Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Monday 11 May 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. $$f(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1.4, 1.5]$

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 1.4$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(2)

(a)

$f(1.4) = -0.435673... < 0$

$f(1.5) = 0.598356... > 0$

Sign change and $f(x)$ is continuous on $[1.4, 1.5]$

$\therefore$ $f(x) = 0$ has a root α in the interval $[1.4, 1.5]$

(b)

$f(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} = x^3 - 10x^{-3/2} + 4x^{-1}$

$f'(x) = 3x^2 + 15x^{-5/2} - 4x^{-2}$

(c)

$f(1.4) = -0.435673...$, $f'(1.4) = 10.307200...$

$x_1 = 1.4 - \frac{f(1.4)}{f'(1.4)} = 1.4 - \frac{-0.435673...}{10.307200...}$

$x_1 = 1.442268...$

$x_1 = 1.442$ (3 d.p.)

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Question 1 continued

Q1

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2. The quadratic equation

$$5x^2 - 2x + 3 = 0$$

has roots α and β.

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine, giving each answer as a simplified fraction, the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(4)

(c) determine a quadratic equation that has roots

$$(\alpha + \beta^2) \text{ and } (\beta + \alpha^2)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers.

(4)

(a)

$\alpha + \beta = \frac{2}{5}, \qquad \alpha\beta = \frac{3}{5}$

(b)(i)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{2}{5}\right)^2 - 2\left(\frac{3}{5}\right) = \frac{4}{25} - \frac{30}{25} = -\frac{26}{25}$

(ii)

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= \left(\frac{2}{5}\right)^3 - 3\left(\frac{3}{5}\right)\left(\frac{2}{5}\right) = \frac{8}{125} - \frac{90}{125} = -\frac{82}{125}$

(c)

Sum $= \alpha + \beta + \alpha^2 + \beta^2 = \frac{2}{5} - \frac{26}{25} = -\frac{16}{25}$

Product $= (\alpha + \beta^2)(\beta + \alpha^2) = \alpha\beta + \alpha^3 + \beta^3 + (\alpha\beta)^2$

$= \frac{3}{5} - \frac{82}{125} + \left(\frac{3}{5}\right)^2 = \frac{75 - 82 + 45}{125} = \frac{38}{125}$

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Question 2 continued

$x^2 - (\text{sum})x + \text{product} = 0 \quad \Rightarrow \quad x^2 + \frac{16}{25}x + \frac{38}{125} = 0$

$\times 125$:

$125x^2 + 80x + 38 = 0$

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Question 2 continued

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Question 2 continued

Q2

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3. $f(z) = z^4 + az^3 + bz^2 + cz + d$

where a, b, c and d are integers.

The complex numbers $3 + i$ and $-1 - 2i$ are roots of the equation $f(z) = 0$

(a) Write down the other roots of this equation.

(2)

(b) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

(c) Determine the values of a, b, c and d.

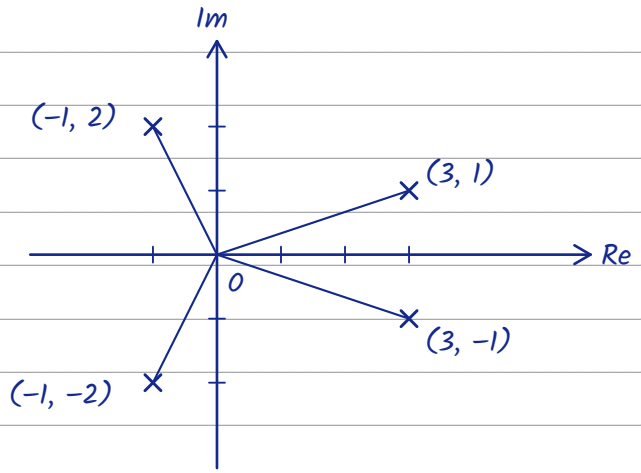
(5)

(a)

f(z) has real coefficients, so complex roots occur in conjugate pairs:

Other roots: $3 - i$ and $-1 + 2i$

(b)



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Question 3 continued

(c)

$z = 3 \pm i$: $(z - (3 + i))(z - (3 - i)) = (z - 3)^2 - i^2 = z^2 - 6z + 10$

$z = -1 \pm 2i$: $(z - (-1 + 2i))(z - (-1 - 2i)) = (z + 1)^2 - (2i)^2$

$= z^2 + 2z + 1 + 4 = z^2 + 2z + 5$

$f(z) = (z^2 - 6z + 10)(z^2 + 2z + 5)$

$= z^4 + 2z^3 + 5z^2 - 6z^3 - 12z^2 - 30z + 10z^2 + 20z + 50$

$= z^4 - 4z^3 + 3z^2 - 10z + 50$

$a = -4, \quad b = 3, \quad c = -10, \quad d = 50$

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Question 3 continued

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Question 3 continued

Q3

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4. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3} n(4n^2 - 1)$$

for all positive integers n.

(5)

(b) Hence find the exact value of the sum of the squares of the odd numbers between 200 and 500

(4)

(a)

$(2r-1)^2 = 4r^2 - 4r + 1$

$\sum_{r=1}^{n} (2r-1)^2 = 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$

$= 4 \times \frac{1}{6} n(n+1)(2n+1) - 4 \times \frac{1}{2} n(n+1) + n$

$= \frac{1}{3} n[2(n+1)(2n+1) - 6(n+1) + 3]$

$= \frac{1}{3} n[4n^2 + 6n + 2 - 6n - 6 + 3]$

$= \frac{1}{3} n(4n^2 - 1)$ as required

(b)

Odd numbers between 200 and 500: 201, 203, ..., 499

$2r - 1 = 201 \Rightarrow r = 101$, $2r - 1 = 499 \Rightarrow r = 250$

$\sum_{r=101}^{250} (2r-1)^2 = \sum_{r=1}^{250} (2r-1)^2 - \sum_{r=1}^{100} (2r-1)^2$

$= \frac{1}{3} \times 250(4 \times 250^2 - 1) - \frac{1}{3} \times 100(4 \times 100^2 - 1)$

$= 20\,833\,250 - 1\,333\,300$

$= 19\,499\,950$

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

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5. The rectangular hyperbola H has equation $xy = 64$

The point $P\left(8p, \frac{8}{p}\right)$, where $p \neq 0$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$p^3x - py = 8(p^4 - 1)$$

(5)

The normal to H at P meets H again at the point Q.

(b) Determine, in terms of p, the coordinates of Q, giving your answers in simplest form.

(4)

(a)

$xy = 64 \Rightarrow y = 64x^{-1} \Rightarrow \frac{dy}{dx} = -64x^{-2} = -\frac{64}{x^2}$

At P, $x = 8p$: $\frac{dy}{dx} = -\frac{64}{64p^2} = -\frac{1}{p^2}$

Gradient of normal $m_N = p^2$

$y - \frac{8}{p} = p^2(x - 8p)$

$\times p$: $py - 8 = p^3x - 8p^4$

$p^3x - py = 8p^4 - 8$

$p^3x - py = 8(p^4 - 1)$ as required

(b)

$y = \frac{64}{x}$: $p^3x - \frac{64p}{x} = 8(p^4 - 1)$

$p^3x^2 - 64p = 8(p^4 - 1)x$

$p^3x^2 + 8(1 - p^4)x - 64p = 0$

$(x - 8p)(p^3x + 8) = 0$

$x = 8p$ (at P) or $x = -\frac{8}{p^3}$

$y = \frac{64}{x} = 64 \times \left(\frac{-p^3}{8}\right) = -8p^3$

Q is $\left(-\frac{8}{p^3}, -8p^3\right)$

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Question 5 continued

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Question 5 continued

Q5

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6. (i) $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents a rotation of 45° clockwise about the origin.

(b) Write down the matrix **B**, giving each element of the matrix in exact form.

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Determine **C**.

(2)

(ii) The trapezium T has vertices at the points $(-2, 0)$, $(-2, k)$, $(5, 8)$ and $(5, 0)$, where k is a positive constant. Trapezium T is transformed onto the trapezium T' by the matrix

$$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}$$

Given that the area of trapezium T' is 510 square units, calculate the exact value of k.

(5)

(i)(a)

Stretch, scale factor 3, parallel to the y-axis

(b)

$B = \begin{pmatrix} \cos 45° & \sin 45° \\ -\sin 45° & \cos 45° \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

(c)

$C = BA = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}$

(ii)

Area of $T = \frac{1}{2}(k + 8)(5 - (-2)) = \frac{7}{2}(k + 8)$

$\det\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix} = 5 \times 3 - (-2) \times 1 = 17$

Area of T' = 17 × area of T

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Question 6 continued

$17 \times \frac{7}{2}(k + 8) = 510$

$k + 8 = \frac{1020}{119} = \frac{60}{7}$

$k = \frac{60}{7} - 8 = \frac{4}{7}$

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Question 6 continued

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Question 6 continued

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Q6

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7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The line l with equation $3x - 4y + 48 = 0$ is a tangent to C at the point P.

(a) Show that $a = 9$

(4)

(b) Hence determine the coordinates of P.

(2)

Given that the point S is the focus of C and that the line l crosses the directrix of C at the point A,

(c) determine the exact area of triangle PSA.

(4)

(a)

$3x - 4y + 48 = 0 \Rightarrow x = \frac{4y - 48}{3}$

$y^2 = 4ax \Rightarrow y^2 = 4a\left(\frac{4y - 48}{3}\right)$

$3y^2 = 16ay - 192a \Rightarrow 3y^2 - 16ay + 192a = 0$

l is a tangent so equal roots: $b^2 = 4ac$

$(16a)^2 = 4 \times 3 \times 192a$

$256a^2 = 2304a \Rightarrow 256a(a - 9) = 0$

$a > 0 \Rightarrow a = 9$ as required

(b)

$a = 9$: $3y^2 - 144y + 1728 = 0 \Rightarrow y^2 - 48y + 576 = 0$

$(y - 24)^2 = 0 \Rightarrow y = 24$

$x = \frac{4(24) - 48}{3} = 16$

P is (16, 24)

(c)

Focus S is (9, 0), directrix is $x = -9$

$x = -9$: $3(-9) - 4y + 48 = 0 \Rightarrow 4y = 21 \Rightarrow y = \frac{21}{4} = 5.25$

A is (−9, 5.25)

Line l meets $x = 9$ at B: $27 - 4y + 48 = 0 \Rightarrow y = 18.75$, B(9, 18.75)

SB is vertical, length 18.75; A is 18 to the left, P is 7 to the right

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Question 7 continued

Area PSA $= \frac{1}{2} \times 18.75 \times 18 + \frac{1}{2} \times 18.75 \times (16 - 9)$

$= 168.75 + 65.625 = 234.375$

Area of triangle PSA $= \frac{1875}{8}$

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Question 7 continued

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Question 7 continued

Q7

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8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2r^2 - 1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2}$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 12^n + 2 \times 5^{n-1}$$

is divisible by 7

(6)

(i)

$n = 1$: LHS $= \frac{2(1)^2 - 1}{1^2(1+1)^2} = \frac{1}{4}$, RHS $= \frac{1^2}{(1+1)^2} = \frac{1}{4}$, true for $n = 1$

Assume true for $n = k$: $\sum_{r=1}^{k} \frac{2r^2 - 1}{r^2(r+1)^2} = \frac{k^2}{(k+1)^2}$

$n = k + 1$: $\sum_{r=1}^{k+1} \frac{2r^2 - 1}{r^2(r+1)^2} = \frac{k^2}{(k+1)^2} + \frac{2(k+1)^2 - 1}{(k+1)^2(k+2)^2}$

$$= \frac{k^2(k+2)^2 + 2(k+1)^2 - 1}{(k+1)^2(k+2)^2}$$

$$= \frac{k^4 + 4k^3 + 4k^2 + 2k^2 + 4k + 2 - 1}{(k+1)^2(k+2)^2} = \frac{k^4 + 4k^3 + 6k^2 + 4k + 1}{(k+1)^2(k+2)^2}$$

$$= \frac{(k+1)^4}{(k+1)^2(k+2)^2} = \frac{(k+1)^2}{(k+2)^2} = \frac{(k+1)^2}{((k+1)+1)^2}$$

This is the result with $n = k + 1$.

If the result is true for $n = k$, then it is true for $n = k + 1$.

As the result has been shown to be true for $n = 1$, the result is true for all $n \in \mathbb{Z}^+$ by mathematical induction.

(ii)

$n = 1$: $f(1) = 12 + 2 \times 5^0 = 12 + 2 = 14 = 2 \times 7$, divisible by 7

Assume true for $n = k$, i.e. $f(k) = 12^k + 2 \times 5^{k-1}$ is divisible by 7

$f(k+1) = 12^{k+1} + 2 \times 5^k$

$f(k+1) - f(k) = 12^{k+1} + 2 \times 5^k - 12^k - 2 \times 5^{k-1}$

$= 11 \times 12^k + 10 \times 5^{k-1} - 2 \times 5^{k-1}$

$= 11 \times 12^k + 8 \times 5^{k-1}$

$= 11(12^k + 2 \times 5^{k-1}) - 14 \times 5^{k-1}$

$= 11f(k) - 14 \times 5^{k-1}$

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Question 8 continued

$\therefore\ f(k+1) = 12f(k) - 14 \times 5^{k-1} = 12f(k) - 7(2 \times 5^{k-1})$

$f(k)$ is divisible by 7 and $7(2 \times 5^{k-1})$ is divisible by 7,

so $f(k+1)$ is divisible by 7.

If the result is true for $n = k$, then it is true for $n = k + 1$.

As the result has been shown to be true for $n = 1$, the result is

true for all $n \in \mathbb{Z}^+$ by mathematical induction.

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

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TOTAL FOR PAPER: 75 MARKS

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