Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 22 May 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1: $$\mathbf{M} = \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix}$$

where a is a constant, $a \neq -\frac{5}{3}$

(a) Determine $\mathbf{M}^{-1}$ in terms of a.

(3)

Given that $\mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a.

(3)

(a) $\det M = (1)(-5) - 3a = -5 - 3a$

$$M^{-1} = \frac{1}{-5-3a}\begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix}$$

(b) $$\frac{1}{-5-3a}\begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix} = 2\begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix} + 8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{1}{-5-3a}\begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 2+8 & 2a \\ 6 & -10+8 \end{pmatrix} = \begin{pmatrix} 10 & 2a \\ 6 & -2 \end{pmatrix}$$

top left elements:

$$\frac{-5}{-5-3a} = 10 \Rightarrow -5 = 10(-5-3a)$$

$$\Rightarrow -5 = -50 - 30a \Rightarrow 30a = -45$$

$$\Rightarrow a = -\frac{3}{2}$$

check (bottom right): $\frac{1}{-5+4.5} = -2$ ✓

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 6 marks)

2:

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The matrix **P** represents the transformation P.

(a) Describe P fully as a single geometrical transformation.

(2)

$$\mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

The matrix **Q** represents the transformation Q.

(b) Describe Q fully as a single geometrical transformation.

(2)

The transformation R is a reflection in the line $y = -x$

(c) Write down the matrix representing R.

(1)

The transformation R maps the point $(4, 3)$ to the point A.

(d) Determine the coordinates of A.

(1)

The transformation P followed by the transformation Q maps the point B to the point A.

(e) Determine the coordinates of B.

(2)

(a) Rotation through 90° clockwise about the origin O
(equivalently 270° anticlockwise about O)

(b) Enlargement, scale factor 5, centre the origin O

(c) $R = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ $A(-3, -4)$

(e) $QP = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow 5y = -3, \quad -5x = -4$

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Question 2 continued

$x = \frac{4}{5}, \quad y = -\frac{3}{5}$

$B\left(\frac{4}{5}, -\frac{3}{5}\right)$

Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

3: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) $f(x) = x^2 + 5 - 8^{5x}$

Given that the equation $f(x) = 0$ has a single root, α, in the interval $[0, 1]$

use interval bisection to determine an interval of width 0.25 that contains α.

(3)

(ii) $g(x) = 3^{\sin x} - 3\cos x$

where x is in radians.

(a) Show that the equation $g(x) = 0$ has a root, β, in the interval $[4, 5]$

(2)

(b) Use linear interpolation once on the interval $[4, 5]$ to determine an approximation for β.
Give your answer to 4 significant figures.

(2)

(i) $f(x) = x^2 + 5 - 8^{5x}$

$f(0) = 4 > 0, \quad f(1) = -32762 < 0$

$f(0.5) = 0.25 + 5 - 8^{2.5} = -175.769... < 0 \Rightarrow \alpha \in [0, 0.5]$

$f(0.25) = 0.0625 + 5 - 8^{1.25} = -8.39184... < 0$

$\Rightarrow \alpha \in [0, 0.25]$

(ii)(a) $g(4) = 3^{\sin 4} - 3\cos 4 = 2.396... > 0$

$g(5) = 3^{\sin 5} - 3\cos 5 = -0.502... < 0$

sign change and $g(x)$ is continuous on $[4, 5]$,

so there is a root β between $x = 4$ and $x = 5$

(b) $\frac{\beta - 4}{2.396...} = \frac{5 - \beta}{0.502...}$

$\beta = 4 + \frac{2.39635...}{2.39635... + 0.50226...} = 4.82672...$

$\beta \approx 4.827$ (4 s.f.)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

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4: The rectangular hyperbola C has equation $xy = 81$

The point $P\left(9t, \frac{9}{t}\right)$, $t \neq 0$, lies on C.

(a) Use calculus to show that the normal to C at P has equation

$$ty = t^3x + 9\left(1 - t^4\right)$$

(4)

The normal to C at P meets the y-axis at the point A.

(b) Determine the exact coordinates of A in terms of t.

(2)

Given that $t = \frac{1}{3}$

(c) find the exact area of triangle OPA, where O is the origin.

(2)

(a) $y = \frac{81}{x} = 81x^{-1} \Rightarrow \frac{dy}{dx} = -\frac{81}{x^2}$

at P, $x = 9t$: $\frac{dy}{dx} = -\frac{81}{81t^2} = -\frac{1}{t^2}$

gradient of normal $= t^2$

$y - \frac{9}{t} = t^2(x - 9t)$

$ty - 9 = t^3x - 9t^4$

$\Rightarrow ty = t^3x + 9(1 - t^4)$ as required

(b) $x = 0$: $ty = 9(1 - t^4) \Rightarrow y = \frac{9(1 - t^4)}{t}$

$A\left(0, \frac{9(1 - t^4)}{t}\right)$

(c) $t = \frac{1}{3}$: $P(3, 27)$, $y_A = 27\left(1 - \frac{1}{81}\right) = \frac{80}{3}$

Area OPA $= \frac{1}{2} \times OA \times x_P = \frac{1}{2} \times \frac{80}{3} \times 3$

$= 40$

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

5: The complex number z_1 is given by

$$z_1 = r\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

where r is a positive constant.

The complex number z_2 has modulus 5

Given that $|z_1 z_2| = 15$

(a) state the value of r.

(1)

Given further that $z_1 + z_2$ is a real number,

(b) determine the possible complex numbers z_2 in the form $a + bi$ where a and b are constants.

(5)

(c) Show z_1 and the possible complex numbers z_2 on a single Argand diagram.

(2)

(a) $|z_1||z_2| = 15 \Rightarrow 5r = 15 \Rightarrow r = 3$

(b) $z_1 = 3\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$

let $z_2 = a + bi$. $z_1 + z_2$ real $\Rightarrow \operatorname{Im}(z_1 + z_2) = 0$

$-\frac{3}{2} + b = 0 \Rightarrow b = \frac{3}{2}$

$|z_2| = 5$: $a^2 + \left(\frac{3}{2}\right)^2 = 25 \Rightarrow a^2 = \frac{91}{4}$

$a = \pm\frac{\sqrt{91}}{2}$

$z_2 = \frac{\sqrt{91}}{2} + \frac{3}{2}i$ or $z_2 = -\frac{\sqrt{91}}{2} + \frac{3}{2}i$

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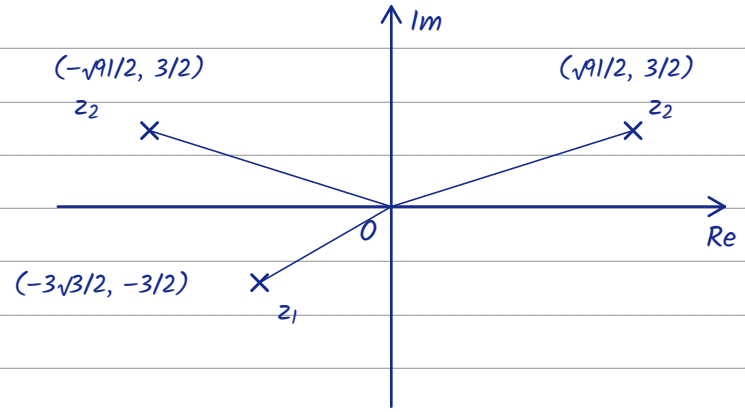
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Question 5 continued

(c)



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Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

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6: $f(x) = 3x^2 + kx - 5$

where k is a constant.

The equation $f(x) = 0$ has roots α and β.

(a) State the value of $\alpha\beta$.

(1)

Given that $\alpha + \beta = 9\alpha\beta$

(b) determine the value of k.

(2)

(c) By first expanding $(\alpha + \beta)^3$ prove that

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

(2)

Without solving the equation $f(x) = 0$

(d) find a quadratic equation with integer coefficients that has roots

$$(\alpha^2 + \beta) \text{ and } (\alpha + \beta^2)$$

(6)

(a) $\alpha\beta = \frac{c}{a} = -\frac{5}{3}$

(b) $\alpha + \beta = -\frac{k}{3}$, $\alpha + \beta = 9\alpha\beta \Rightarrow -\frac{k}{3} = 9 \times \left(-\frac{5}{3}\right) = -15$

$\Rightarrow k = 45$

(c) $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$

$\Rightarrow \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$

$= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ as required

(d) $\alpha + \beta = -15$, $\alpha\beta = -\frac{5}{3}$

Sum: $(\alpha^2 + \beta) + (\alpha + \beta^2) = (\alpha + \beta) + (\alpha + \beta)^2 - 2\alpha\beta$

$= -15 + (-15)^2 - 2\left(-\frac{5}{3}\right) = -15 + 225 + \frac{10}{3} = \frac{640}{3}$

Product: $(\alpha^2 + \beta)(\alpha + \beta^2) = \alpha^3 + \alpha^2\beta^2 + \alpha\beta + \beta^3$

$= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) + \alpha\beta + (\alpha\beta)^2$

$= (-15)^3 - 3\left(-\frac{5}{3}\right)(-15) - \frac{5}{3} + \left(-\frac{5}{3}\right)^2$

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Question 6 continued

$$= -3375 - 75 - \frac{5}{3} + \frac{25}{9} = -\frac{31040}{9}$$

$$x^2 - \frac{640}{3}x - \frac{31040}{9} = 0$$

$$9x^2 - 1920x - 31040 = 0$$

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 11 marks)

7: $f(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68$

where P and Q are real constants.

Given that $3 + 5i$ is a root of the equation $f(z) = 0$

(a) write down another root of the equation $f(z) = 0$

(1)

(b) Hence determine a quadratic factor, with real coefficients, of $f(z)$

(2)

(c) Determine the value of P and the value of Q.

(5)

Without using a calculator,

(d) determine the other roots of the equation $f(z) = 0$ giving your answers in simplest form.

(2)

(a) $3 - 5i$

(b) $(z - (3 + 5i))(z - (3 - 5i)) = (z - 3)^2 - (5i)^2$

$= z^2 - 6z + 9 + 25 = z^2 - 6z + 34$

(c) $Pz^4 - 36z^3 + Qz^2 + 192z + 68 = (z^2 - 6z + 34)(az^2 + bz + c)$

$= az^4 + (b - 6a)z^3 + (c - 6b + 34a)z^2 + (34b - 6c)z + 34c$

constant: $34c = 68 \Rightarrow c = 2$

z: $34b - 6c = 192 \Rightarrow 34b - 12 = 192 \Rightarrow b = 6$

z^3: $b - 6a = -36 \Rightarrow 6 - 6a = -36 \Rightarrow a = 7$, so $P = 7$

z^2: $Q = c - 6b + 34a = 2 - 36 + 238 = 204$

$P = 7, \quad Q = 204$

(d) $7z^2 + 6z + 2 = 0$

$$z = \frac{-6 \pm \sqrt{(6^2 - 4 \times 7 \times 2)}}{2 \times 7} = \frac{-6 \pm \sqrt{(-20)}}{14} = \frac{-6 \pm 2\sqrt{5}\,i}{14}$$

$$z = -\frac{3}{7} + \frac{\sqrt{5}}{7}i \quad \text{and} \quad z = -\frac{3}{7} - \frac{\sqrt{5}}{7}i$$

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

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8: (a) Using the standard summation formulae show that

$$\sum_{r=1}^{2n}\left(2r^2-1\right)=\frac{4}{3}n(n+1)(an+b)$$

where a and b are integers to be determined.

(5)

(b) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r(3r-2)^2=\frac{n^2(n+1)(9n-7)}{4}$$

(5)

Using the results from parts (a) and (b) and showing all stages of your working,

(c) determine the value of n for which

$$8\sum_{r=1}^{n} r(3r-2)^2=15\sum_{r=1}^{2n}\left(2r^2-1\right)$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

(a) $\sum_{r=1}^{2n}(2r^2-1)=2\sum_{r=1}^{2n}r^2-\sum_{r=1}^{2n}1$

$=2\times\frac{2n}{6}(2n+1)(4n+1)-2n$

$=\frac{2n}{3}[(2n+1)(4n+1)-3]$

$=\frac{2n}{3}(8n^2+6n-2)=\frac{4n}{3}(4n^2+3n-1)$

$=\frac{4}{3}n(n+1)(4n-1)$ $\quad a=4,\ b=-1$

(b) $n=1$: LHS $=1(3-2)^2=1$, RHS $=\frac{1^2\times2\times2}{4}=1$ ✓ true for $n=1$

Assume true for $n=k$: $\sum_{r=1}^{k}r(3r-2)^2=\frac{k^2(k+1)(9k-7)}{4}$

$\sum_{r=1}^{k+1}r(3r-2)^2=\frac{k^2(k+1)(9k-7)}{4}+(k+1)(3k+1)^2$

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Question 8 continued

$= \frac{k+1}{4}[k^2(9k-7) + 4(9k^2+6k+1)]$

$= \frac{k+1}{4}(9k^3+29k^2+24k+4)$

$= \frac{k+1}{4}(k+1)(k+2)(9k+2)$

$= \frac{(k+1)^2((k+1)+1)(9(k+1)-7)}{4}$

which is the result with $n = k+1$.

The result is true for $n = 1$, and if it is true for $n = k$ then it is also true for $n = k+1$. Hence, by mathematical induction, it is true for all $n \in \mathbb{N}$.

(c) $8 \times \frac{n^2(n+1)(9n-7)}{4} = 15 \times \frac{4}{3} n(n+1)(4n-1)$

$2n^2(n+1)(9n-7) = 20n(n+1)(4n-1)$

$n > 0$: $2n(9n-7) = 20(4n-1)$

$18n^2 - 14n = 80n - 20 \Rightarrow 18n^2 - 94n + 20 = 0$

$9n^2 - 47n + 10 = 0 \Rightarrow (9n-2)(n-5) = 0$

$n = \frac{2}{9}$ (not an integer, reject) or $n = 5$

$n = 5$

Question 8 continued

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Question 8 continued

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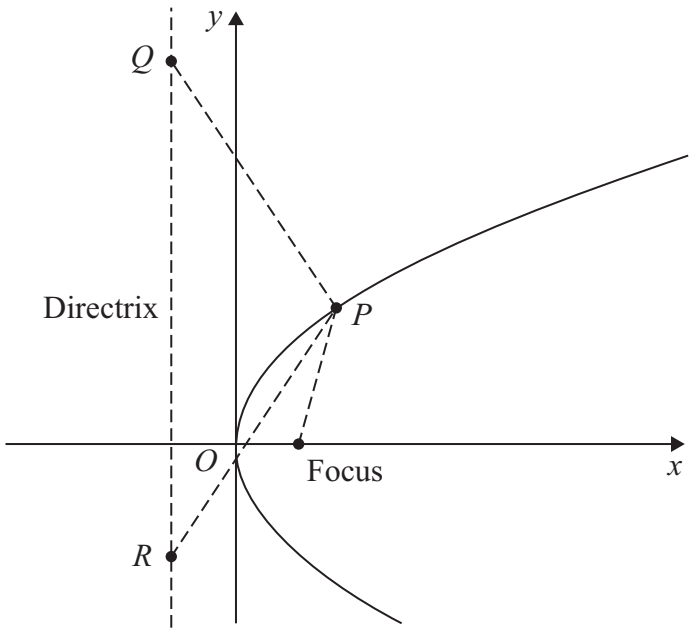


Figure 1

Figure 1 shows a sketch of a parabola C.

The point P lies on C at a distance 5 units from the focus of C.

The points Q and R lie on the directrix of C, where $PQ = PR = 10$ units.

Determine the exact length of QR giving the answer in simplest form.

(4)

Let M be the foot of the perpendicular from P to the directrix.

By the focus-directrix property, $PM = PS = 5$

Triangle PMQ is right-angled at M:

$QM = \sqrt{(PQ^2 - PM^2)} = \sqrt{(10^2 - 5^2)} = \sqrt{75} = 5\sqrt{3}$

similarly $RM = 5\sqrt{3}$, with Q and R on opposite sides of M

$QR = 2 \times 5\sqrt{3} = 10\sqrt{3}$

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Question 9 continued

Question 9 continued

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TOTAL FOR PAPER IS 75 MARKS

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