Pearson Edexcel International Advanced Level

Thursday 17 January 2019

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WDM01/01**

Decision Mathematics D1

Advanced/Advanced Subsidiary

You must have:
D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 5 4 9 5 2 A

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Write your answers in the D1 answer book for this paper.

1. (a) Define the term 'bipartite graph'.

(2)

Six workers, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6

The table below shows the tasks that each worker is able to do.

Worker	Tasks
A	2, 4
B	5
C	3, 4
D	2
E	4, 5, 6
F	1, 6

(b) Using Diagram 1 in the answer book, draw a bipartite graph to show the possible allocation of workers to tasks.

(1)

Initially, workers A, C, E and F are allocated to tasks 2, 4, 5 and 6 respectively.

(c) Starting from the given initial matching, apply the maximum matching algorithm to obtain a complete matching. You must state the alternating paths that you use, and state your improved matching after each iteration.

(6)

(Total 9 marks)

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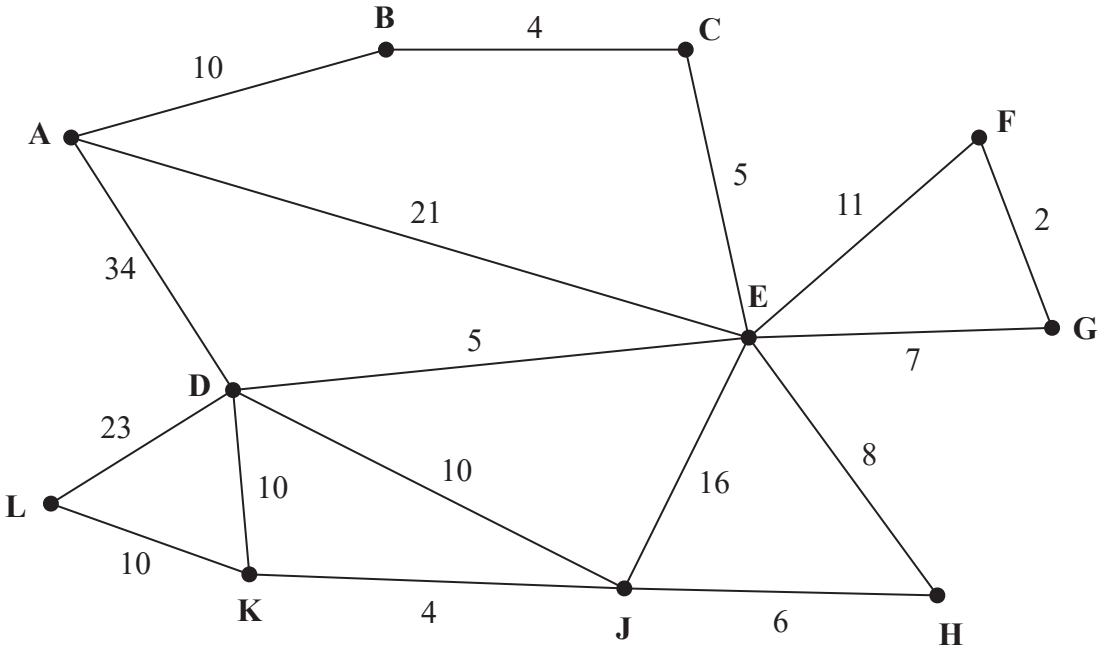


Figure 1

Figure 1 represents a network of roads. The number on each arc is the length, in km, of the corresponding road.

(a) Use Dijkstra's algorithm to find the shortest route from A to L. State the shortest route and its length.

(6)

(b) Explain how you determined the shortest route from your labelled diagram.

(2)

(c) Find the length of the shortest route from J to F via A, and state your route.

(2)

(Total 10 marks)

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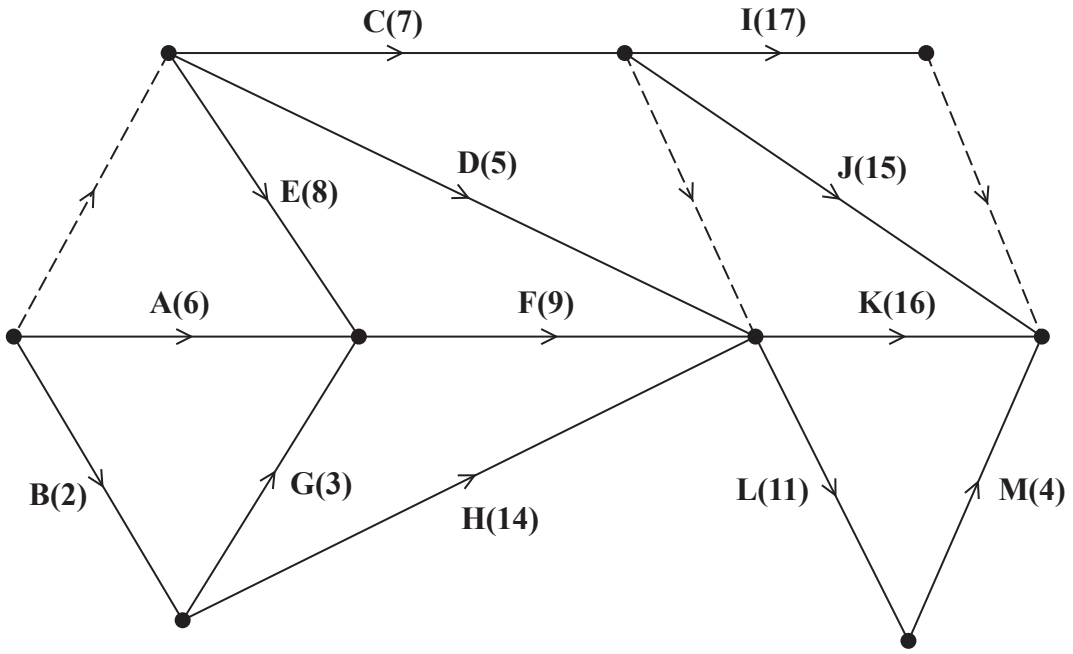


Figure 2

A project is modelled by the activity network shown in Figure 2. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the corresponding activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(b) State the critical activities.

(1)

(c) Draw a cascade (Gantt) chart for this project on Grid 1 in the answer book.

(4)

(d) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities. (You do not need to provide a schedule of the activities.)

(2)

(Total 11 marks)

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4.

180 80 250 115 100 230 150 95 105 90 390

The numbers in the list above represent the weights, in kilograms, of 11 boxes. John must transport all the boxes using his van. You may assume the van has sufficient space for any combination of boxes. Each van load of boxes must weigh at most 475 kg.

(a) Calculate a lower bound for the number of van loads needed to transport all 11 boxes.

(2)

(b) Use the first-fit bin packing algorithm to show how the boxes could be put into van loads. State the number of van loads needed according to this solution.

(3)

(c) Carry out a quick sort on the numbers in the list given above to produce a list of the weights in descending order. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Use the first-fit decreasing bin packing algorithm on your ordered list to show how the boxes could be put into van loads. State the number of van loads needed according to this solution.

(3)

Due to volume restrictions, the van cannot transport more than three boxes at any one time.

(e) Show how the boxes could now be put into the minimum number of van loads.

(2)

(Total 14 marks)

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5.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	B
E	A, D
F	B
G	B, C
H	E, F, G
I	F, G
J	G
K	H, I

(a) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain only the minimum number of dummies.

(5)

Given that D is a critical activity,

(b) state which other activities must also be critical.

(1)

(Total 6 marks)

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6.

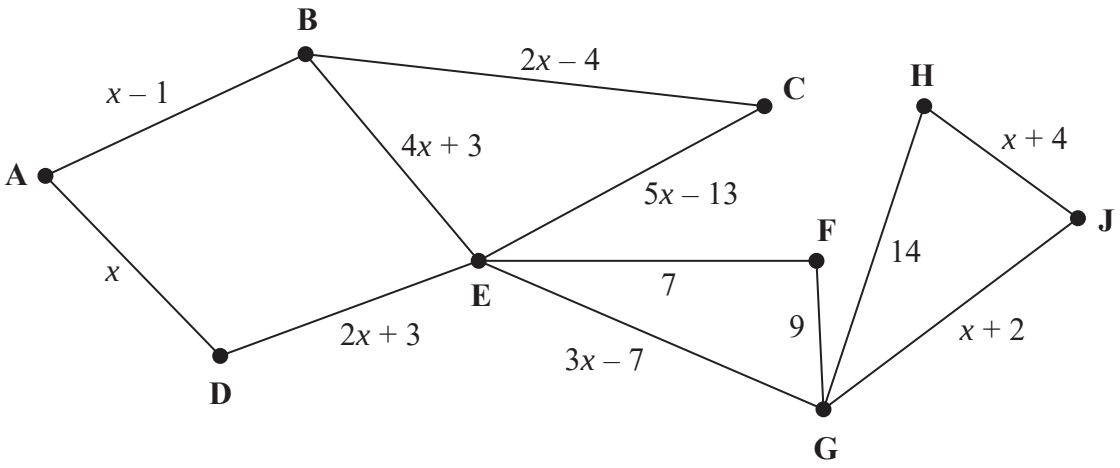


Figure 3

[*The weight of the network is* $20x + 17$]

(a) Explain why it is not possible to draw a network with an odd number of vertices of odd valency.

(2)

Figure 3 represents a network of 12 roads in a city. The expression on each arc gives the time, in minutes, to travel along the corresponding road.

(b) During rush hour one day $x = 9$

(i) Starting at A, use Prim's algorithm to find a minimum spanning tree. You must state the order in which you select the arcs of your tree.

(ii) Calculate the weight of the minimum spanning tree.

(4)

You are now given that $x > 3$

A route that minimises the total time taken to traverse each road at least once needs to be found. The route must start and finish at the same vertex.

The route inspection algorithm is applied to the network in Figure 3 and the time taken for the route is 162 minutes.

(c) Determine the value of x, showing your working clearly.

(6)

(Total 12 marks)

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7. A company makes two types of wooden bookcase, the *Manhattan* and the *Brooklyn*. The pieces of wood used for each bookcase go through three stages. They must be cut, assembled and packaged.

The table below shows the time, in hours, needed to complete each of the three stages for a single bookcase, and the profit made, in pounds, when each type of bookcase is sold. The table also shows the amount of time, in hours, that is available each week for each of the three stages.

	Manhattan	*Brooklyn*	Time available
Cutting (hrs)	1	4	30
Assembling (hrs)	1.5	4	35
Packaging (hrs)	2.5	3	45
Profit (£)	50	150	

Let x be the number of *Manhattan* bookcases made each week and let y be the number of *Brooklyn* bookcases made each week.

The company wishes to maximise its weekly profit. You may assume that all of the bookcases made each week will be sold.

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(3)

(b) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(c) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label V.

(2)

(d) State the number of each type of bookcase the company should make and calculate the maximum weekly profit.

(2)

When the optimal solution to the linear programming problem is adopted, the time needed for one of the three stages of production is less than that available.

(e) (i) Identify the stage for which the time needed is less than that available.

(ii) Calculate the difference between the time available and the time used for this stage.

(2)

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

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Answer Book
Do not return the question paper with the answer book.

Total Marks

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1. (a)

A bipartite graph consists of two sets of vertices, X and Y.

The edges only join a vertex in set X to a vertex in set Y;

no edge joins two vertices in the same set.

(b)

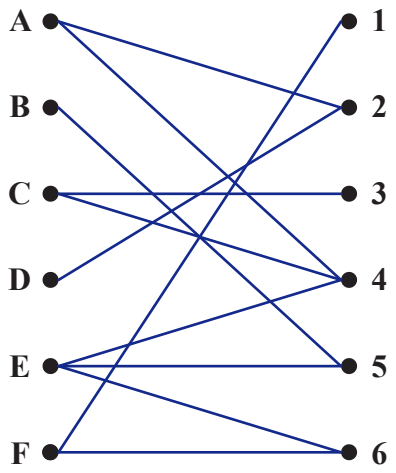


Diagram 1

(c) Initial matching: A = 2, C = 4, E = 5, F = 6

Alternating path: B – 5 = E – 6 = F – 1

Change status: B = 5 – E = 6 – F = 1

Improved matching: A = 2, B = 5, C = 4, E = 6, F = 1

(D and 3 unmatched)

Alternating path: D – 2 = A – 4 = C – 3

Change status: D = 2 – A = 4 – C = 3

Complete matching:

A = 4, B = 5, C = 3, D = 2, E = 6, F = 1

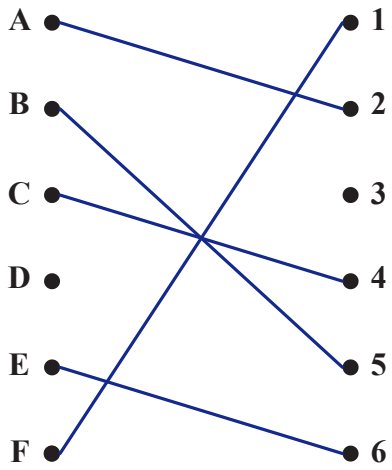
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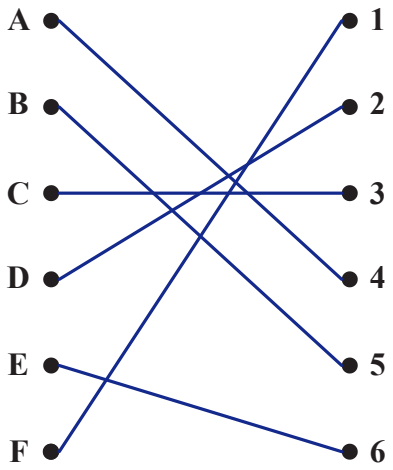
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Question 1 continued





Left: improved matching after 1st path. Right: complete matching.

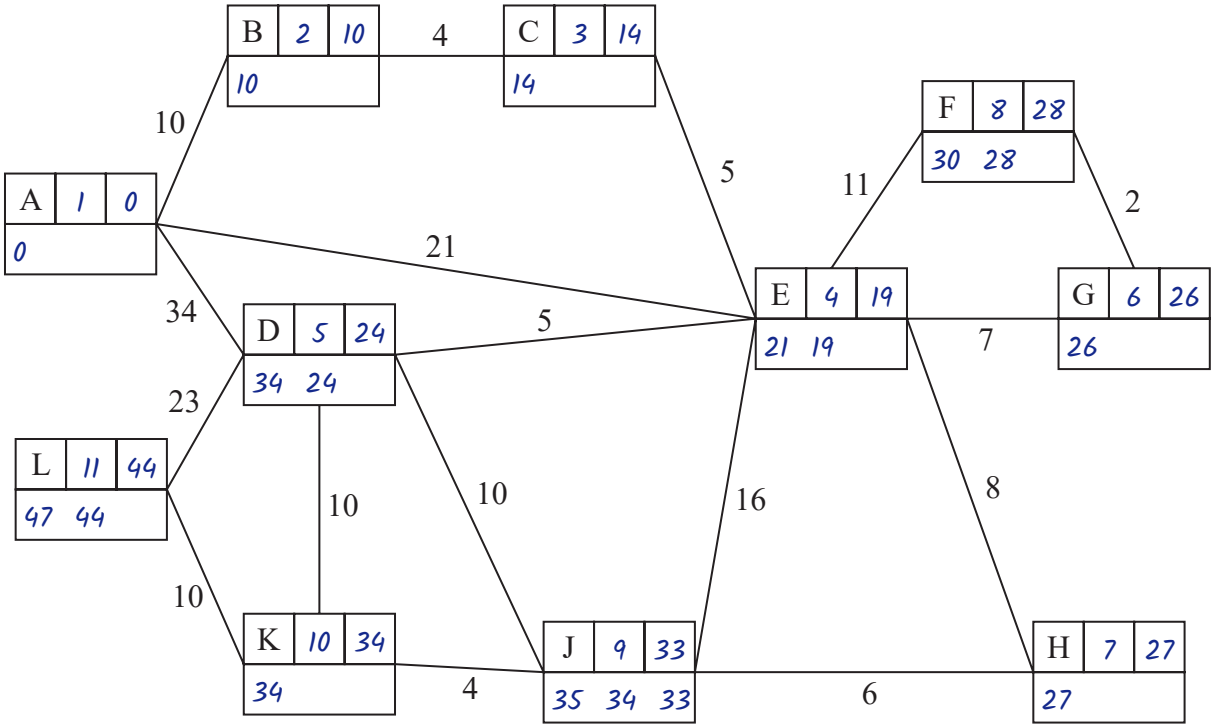
Q1

(Total 9 marks)

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2.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest route: A – B – C – E – D – K – L

Length of shortest route: 44 km

(b) Trace back from L: arc XY is on the route if the final value of Y – final value of X = weight of arc XY.

L: 44 – 10 = 34 = K, K: 34 – 10 = 24 = D, D: 24 – 5 = 19 = E, E: 19 – 5 = 14 = C, C: 14 – 4 = 10 = B, B: 10 – 10 = 0 = A

(c) J to A: J – H – E – C – B – A = 33 (final value at J)

A to F: A – B – C – E – G – F = 28 (final value at F)

Route: J – H – E – C – B – A – B – C – E – G – F

Length = 33 + 28 = 61 km

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Question 2 continued

Q2

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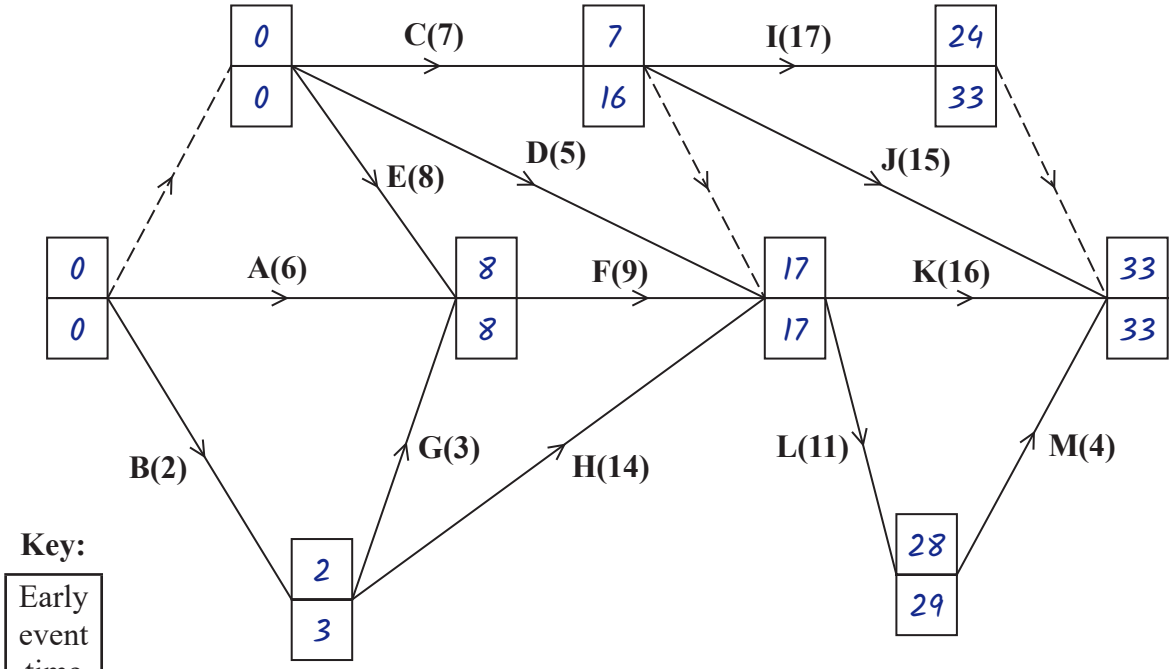


Diagram 1

(b) Critical activities: E, F and K

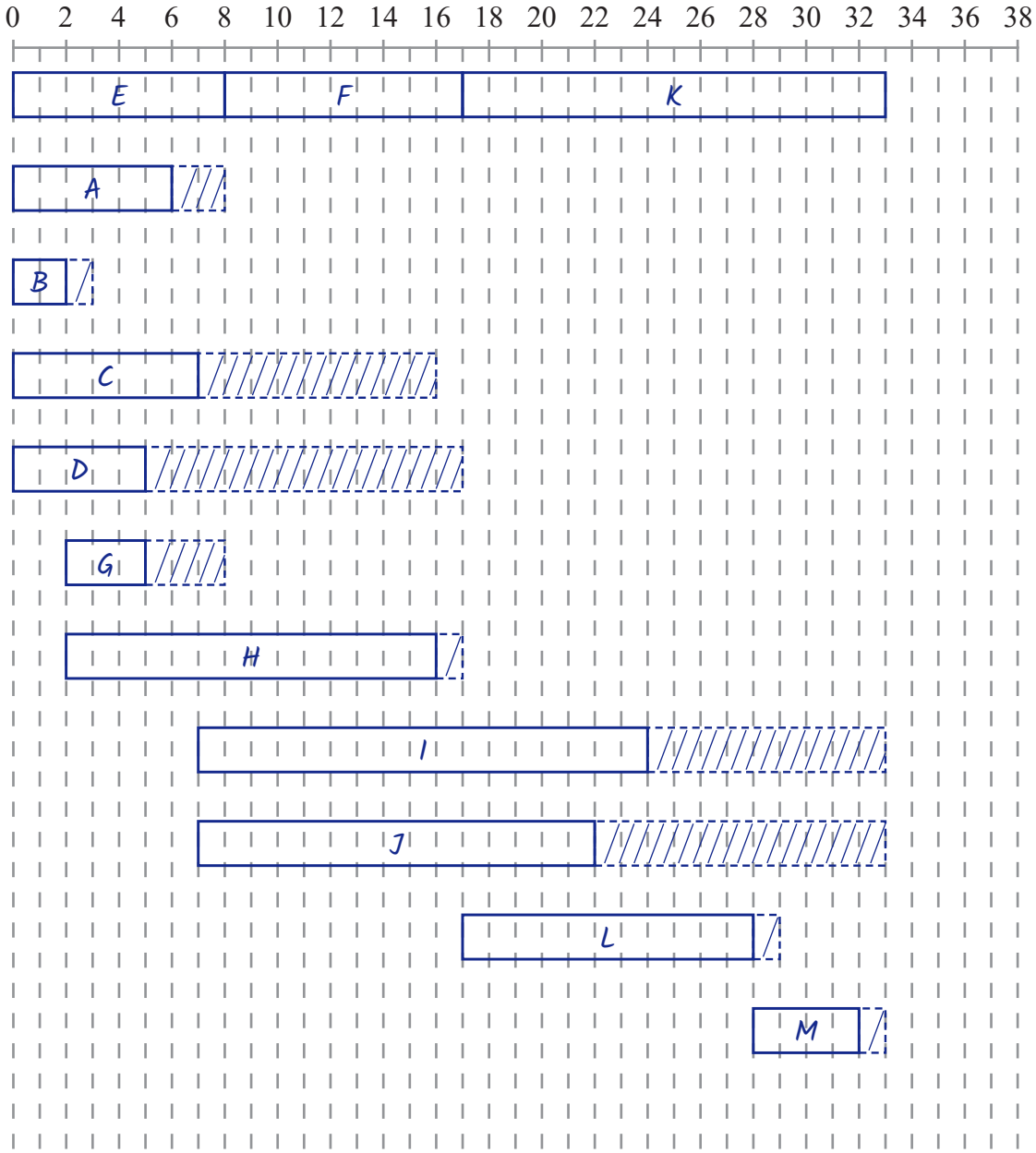
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Question 3 continued



Grid 1

(d) At time 19 (any time with 18 < t < 22) activities I, J, K and L must all be happening at the same time:

I must occupy 16 to 24, J must occupy 18 to 22,

K occupies 17 to 33 and L must occupy 18 to 28.

So the minimum number of workers needed is 4

Q3

(Total 11 marks)

P 5 4 9 5 2 A 0 7 2 0
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4.

180 80 250 115 100 230 150 95 105 90 390

(a) Total = 180 + 80 + 250 + 115 + 100 + 230 + 150 + 95 + 105 + 90 + 390 = 1785 kg

Lower bound = 1785 ÷ 475 = 3.757... so 4 van loads

(b) First-fit:

Van 1: 180 80 115 100

Van 2: 250 150

Van 3: 230 95 105

Van 4: 90

Van 5: 390

The van is needed 5 times (5 van loads).

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Question 4 continued

180 80 250 115 100 230 150 95 105 90 390

(c) Quick sort (descending), pivot = middle item (right of centre):

180 80 250 115 100 230 150 95 105 90 390 pivot 230

250 390 230 180 80 115 100 150 95 105 90 pivots 390, 150

390 250 230 180 150 80 115 100 95 105 90 pivots 250, 180, 95

390 250 230 180 150 115 100 105 95 80 90 pivots 100, 90

390 250 230 180 150 115 105 100 95 90 80 pivots 105, 80

390 250 230 180 150 115 105 100 95 90 80 sort complete

Sorted: 390 250 230 180 150 115 105 100 95 90 80

(underlined = pivots used in that pass, boxed = fixed pivots)

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Question 4 continued

(d) First-fit decreasing:

Van 1: 390 80

Van 2: 250 180

Van 3: 230 150 95

Van 4: 115 105 100 90

The van is needed 4 times (4 van loads).

(e) At most three boxes per van load:

Van 1: 390 80 (470 kg)

Van 2: 250 115 105 (470 kg)

Van 3: 230 150 95 (475 kg)

Van 4: 180 100 90 (370 kg)

So 4 van loads are still enough.

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Question 4 continued

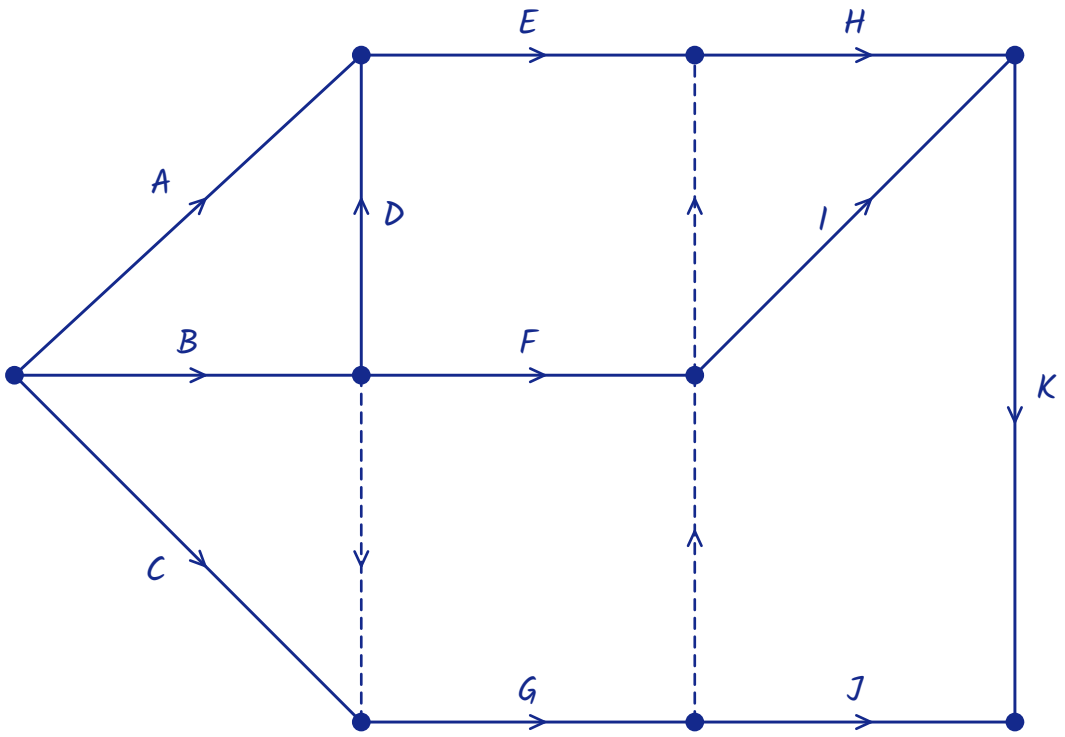
Q4

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5.



(dashed arcs are dummies)

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Question 5 continued

(b) B, E, H and K must also be critical.

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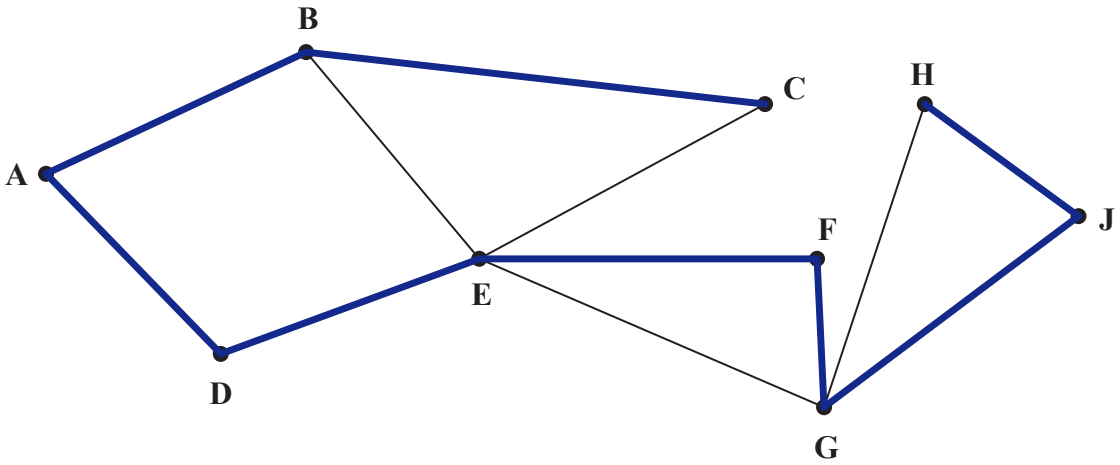
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6. (a)

Each arc contributes 1 to the valency of two vertices, so the sum of the valencies of all the vertices = 2 × number of arcs, which is even. Vertices of even valency add an even amount, so the total of the odd valencies must be even, which needs an even number of odd vertices. So an odd number of vertices of odd valency is impossible.



(b)(i) x = 9: AB 8, AD 9, BC 14, BE 39, CE 32, DE 21, EF 7, EG 20, FG 9, GH 14, GJ 11, HJ 13

Prim from A, arcs in order selected:

AB, AD, BC, DE, EF, FG, GJ, HJ

(tree shown in bold on the diagram above)

(ii) Weight = 8 + 9 + 14 + 21 + 7 + 9 + 11 + 13

= 92 minutes

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Question 6 continued

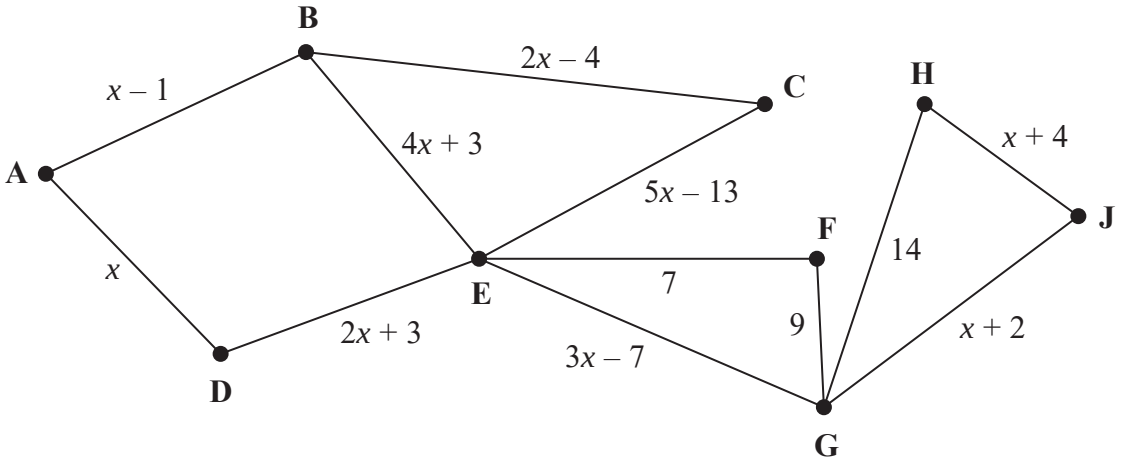


Figure 3

[*The weight of the network is* $20x + 17$]

(c) Valencies: A 2, B 3, C 2, D 2, E 5, F 2, G 4, H 2, J 2

Only odd vertices are B and E, so the shortest route between

B and E must be repeated: $20x + 17 + (\text{shortest BE}) = 162$

Routes B to E: BE $= 4x + 3$

BADE $= (x - 1) + x + (2x + 3) = 4x + 2$

BCE $= (2x - 4) + (5x - 13) = 7x - 17$

$4x + 3 > 4x + 2$ for all x, so BE is never the shortest;

the repeat is BADE or BCE.

BADE = BCE when $4x + 2 = 7x - 17 \Rightarrow x = 19/3 = 6.333...$

If $x < 19/3$, BCE is shortest:

$20x + 17 + 7x - 17 = 162 \Rightarrow 27x = 162 \Rightarrow x = 6$

$6 < 19/3$, so this is consistent.

If $x > 19/3$, BADE is shortest:

$20x + 17 + 4x + 2 = 162 \Rightarrow 24x = 143$

$\Rightarrow x = 143/24 = 5.958...$

but $5.958... < 19/3 = 6.333...$, a contradiction,

so $x \neq 143/24$.

$\therefore x = 6$

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Question 6 continued

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Question 6 continued

Q6

(Total 12 marks)

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7.

(a) Let x = number of Manhattan, y = number of Brooklyn

Maximise $P = 50x + 150y$

subject to:

$x + 4y \leq 30$ (cutting)

$1.5x + 4y \leq 35 \Rightarrow 3x + 8y \leq 70$ (assembling)

$2.5x + 3y \leq 45 \Rightarrow 5x + 6y \leq 90$ (packaging)

$x \geq 0, \ y \geq 0$

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Question 7 continued

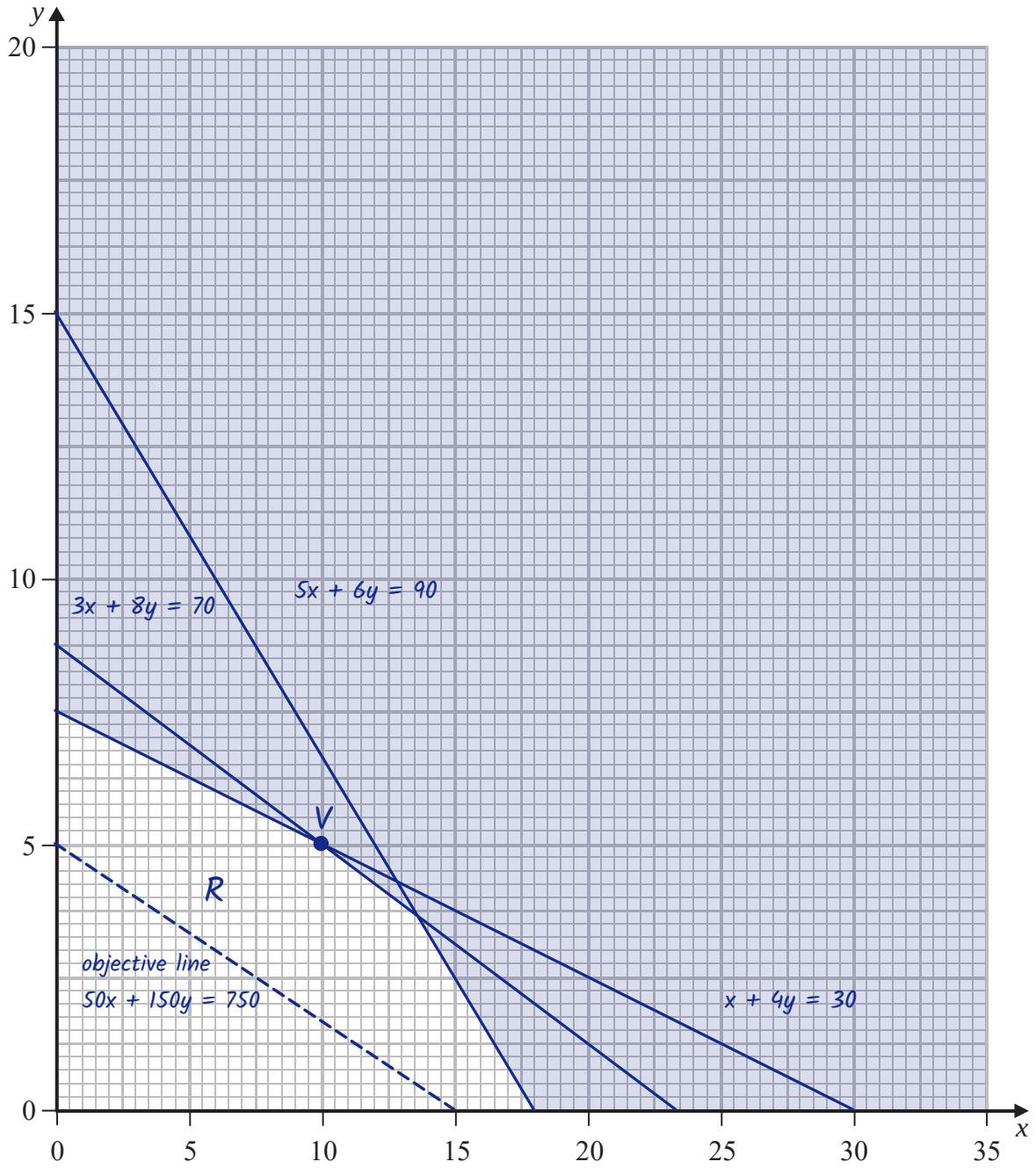


Diagram 1

(c) Objective line $50x + 150y = 750$ $(x + 3y = 15)$ drawn dashed;

sliding it away from O, the last point of R is V where

$x + 4y = 30$ and $3x + 8y = 70 \Rightarrow x = 10, y = 5,$ V = (10, 5)

(d) Make 10 Manhattan and 5 Brooklyn bookcases each week.

Maximum weekly profit = $50(10) + 150(5) = £1250$

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Question 7 continued

(e) At V(10, 5):

Cutting: $10 + 4(5) = 30$ hours = 30 available

Assembling: $1.5(10) + 4(5) = 35$ hours = 35 available

Packaging: $2.5(10) + 3(5) = 40$ hours < 45 available

(i) Packaging

(ii) $45 - 40 = 5$ hours

Q7

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

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