)^2$

$= t^2 - 8t + 16 + 25 - 10t + t^2$

$= 2t^2 - 18t + 41$

$= 2(t - 4.5)^2 - 40.5 + 41$

$= 2(t - 4.5)^2 + 0.5$

Minimum when $t = 4.5$: $AB^2 = 0.5$

Minimum distance $= \sqrt{0.5} = 0.7071\ldots = 0.71$ m

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Question 6 continued

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Question 6 continued

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Q6

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7. A helicopter is hovering at rest above horizontal ground at the point H. A parachutist steps out of the helicopter and immediately falls vertically and freely under gravity from rest for 2.5 s. His parachute then opens and causes him to immediately decelerate at a constant rate of $3.9\,\text{m s}^{-2}$ for T seconds ($T < 6$), until his speed is reduced to $V\,\text{m s}^{-1}$. He then moves with this constant speed $V\,\text{m s}^{-1}$ until he hits the ground. While he is decelerating, he falls a distance of 73.75 m. The total time between the instant when he leaves H and the instant when he hits the ground is 20 s.

The parachutist is modelled as a particle.

(a) Find the speed of the parachutist at the instant when his parachute opens.

(1)

(b) Sketch a speed-time graph for the motion of the parachutist from the instant when he leaves H to the instant when he hits the ground.

(2)

(c) Find the value of T.

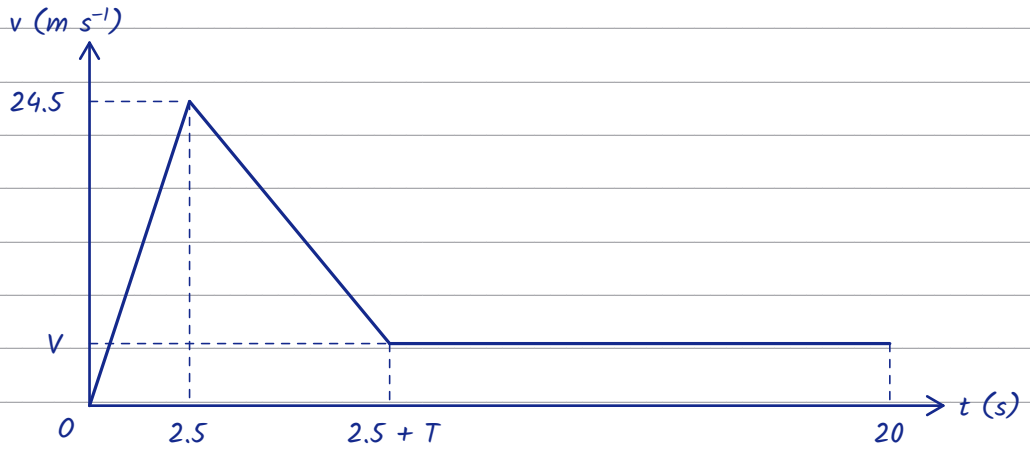
(5)

(d) Find, to the nearest metre, the height of the point H above the ground.

(4)

(a) $v = u + at = 0 + 9.8 \times 2.5 = 24.5\ \text{m s}^{-1}$

(b)



(c) Deceleration phase: $u = 24.5$, $a = -3.9$, $t = T$, $s = 73.75$

$s = ut + \frac{1}{2}at^2 \Rightarrow 73.75 = 24.5T - \frac{1}{2} \times 3.9T^2$

$73.75 = 24.5T - 1.95T^2$

$1.95T^2 - 24.5T + 73.75 = 0$

$39T^2 - 490T + 1475 = 0$

$(T - 5)(39T - 295) = 0$

$T = 5$ or $T = \frac{295}{39} = 7.56...$ (reject since $T < 6$)

$T = 5$

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Question 7 continued

(d) $V = 24.5 - 3.9 \times 5 = 5 \text{ m s}^{-1}$

Constant speed for $20 - 2.5 - 5 = 12.5$ s

Height = total area under the graph

$= \frac{1}{2} \times 2.5 \times 24.5 + 73.75 + 12.5 \times 5$

$= 30.625 + 73.75 + 62.5 = 166.875$

Height of H = 167 m (nearest metre)

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Question 7 continued

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Question 7 continued

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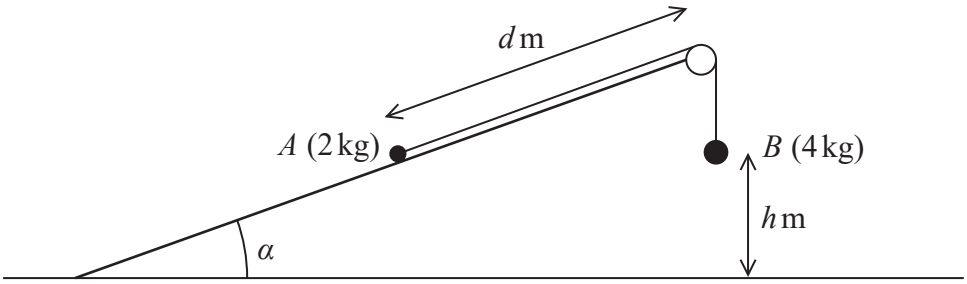


Figure 3

Two particles, A and B, have masses 2 kg and 4 kg respectively. The particles are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal ground at an angle α where $\tan\alpha = \frac{3}{4}$. The particle A is held at rest on the plane at a distance d metres from the pulley. The particle B hangs freely at rest, vertically below the pulley, at a distance h metres above the ground, as shown in Figure 3. The part of the string between A and the pulley is parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{4}$

The system is released from rest with the string taut and B descends.

(a) Find the tension in the string as B descends.

(9)

On hitting the ground, B immediately comes to rest.

Given that A comes to rest before reaching the pulley,

(b) find, in terms of h, the range of possible values of d.

(7)

(c) State one physical factor, other than air resistance, that could be taken into account to make the model described above more realistic.

(1)

(a) $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$

For A, perpendicular to the plane: $R = 2g\cos\alpha = 1.6g$

Friction (limiting, A moving): $F = 0.25R = 0.4g$

N2L for A up the plane: $T - 2g\sin\alpha - F = 2a$

$T - 1.2g - 0.4g = 2a \Rightarrow T - 1.6g = 2a$ (1)

N2L for B downwards: $4g - T = 4a$ (2)

$2 \times (1)$: $2T - 3.2g = 4a$

$2T - 3.2g = 4g - T$

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Question 8 continued

$3T = 7.2g$

$T = 2.4g = \frac{12g}{5} = 23.52$

Tension = 23.5 N

(b) From (2): $4g - 2.4g = 4a \Rightarrow a = 0.4g$

Speed of A and B when B hits the ground (B falls h):

$v^2 = 0 + 2(0.4g)h \Rightarrow v^2 = \frac{4gh}{5}$

After B stops the string is slack. N2L for A up the plane:

$-2g \sin\alpha - F = 2a' \Rightarrow -1.2g - 0.4g = 2a'$

$a' = -0.8g$

A comes to rest after a further distance s:

$0 = v^2 + 2a's \Rightarrow 0 = \frac{4gh}{5} - 1.6gs$

$s = \frac{4gh}{5 \times 1.6g} = \frac{1}{2}h$

Total distance moved by A $= h + \frac{1}{2}h = \frac{3}{2}h$

A stops before reaching the pulley, so

$d > \frac{3}{2}h \quad (d > 1.5h)$

(c) Friction at the pulley (the pulley may not be smooth).

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

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TOTAL FOR PAPER: 75 MARKS

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