Pearson Edexcel International Advanced Level

Monday 20 January 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Write your answers in the D1 answer book for this paper.

1. The table below shows the distances, in km, between six data collection points, A, B, C, D, E and F.

	A	**B**	**C**	**D**	**E**	**F**
A	–	35	42	55	48	50
B	35	–	40	49	52	31
C	42	40	–	47	53	49
D	55	49	47	–	39	44
E	48	52	53	39	–	52
F	50	31	49	44	52	–

Ferhana must visit each data collection point. She will start and finish at A and wishes to minimise the total distance she travels.

(a) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the distance Ferhana must travel. Make your method clear.

(2)

(b) Starting by deleting B, and all of its arcs, find a lower bound for the distance Ferhana must travel. Make your calculation clear.

(3)

(Total 5 marks)

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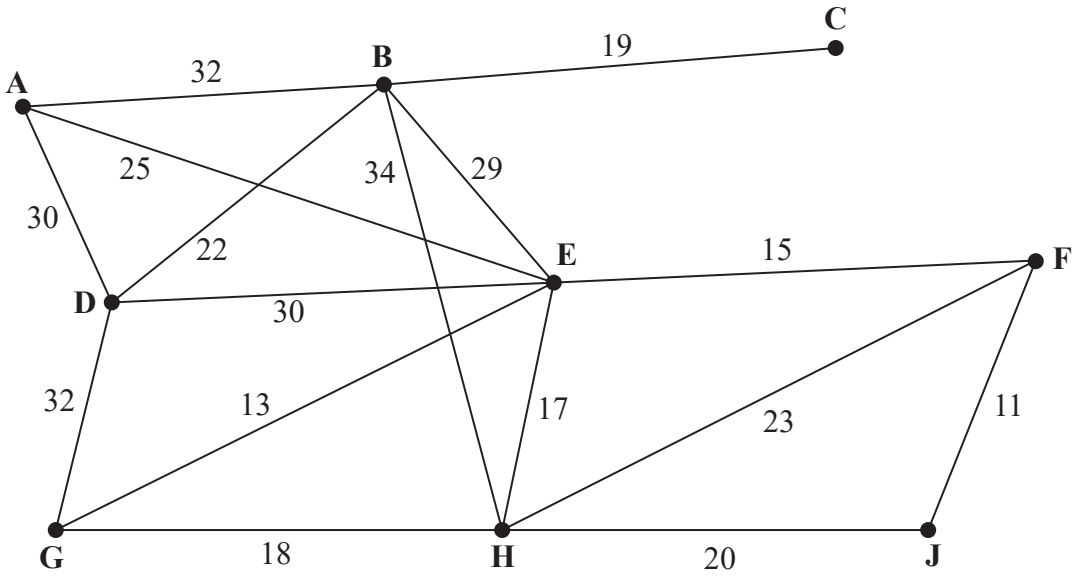


Figure 1

(a) Define the terms

(i) tree,

(ii) minimum spanning tree.

(3)

(b) Use Kruskal's algorithm to find the minimum spanning tree for the network shown in Figure 1. You must clearly show the order in which you consider the edges. For each edge, state whether or not you are including it in the minimum spanning tree.

(3)

(c) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book and state the weight of the minimum spanning tree.

(2)

(Total 8 marks)

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3.

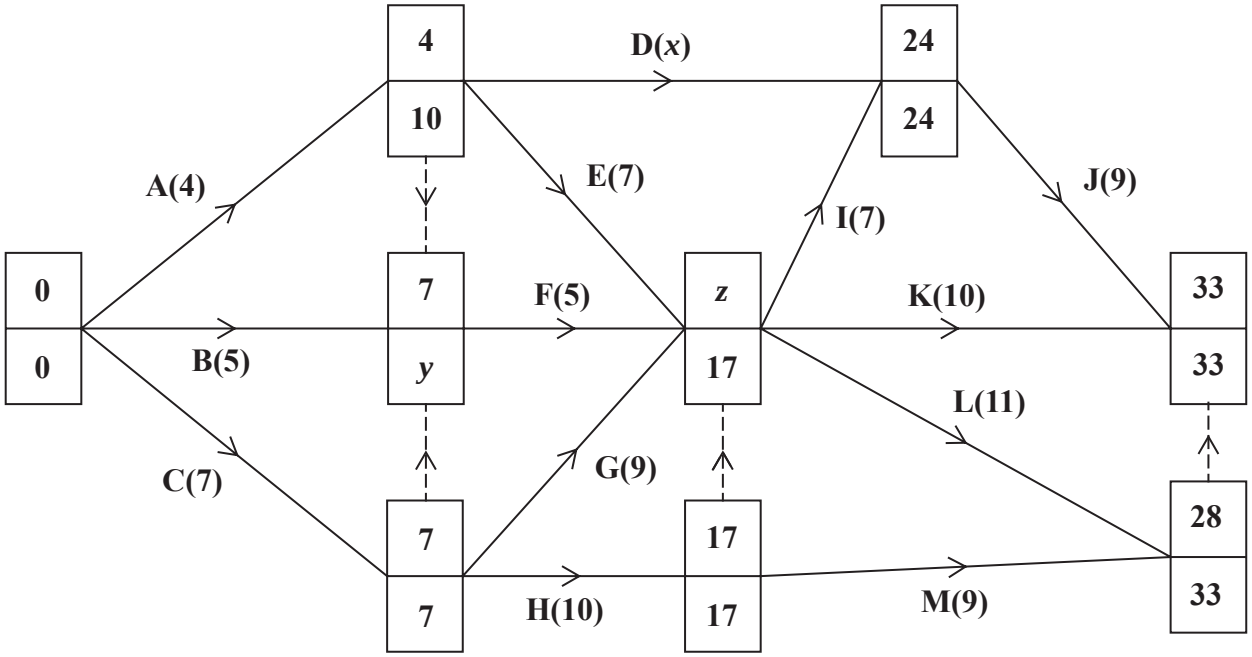


Figure 2

The network in Figure 2 shows the activities that need to be undertaken by a company to complete a project. Each activity is represented by an arc and the duration, in days, is shown in brackets. Each activity requires one worker. The early event times and late event times are shown at each vertex.

The total float on activity D is twice the total float on activity E.

(a) Find the values of x, y and z.

(3)

(b) Draw a cascade chart for this project on Grid 1 in the answer book.

(4)

(c) Use your cascade chart to determine a lower bound for the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities. (You do not need to provide a schedule of the activities.)

(2)

(Total 9 marks)

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4.

35 17 10 7 28 23 41 15 20 29

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 60

(3)

(b) The list of numbers is to be sorted into descending order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

(c) Use the first-fit decreasing bin packing algorithm on your ordered list to pack the numbers into bins of size 60

(3)

The ten distinct numbers below are to be sorted into descending order.

20 24 17 26 8 15 x y 19 12

A bubble sort, starting at the left-hand end of the list, is to be used to obtain the sorted list.

After the second complete pass the list is

24 26 20 17 15 y 19 12 x 8

(d) Find the constraints on the values of x and y.

(3)

(Total 13 marks)

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5.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	C
F	A, B, C
G	A, B, C
H	D, F, G
I	A, B, C
J	D, F, G
K	H
L	D, E, F, G, I

(a) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain only the minimum number of dummies.

(5)

Given that all critical paths for the network include activity H,

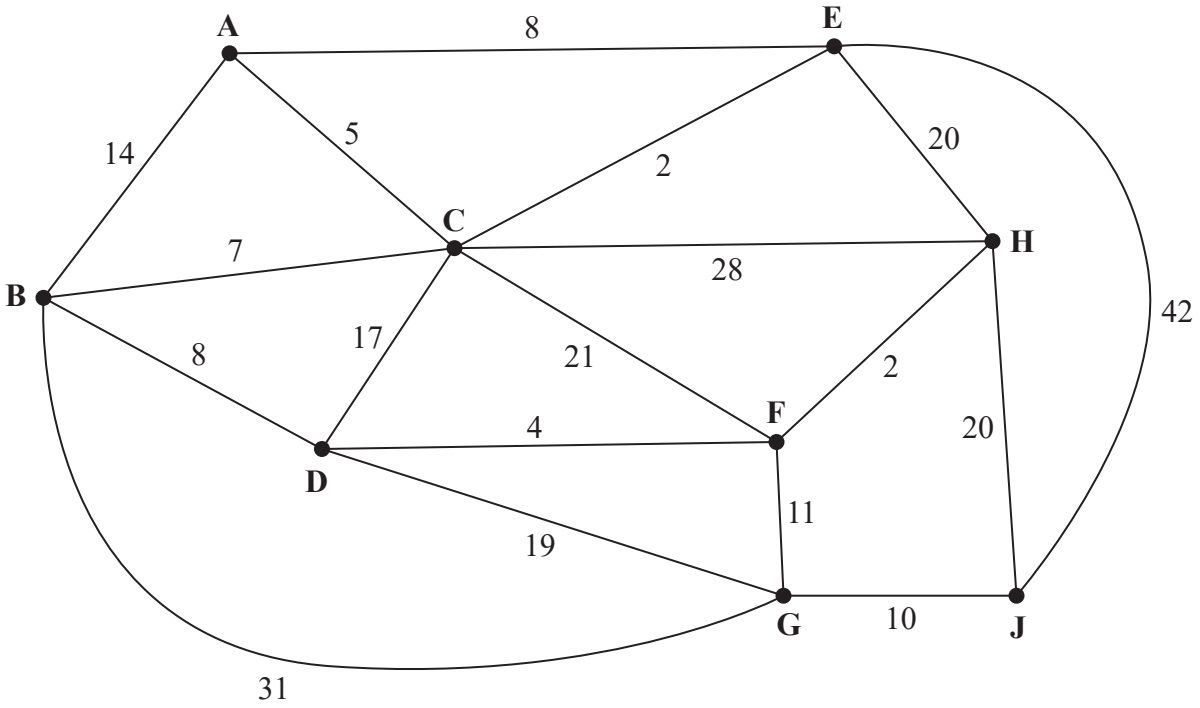
(b) state which activities cannot be critical.

(2)

(Total 7 marks)

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6.



[*The total weight of the network is 269*]

Figure 3

Figure 3 models a network of roads. The number on each edge gives the time taken, in minutes, to travel along the corresponding road.

(a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to J. State the quickest route.

(6)

Alan needs to travel along all the roads to check that they are in good repair. He wishes to complete his route as quickly as possible and will start at his home, H, and finish at his workplace, D.

(b) By considering the pairings of all relevant nodes, find the arcs that will need to be traversed twice in Alan's inspection route from H to D. You must make your method and working clear.

(5)

For Alan's inspection route from H to D

(c) (i) state the number of times vertex C will appear,

(ii) state the number of times vertex D will appear.

(2)

(d) Determine whether it would be quicker for Alan to start and finish his inspection route at H, instead of starting at H and finishing at D. You must explain your reasoning and show all your working.

(2)

(Total 15 marks)

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7. A school is planning to run several training days next year for its new teachers, middle leaders and senior leaders.

Next year, the school will need to run

- at least 20 training days in total,
- at most twice as many training days for new teachers when compared to the total number of training days required for both middle and senior leaders,
- at most 25% of the training days for senior leaders.

The costs of running a training day for new teachers, middle leaders and senior leaders are £400, £550 and £750 respectively.

The school wants to minimise the total cost of running the training days.

Let x be the number of training days required for new teachers.
Let y be the number of training days required for middle leaders.
Let z be the number of training days required for senior leaders.

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(6)

The school decides that the number of training days for middle leaders and the number of training days for senior leaders should be in the ratio 5 : 3
This reduces two of the constraints to $5x \leqslant 16y$ and $4y \leqslant 5x$.

(b) (i) Express the constraint representing the requirement for a total of at least 20 training days as a simplified inequality with integer coefficients in terms of x and y only.

(ii) Express the objective in the form $ax + by$ where a and b are integers.

(3)

(c) Represent the constraints in x and y only on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(d) Use the objective line method to locate the optimal vertex, V, of the feasible region. You must make your objective line clear and label the optimal vertex, V.

(3)

(e) Hence, determine

(i) the total cost of running the training days,

(ii) the number of training days required for senior leaders.

(2)

(Total 18 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Please check the examination details below before entering your candidate information

Candidate surname	StemUp Model Solution	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

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Mathematics

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Decision Mathematics D1

Answer Book
Do not return the question paper with the answer book.

Total Marks

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1.

	A	B	C	D	E	F
A	–	35	42	55	48	50
B	35	–	40	49	52	31
C	42	40	–	47	53	49
D	55	49	47	–	39	44
E	48	52	53	39	–	52
F	50	31	49	44	52	–

(a) Nearest neighbour starting at A:

A → B (35), B → F (31), F → D (44),

D → E (39), E → C (53), C → A (42)

Route: A B F D E C A

Upper bound = 35 + 31 + 44 + 39 + 53 + 42 = 244 km

(b) Delete B. Minimum spanning tree on A, C, D, E, F (Prim from A):

AC (42), CD (47), DE (39), DF (44)

RMST weight = 42 + 47 + 39 + 44 = 172

Add the two least arcs from B: BF (31) and BA (35)

Lower bound = 172 + 31 + 35 = 238 km

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Question 1 continued

	A	B	C	D	E	F
A	–	35	42	55	48	50
B	35	–	40	49	52	31
C	42	40	–	47	53	49
D	55	49	47	–	39	44
E	48	52	53	39	–	52
F	50	31	49	44	52	–

Q1

(Total 5 marks)

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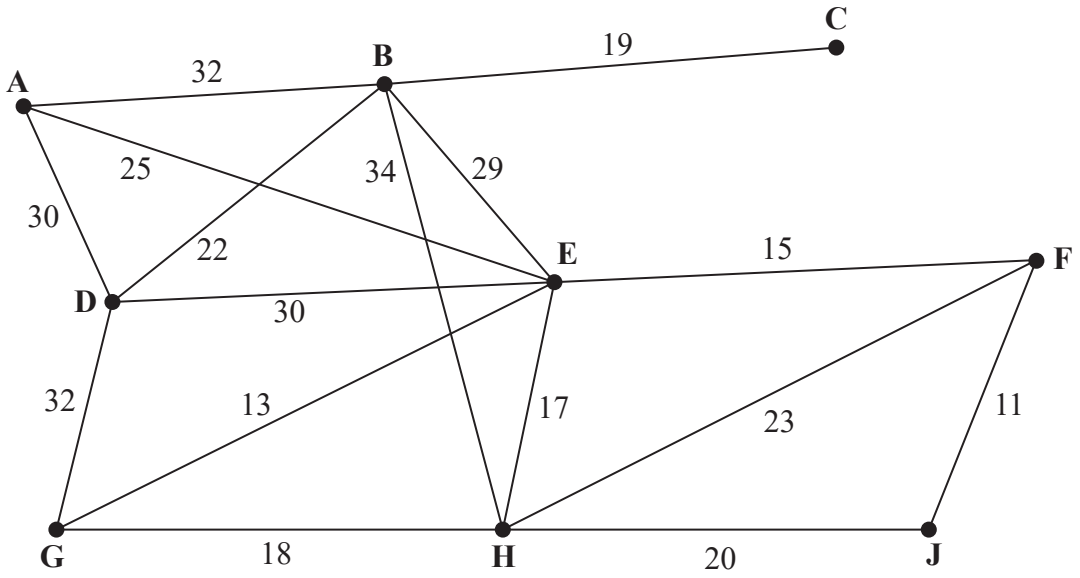


Figure 1

(a)(i) A tree is a connected graph with no cycles.

(ii) A minimum spanning tree is a tree that contains all the vertices of the network, such that the total length (weight) of its arcs is as small as possible.

(b) Kruskal's algorithm, arcs in order of weight:

FJ (11) include	BC (19) include
EG (13) include	HJ (20) reject
EF (15) include	BD (22) include
EH (17) include	FH (23) reject
GH (18) reject	AE (25) include
	BE (29) include

All 9 vertices now connected (8 arcs), so stop.

Arcs in MST: FJ, EG, EF, EH, BC, BD, AE, BE

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Question 2 continued

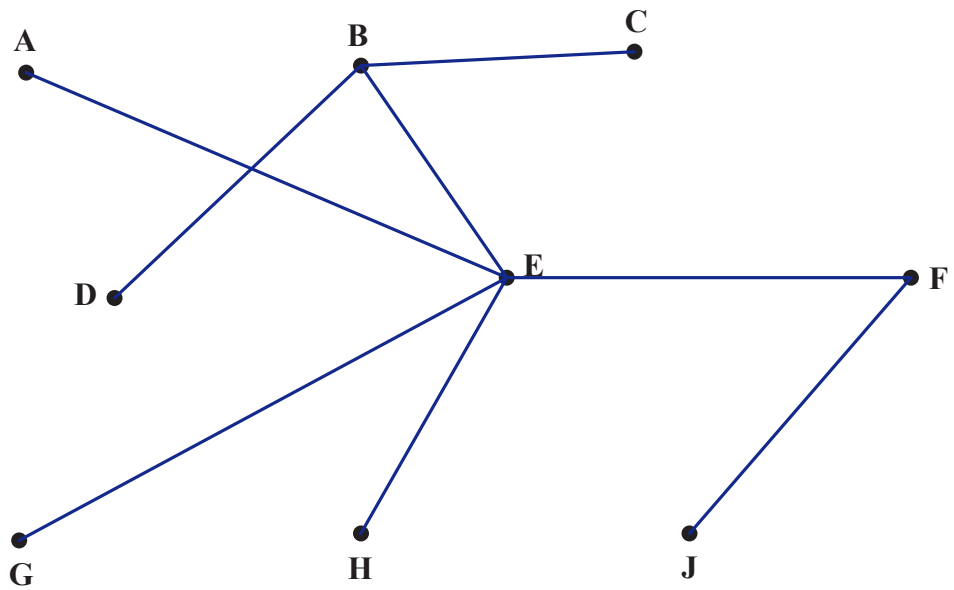


Diagram 1

Weight of the minimum spanning tree 151

Q2

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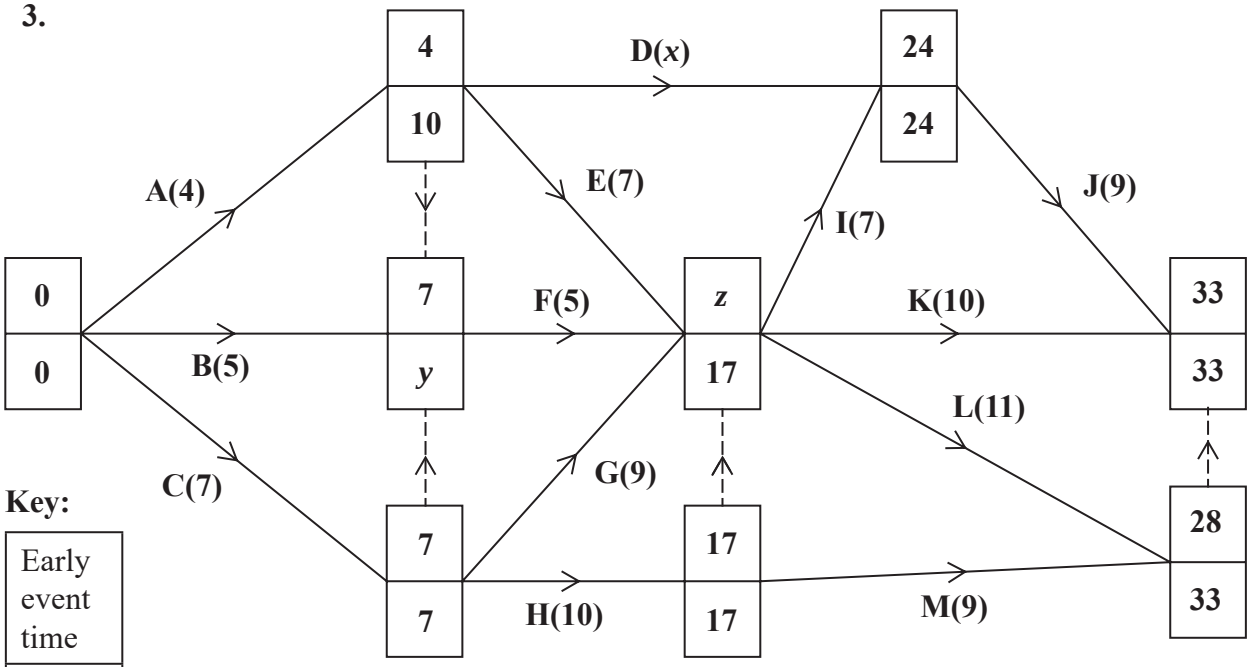


Figure 2

(a) Total float on E = 17 − 4 − 7 = 6

Total float on D = 24 − 4 − x = 20 − x

20 − x = 2 × 6 = 12 ⇒ x = 8

y = late time of the event at the start of F = 17 − 5 ⇒ y = 12

z = max(4 + 7, 7 + 5, 7 + 9, 17 + 0) = 17 ⇒ z = 17

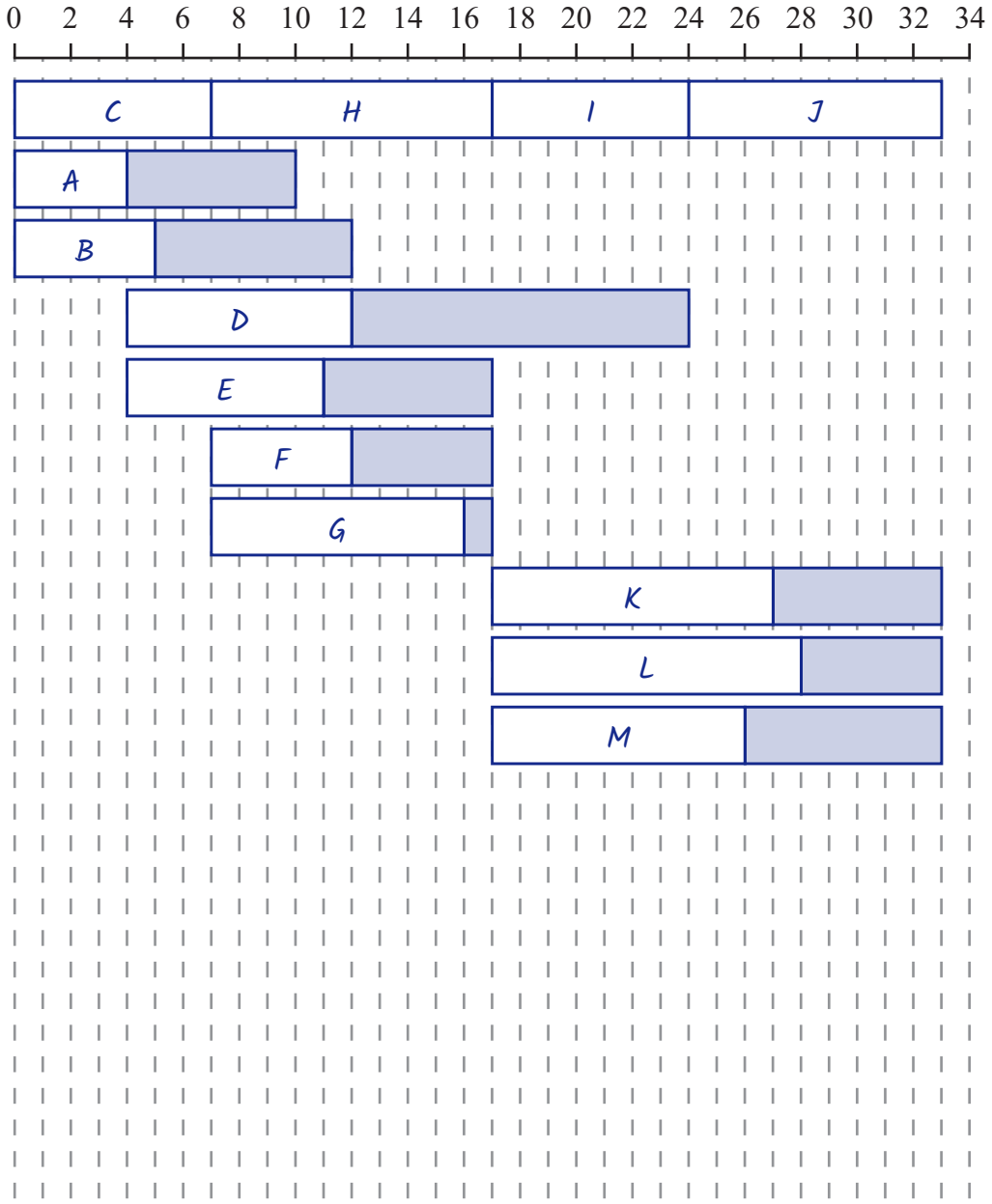
x = 8, y = 12, z = 17

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Question 3 continued



Grid 1

(c) At any time t with 24 < t < 26 (e.g. during day 25) activities J, K, L and M must all be happening at the same time, so a lower bound is 4 workers.

Q3

(Total 9 marks)

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4.

35 17 10 7 28 23 41 15 20 29

(a) First-fit, bins of size 60:

Bin 1: 35 17 7

Bin 2: 10 28 15

Bin 3: 23 20

Bin 4: 41

Bin 5: 29

(b) Quick sort, pivots ringed (middle right item of each sublist):

	Pivots
35 17 10 7 28 (23) 41 15 20 29	23
35 28 (41) 29 23 17 10 (7) 15 20	41, 7
41 35 (28) 29 23 17 10 (15) 20 7	28, 15
41 35 (29) 28 23 17 (20) 15 (10) 7	29, 20, (10)
41 35 29 28 23 20 17 15 10 7	sort complete

Sorted list: 41 35 29 28 23 20 17 15 10 7

(c) First-fit decreasing, bins of size 60:

Bin 1: 41 17

Bin 2: 35 23

Bin 3: 29 28

Bin 4: 20 15 10 7

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Question 4 continued

(d) Pass 1: 24 20 26 17 15 x y 19 12 8

Pass 2: 24 26 20 17 15 y 19 12 x 8

In pass 2: 15 and x are not swapped, x and y are swapped $\Rightarrow y > x$

x is then swapped with 19 and with 12 $\Rightarrow x < 12$

x is not swapped with 8 $\Rightarrow x > 8$

$8 < x < 12$ and $y > x$

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Question 4 continued

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Question 4 continued

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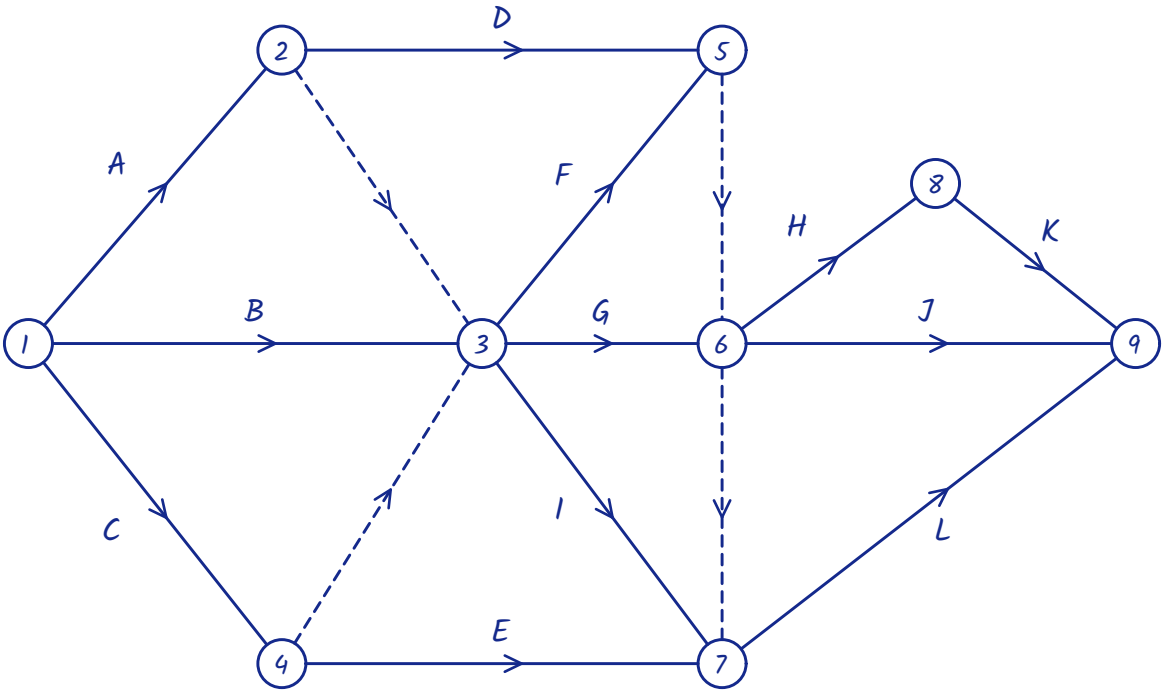
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5.

(a)



(dashed arcs are dummies)

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Question 5 continued

(b) Every critical path passes through H, i.e. along the dummy into event 6, then H and K. Activities that bypass H cannot be critical:
J and L run in parallel with H, and E and I lead only into L.

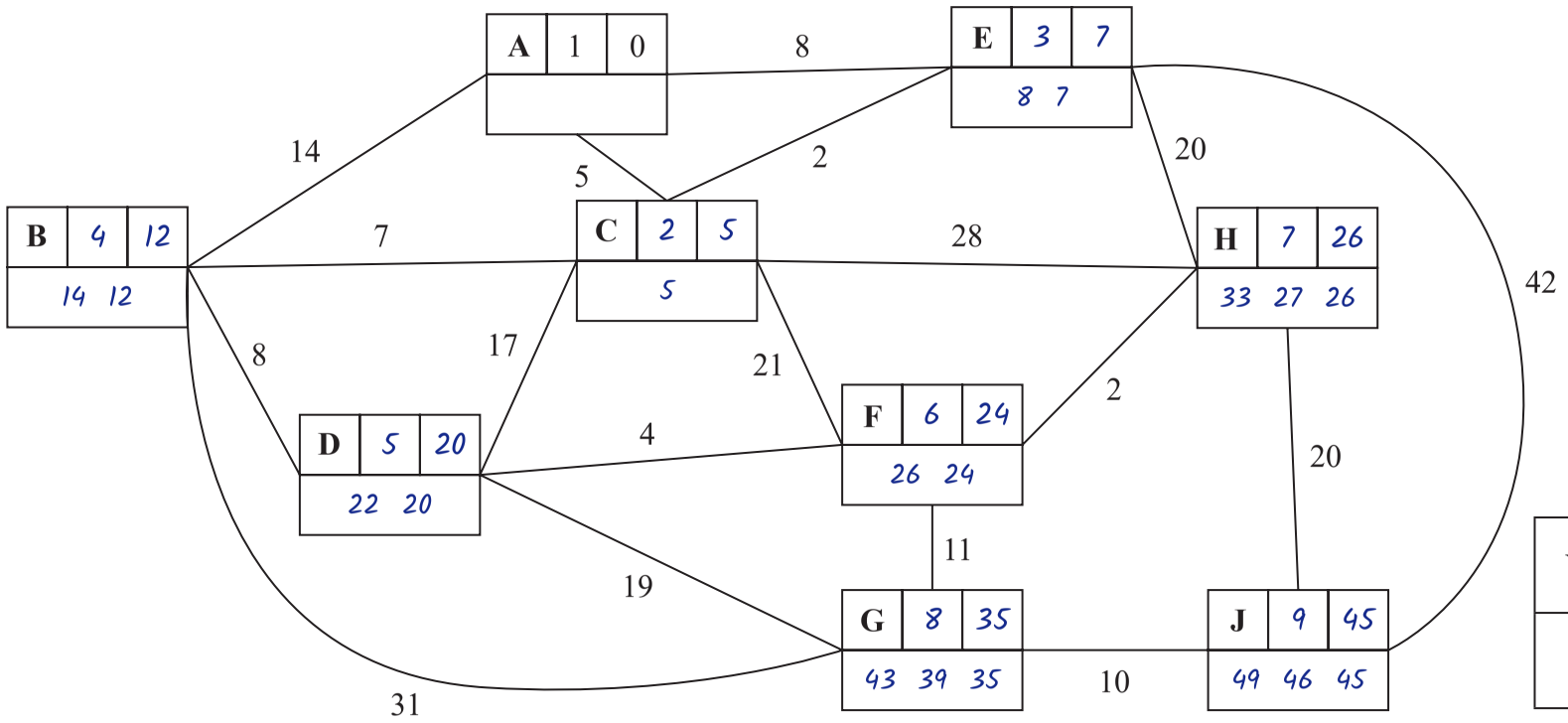
Activities E, I, J and L cannot be critical.

Q5

(Total 7 marks)

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6.



Shortest time: 45 minutes

Quickest route: A C B D F G J

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Question 6 continued

(b) Odd vertices: A (3) and J (3). The route starts at H and finishes at D, so consider pairings of A, D, H and J:

AD + HJ = 20 + 20 = 40 (AD via ACBD)

AH + DJ = 26 + 25 = 51 (AH via ACBDFH, DJ via DFGJ)

AJ + DH = 45 + 6 = 51 (AJ via ACBDFGJ, DH via DFH)

Minimum is AD + HJ = 40

Arcs traversed twice: AC, BC, BD and HJ

Length of route = 269 + 40 = 309 minutes

(c)(i) C has valency 6, plus repeats AC and BC gives 8, so C appears 8 ÷ 2 = 4 times.

(ii) D has valency 4, plus repeat BD gives 5; D is the finish vertex, so D appears (5 + 1) ÷ 2 = 3 times.

(d) From H back to H: H and D are then even, so the only odd vertices are A and J, and only the shortest path A to J (45) is repeated.

Length = 269 + 45 = 314 minutes

314 > 309, so it would not be quicker: starting at H and finishing at D is quicker (by 5 minutes).

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Question 6 continued

Q6

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7.

(a) Minimise $C = 400x + 550y + 750z$

subject to

$x + y + z \geqslant 20$

$x \leqslant 2(y + z) \Rightarrow x \leqslant 2y + 2z$

$z \leqslant \frac{1}{4}(x + y + z) \Rightarrow 4z \leqslant x + y + z$

$3z \leqslant x + y$

$x, y, z \geqslant 0$ and integers

(b)(i) $y : z = 5 : 3 \Rightarrow 3y = 5z \Rightarrow z = \frac{3y}{5}$

$x + y + \frac{3y}{5} \geqslant 20 \Rightarrow 5x + 5y + 3y \geqslant 100$

$5x + 8y \geqslant 100$

(ii) $400x + 550y + 750 \times \frac{3y}{5} = 400x + 550y + 450y$

Objective: minimise $400x + 1000y$

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Question 7 continued

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Question 7 continued

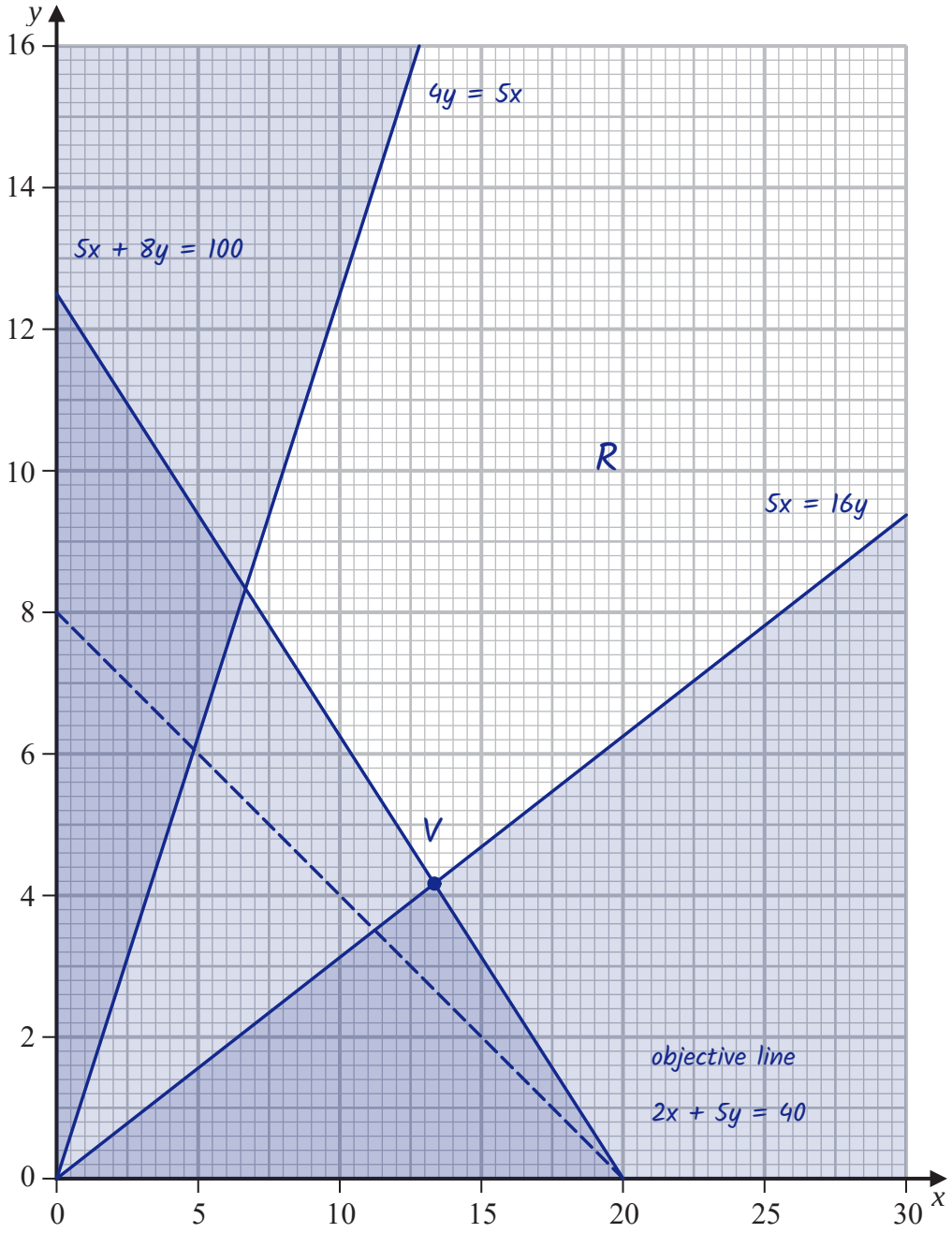


Diagram 1

(c) Lines $5x + 8y = 100$, $4y = 5x$ and $5x = 16y$ drawn; unwanted regions shaded; feasible region R labelled.

(d) Objective line $2x + 5y = 40$ ($400x + 1000y = 8000$) drawn.

Minimising: the line is moved towards O; the last point of R it touches is V.

V: $5x + 8y = 100$ and $5x = 16y \Rightarrow 24y = 100 \Rightarrow y = 4.17, x = 13.3$

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Question 7 continued

(e) The numbers of days must be integers and $y : z = 5 : 3$, so y must be a multiple of 5. $y = 0$ is not feasible (it forces $x = 0$).

Near V, take $y = 5$, $z = 3$: $5x \geq 100 - 40 = 60 \Rightarrow x \geq 12$, so $x = 12$

(check: $5(12) = 60 \leq 16(5) = 80$ and $4(5) = 20 \leq 5(12) = 60$)

($y = 10$ would cost at least $400(8) + 1000(10) = £13200$)

(i) Cost $= 400(12) + 550(5) + 750(3) = 4800 + 2750 + 2250$

$= £9800$

(ii) Senior leaders: $z = 3$ training days

Q7

(Total 18 marks)

TOTAL FOR PAPER: 75 MARKS

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