Pearson Edexcel International Advanced Level

Wednesday 20 January 2021

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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Write your answers in the D1 answer book for this paper.

1. Use the binary search algorithm to try to locate the word "Parallelogram" in the following alphabetical list. Clearly indicate how you choose your pivots and which part of the list you are rejecting at each stage.

Arc

Centre

Chord

Circle

Circumference

Diameter

Radius

Sector

Segment

Tangent

(Total 4 marks)

2. A restaurant sells two sizes of pizza, small and large. The restaurant owner knows that, each evening, she needs to make

- at least 85 pizzas in total
- at least twice as many large pizzas as small pizzas

In addition, at most 80% of the pizzas must be large.

Each small pizza costs £2 to make and each large pizza costs £3 to make.

The restaurant owner wants to minimise her costs.

Let x represent the number of small pizzas made each evening and let y represent the number of large pizzas made each evening.

Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients. You should **not** attempt to solve the problem.

(Total 5 marks)

3. 2.6 0.8 2.1 1.2 0.9 1.7 2.3 0.3 1.8 2.7

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

(3)

The list is to be sorted into **descending** order.

(b) (i) Starting at the left-hand end of the above list, perform **two** passes through the list using a bubble sort. Write down the lists that result at the end of the first pass and the second pass.

(ii) Write down, in the table in the answer book, the number of comparisons and the number of swaps performed during each of these two passes.

(4)

After a third pass using this bubble sort, the updated list is

2.6 2.1 1.7 2.3 1.2 1.8 2.7 0.9 0.8 0.3

(c) Use a quick sort on this updated list to obtain the fully sorted list. You must make your pivots clear.

(3)

(d) Apply the first-fit decreasing bin packing algorithm to the fully sorted list to pack the numbers into bins of size 5

(3)

(Total 13 marks)

4. (a) Explain the difference between the classical and the practical travelling salesperson problems.

(2)

The table below shows the distances, in km, between seven museums, A, B, C, D, E, F and G.

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	–	25	31	28	35	30	32
B	25	–	34	24	27	32	39
C	31	34	–	40	35	27	29
D	28	24	40	–	37	35	36
E	35	27	35	37	–	28	31
F	30	32	27	35	28	–	33
G	32	39	29	36	31	33	–

Fran must visit each museum. She will start and finish at A and wishes to minimise the total distance travelled.

(b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the length of Fran's route. Make your method clear.

(2)

Starting at D, a second upper bound of 203 km was found.

(c) State whether this is a better upper bound than the answer to (b), giving a reason for your answer.

(1)

A reduced network is formed by deleting G and all the arcs that are directly joined to G.

(d) (i) Use Prim's algorithm, starting at A, to construct a minimum spanning tree for the reduced network. You must clearly state the order in which you select the arcs of your tree.

(ii) Hence calculate a lower bound for the length of Fran's route.

(4)

By deleting A, a second lower bound was found to be 188 km.

(e) State whether this is a better lower bound than the answer to (d)(ii), giving a reason for your answer.

(1)

(f) Using only the results from (c) and (e), write down the smallest interval that you can be confident contains the length of Fran's optimal route.

(2)

(Total 12 marks)

5.

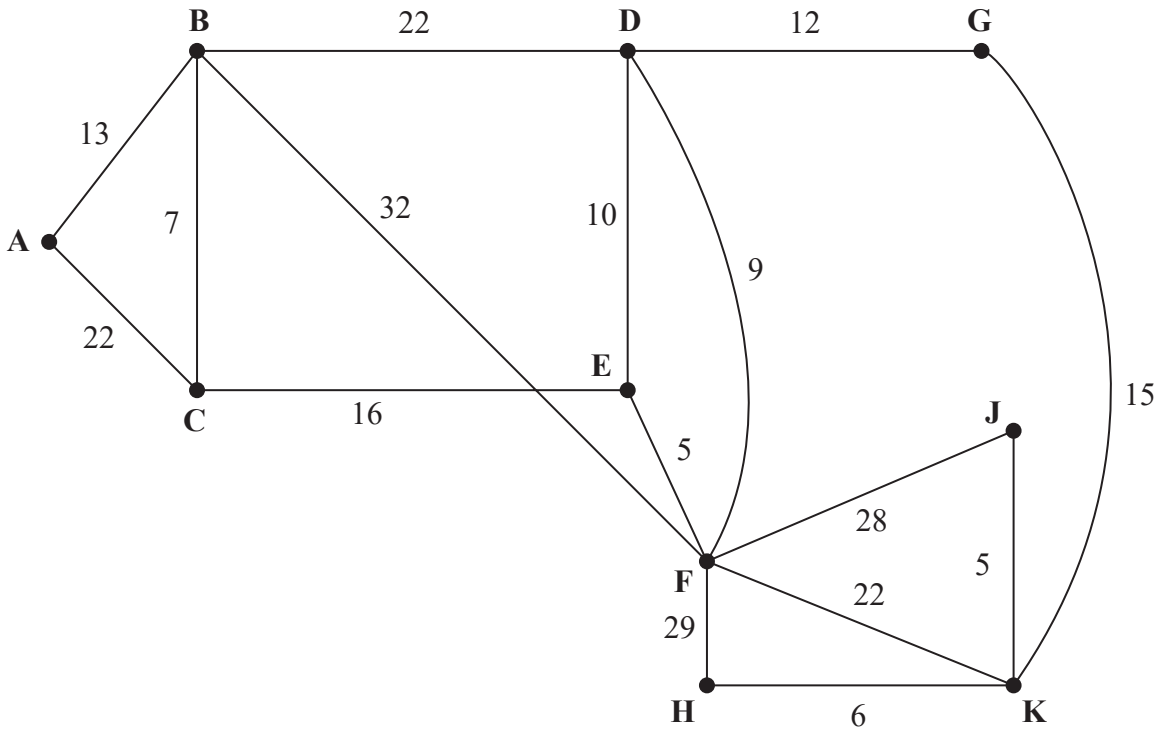


Figure 1

[*The total weight of the network is* 253]

Figure 1 represents a network of roads between 10 cities, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in miles, of the corresponding road.

One day, Mabintou wishes to travel from A to H. She wishes to minimise the distance she travels.

(a) Use Dijkstra's algorithm to find the shortest path from A to H. State your path and its length.

(6)

On another day, Mabintou wishes to travel from F to K via A.

(b) Find a route of minimum length from F to K via A and state its length.

(2)

The roads between the cities need to be inspected. James must travel along each road at least once. He wishes to minimise the length of his inspection route. James will start his inspection route at A and finish at J.

(c) By considering the pairings of all relevant nodes, find the length of James' route. State the arcs that will need to be traversed twice. You must make your method and working clear.

(6)

(d) State the number of times that James will pass through F.

(1)

It is now decided to start the inspection route at D. James must minimise the length of his route. He must travel along each road at least once but may finish at any vertex.

(e) State the vertex where the new inspection route will finish.

(1)

(f) Calculate the difference between the lengths of the two inspection routes.

(1)

(Total 17 marks)

6.

Activity	Duration (days)	Immediately preceding activities
A	4	–
B	7	–
C	6	–
D	10	A
E	5	A
F	7	C
G	6	B, C, E
H	6	B, C, E
I	7	B, C, E
J	9	D, H
K	8	B, C, E
L	4	F, G, K
M	6	F, G, K
N	7	F, G
P	5	M, N

The table above shows the activities required for the completion of a building project. For each activity the table shows the duration, in days, and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

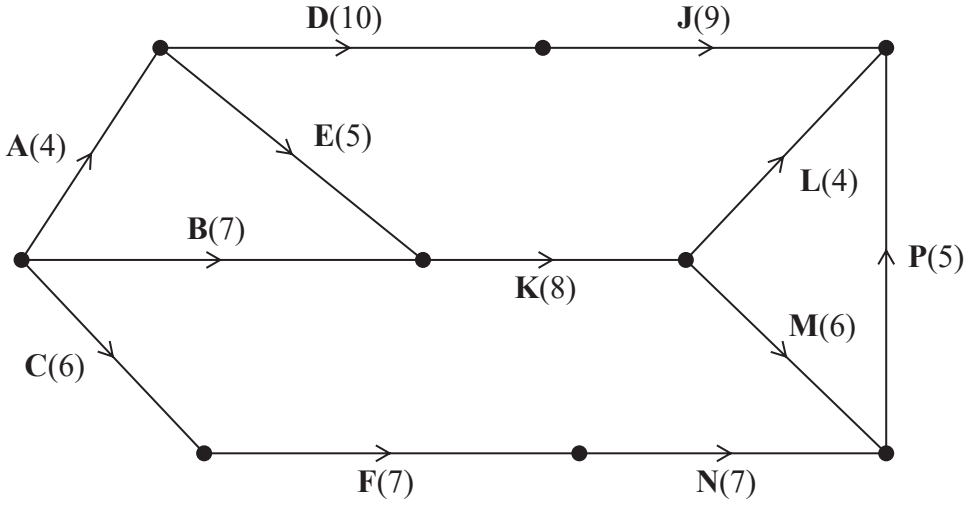


Figure 2

Figure 2 shows a **partially completed** activity network used to model the project. The activities are represented by the arcs and the numbers in brackets on the arcs are the times taken, in days, to complete each activity.

(a) Complete the network in Diagram 1 in the answer book by adding activities G, H and I and the minimum number of dummies.

(3)

(b) Add the early event times and the late event times to Diagram 1 in the answer book.

(4)

(c) State the critical activities.

(1)

(d) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. You must show your working.

(2)

(e) Schedule the activities on Grid 1 in the answer book, using the minimum number of workers, so that the project is completed in the minimum time.

(3)

(Total 13 marks)

7.

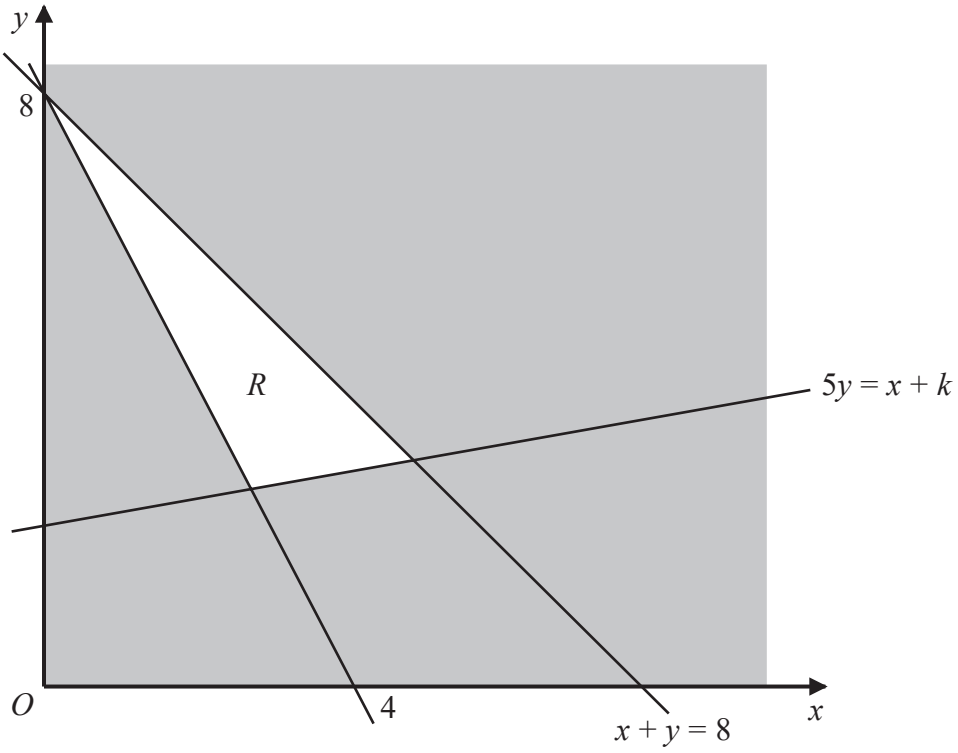


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y, where R is the feasible region. The equations of two of the lines have been shown in Figure 3.

Given that k is a positive constant,

(a) determine, in terms of k where necessary, the inequalities that define R.

(4)

The objective is to maximise $P = 5x + ky$

Given that the value of P is 38 at the optimal vertex of R,

(b) determine the possible value(s) of k. You must show algebraic working and make your method clear.

(7)

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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Please check the examination details below before entering your candidate information

Candidate surname	Other names
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Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

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Decision Mathematics D1

Answer Book
Do not return the question paper with the answer book.

Total Marks

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1. 1 Arc 2 Centre 3 Chord 4 Circle 5 Circumference
6 Diameter 7 Radius 8 Sector 9 Segment 10 Tangent

Pivot 1: (1 + 10) ÷ 2 = 5.5 → 6th item: Diameter
Parallelogram comes after Diameter, so reject items 1 to 6
Pivot 2: (7 + 10) ÷ 2 = 8.5 → 9th item: Segment
Parallelogram comes before Segment, so reject items 9 to 10
Pivot 3: (7 + 8) ÷ 2 = 7.5 → 8th item: Sector
Parallelogram comes before Sector, so reject item 8
Pivot 4: (7 + 7) ÷ 2 = 7 → 7th item: Radius
Parallelogram ≠ Radius, so reject item 7: list is now empty
∴ Parallelogram is not in the list

Q1

(Total 4 marks)

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2. Minimise $C = 2x + 3y$

subject to:

$x + y \geq 85$

$y \geq 2x$

$y \leq 0.8(x + y) \Rightarrow 5y \leq 4x + 4y \Rightarrow y \leq 4x$

$x \geq 0, \quad y \geq 0$

Constraints: $x + y \geq 85, \quad y \geq 2x, \quad y \leq 4x$

Q2

(Total 5 marks)

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3. (a) First-fit (bins of size 5):

Bin 1: 2.6 0.8 1.2 0.3

Bin 2: 2.1 0.9 1.7

Bin 3: 2.3 1.8

Bin 4: 2.7

(b)(i) First pass: 2.6 2.1 1.2 0.9 1.7 2.3 0.8 1.8 2.7 0.3

Second pass: 2.6 2.1 1.2 1.7 2.3 0.9 1.8 2.7 0.8 0.3

(ii) Comparisons and swaps: see table on next page

(c) Quick sort, pivot = middle right item of each sublist:

2.6 2.1 1.7 2.3 1.2 1.8 2.7 0.9 0.8 0.3	pivot 1.8
2.6 2.1 2.3 2.7 1.8 1.7 1.2 0.9 0.8 0.3	pivots 2.3, 0.9
2.6 2.7 2.3 2.1 1.8 1.7 1.2 0.9 0.8 0.3	pivots 2.7, 2.1, 1.2, 0.3
2.7 2.6 2.3 2.1 1.8 1.7 1.2 0.9 0.8 0.3	sort complete

(d) First-fit decreasing (bins of size 5):

Bin 1: 2.7 2.3

Bin 2: 2.6 2.1 0.3

Bin 3: 1.8 1.7 1.2

Bin 4: 0.9 0.8

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Question 3 continued

	Number of comparisons	Number of swaps
First pass	9	7
Second pass	8	4

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Question 3 continued

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Question 3 continued

(Total 13 marks)

Q3

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4. (a) *In the classical problem each vertex must be visited exactly once.*
In the practical problem each vertex must be visited at least once
(so a vertex may be visited more than once).

	A	B	C	D	E	F	G
A	–	25	31	28	35	30	32
B	25	–	34	24	27	32	39
C	31	34	–	40	35	27	29
D	28	24	40	–	37	35	36
E	35	27	35	37	–	28	31
F	30	32	27	35	28	–	33
G	32	39	29	36	31	33	–

(b) Nearest neighbour starting at A:
A – B (25), B – D (24), D – F (35), F – C (27), C – G (29),
G – E (31), E – A (35)
Route: A – B – D – F – C – G – E – A
Upper bound = 25 + 24 + 35 + 27 + 29 + 31 + 35 = 206 km

(c) Yes: 203 < 206, so the upper bound starting at D is better
as it is smaller (lower).

(d)(i) Prim's algorithm starting at A (G deleted):
Order of arcs: AB (25), BD (24), BE (27), EF (28), CF (27)
Weight of RMST = 25 + 24 + 27 + 28 + 27 = 131
(ii) Two least arcs at G: CG (29) and EG (31)
Lower bound = 131 + 29 + 31 = 191 km

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Question 4 continued

	A	B	C	D	E	F	G
A	–	25	31	28	35	30	32
B	25	–	34	24	27	32	39
C	31	34	–	40	35	27	29
D	28	24	40	–	37	35	36
E	35	27	35	37	–	28	31
F	30	32	27	35	28	–	33
G	32	39	29	36	31	33	–

(e) No: 188 < 191, so the lower bound found by deleting G (191 km) is better as it is larger (higher).

(f) 191 km ≤ length of optimal route ≤ 203 km

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Question 4 continued

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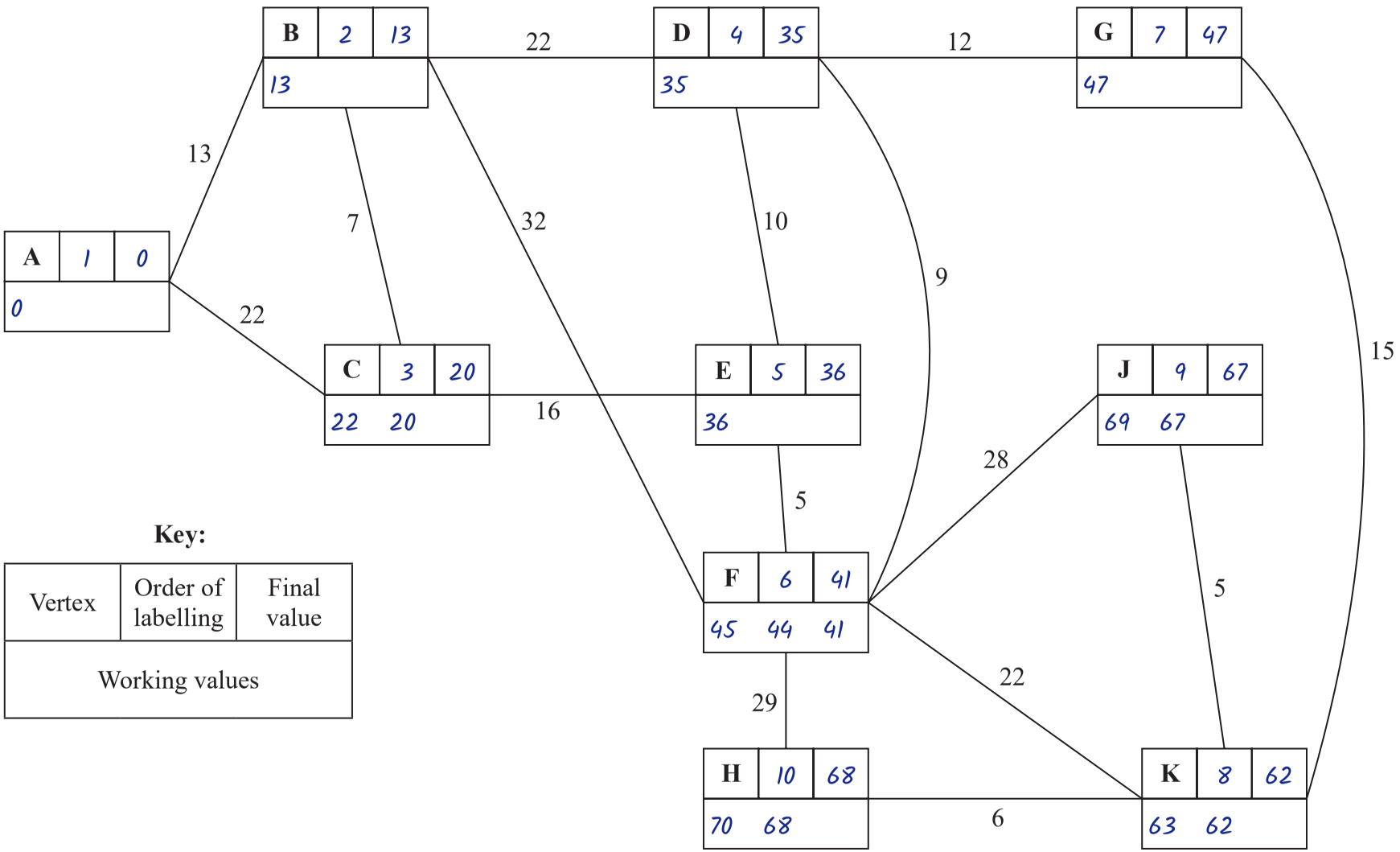
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Question 4 continued

Q4

(Total 12 marks)

5.



Shortest path: A – B – D – G – K – H

Length of shortest path: 68 miles

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Question 5 continued

(b) F to A: F – E – C – B – A = 41 (final value at F)

A to K: A – B – D – G – K = 62 (final value at K)

Route: F – E – C – B – A – B – D – G – K

Length = 41 + 62 = 103 miles

(c) Odd vertices are C and E. The route starts at A and finishes at J, so the relevant nodes are A, C, E and J.

AC + EJ = 20 + 32 = 52

AE + CJ = 36 + 48 = 84

AJ + CE = 67 + 16 = 83

Least is AC + EJ = 52 (AC: A – B – C, EJ: E – F – K – J)

Arcs traversed twice: AB, BC, EF, FK, JK

Length of route = 253 + 52 = 305 miles

(d) F has degree 6, plus repeated arcs EF and FK gives 8, so James passes through F 8 ÷ 2 = 4 times

(e) Starting at D (even), odd vertices C and E remain:

finish at C and repeat DE = 10 (finish at E needs DC = 26)

So the route finishes at C

(f) New route = 253 + 10 = 263 miles

Difference = 305 – 263 = 42 miles

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Question 5 continued

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Question 5 continued

(Total 17 marks)

Q5

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6. (a) and (b)

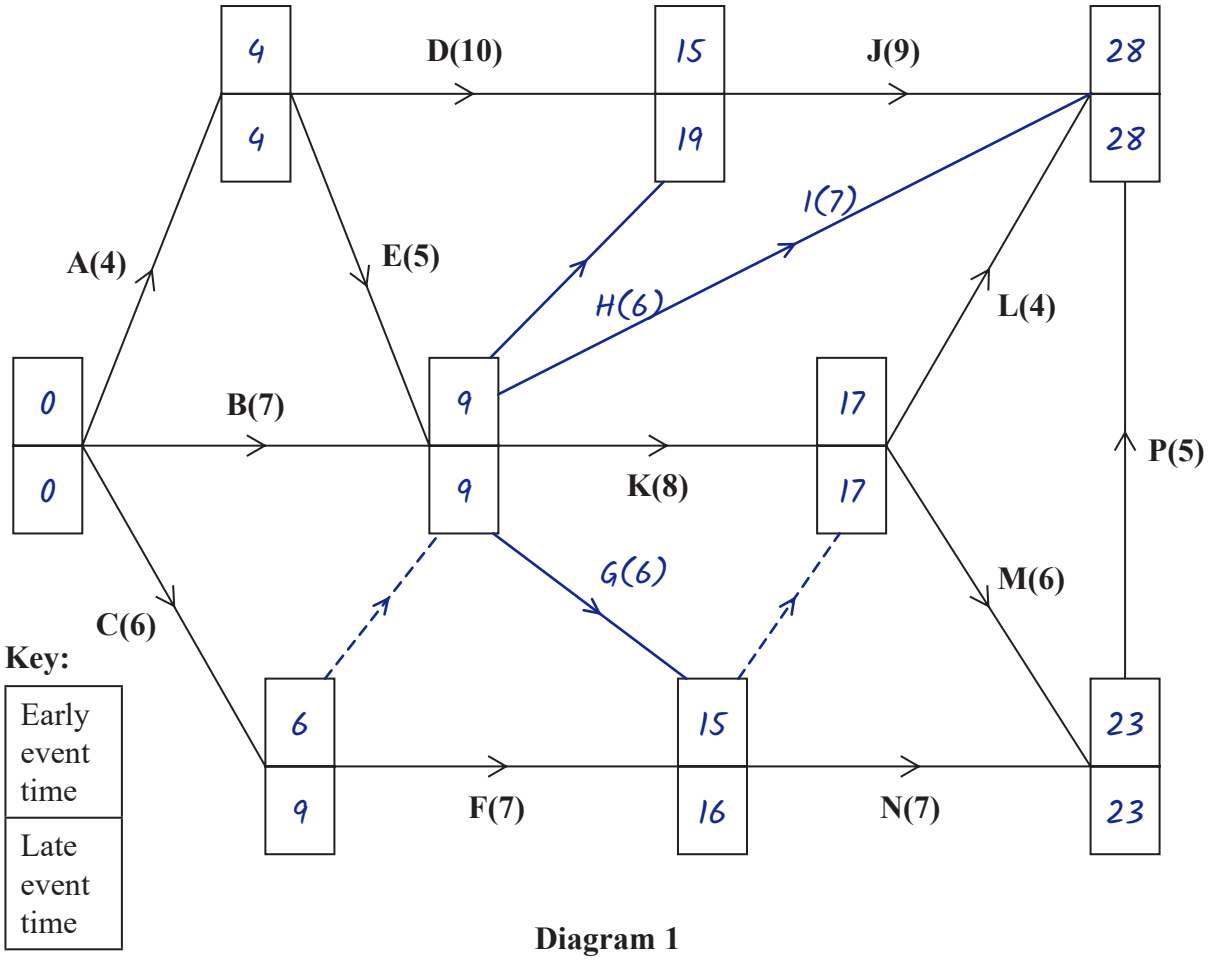


Diagram 1

(c) Critical activities: A, E, K, M, P

(d) Total of durations = 4 + 7 + 6 + 10 + 5 + 7 + 6 + 6 + 7 + 9 + 8 + 4 + 6 + 7 + 5 = 97

Lower bound = 97 ÷ 28 = 3.464... so 4 workers

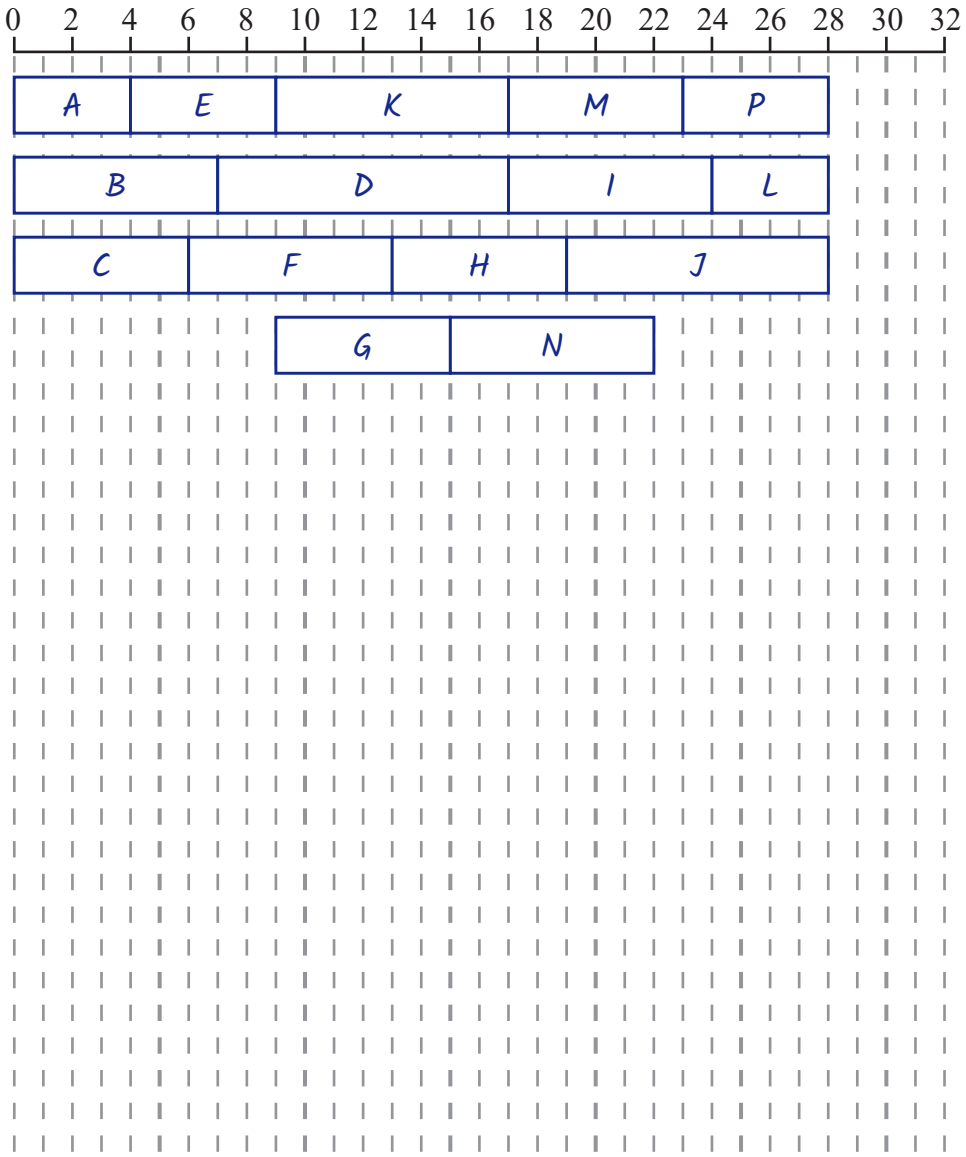
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Question 6 continued

(e) Each row of Grid 1 is one worker (4 workers), project finishes at 28



Grid 1

Q6

(Total 13 marks)

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7. (a) $x + y \leqslant 8$

$5y \geqslant x + k$

Line through (0, 8) and (4, 0): gradient $= -8 \div 4 = -2$

$y = -2x + 8$, R is on the side away from O, so $2x + y \geqslant 8$

Inequalities: $x + y \leqslant 8$, $2x + y \geqslant 8$, $5y \geqslant x + k$

(b) $P = 5x + ky$. Since $k > 0$ the optimal vertex is either (0, 8) or the intersection of $x + y = 8$ and $5y = x + k$.

If (0, 8) is optimal: $P = 8k = 38 \Rightarrow k = 19/4\ (= 4.75)$

Intersection of $x + y = 8$ and $5y = x + k$:

$5(8 - x) = x + k \Rightarrow 40 - 5x = x + k$

$x = (40 - k)/6$, $y = 8 - x = (8 + k)/6$

$P = 5(40 - k)/6 + k(8 + k)/6 = 38$

$200 - 5k + 8k + k^2 = 228$

$k^2 + 3k - 28 = 0$

$(k - 4)(k + 7) = 0$

$k = 4$ or $k = -7$ (reject as $k > 0$)

Check $k = 4$: vertex (6, 2), $P = 5(6) + 4(2) = 38$

and at (0, 8), $P = 4(8) = 32 < 38$, so (6, 2) is optimal ✓

Check $k = 19/4$: vertex $= (47/8, 17/8)$,

$P = 5(47/8) + (19/4)(17/8) = 1263/32 = 39.47 > 38$

so (0, 8) would not be optimal: reject $k = 19/4$

$\therefore$ $k = 4$ only

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Question 7 continued

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Question 7 continued

Q7

(Total 11 marks)

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