Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

P66060A

©2021 Pearson Education Ltd.

1/1/1/

P 6 6 0 6 0 A 0 1 3 2

Pearson

Leave blank

1. (i) $f(x) = x^3 + 4x - 6$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 1.5]$

(2)

(b) Taking 1.5 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places. Show your working clearly.

(4)

(ii) $g(x) = 4x^2 + x - \tan x$

where x is measured in radians.

The equation $g(x) = 0$ has a single root β in the interval $[1.4, 1.5]$

Use linear interpolation on the values at the end points of this interval to obtain an approximation to β. Give your answer to 3 decimal places.

(4)

(i)(a)

$f(1) = 1 + 4 - 6 = -1 < 0$

$f(1.5) = 3.375 + 6 - 6 = 3.375 > 0$

Sign change and $f(x)$ is continuous on $[1, 1.5]$

$\therefore$ $f(x) = 0$ has a root α in the interval $[1, 1.5]$

(b)

$f'(x) = 3x^2 + 4$

$x_2 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{3.375}{10.75} = 1.186046...$

$f(1.186046...) = 0.412605...$, $f'(1.186046...) = 8.220118...$

$x_3 = 1.186046... - \frac{0.412605...}{8.220118...} = 1.135851...$

$\alpha \approx 1.136$ (3 d.p.)

(ii)

$g(1.4) = 4(1.4)^2 + 1.4 - \tan 1.4 = 3.442116...$

$g(1.5) = 4(1.5)^2 + 1.5 - \tan 1.5 = -3.601419...$

$\frac{\beta - 1.4}{3.442116...} = \frac{1.5 - \beta}{3.601419...}$

$3.601419...(\beta - 1.4) = 3.442116...(1.5 - \beta)$

$\beta = \frac{1.4 \times 3.601419... + 1.5 \times 3.442116...}{3.601419... + 3.442116...} = \frac{10.205162...}{7.043536...}$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 1 continued

$\beta = 1.448869...$

$\beta \approx 1.449$ (3 d.p.)

Leave blank

Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 1 continued

(Total 10 marks)

Q1

Leave blank

2. The complex numbers z_1, z_2 and z_3 are given by

$$z_1 = 2 - \mathrm{i} \quad z_2 = p - \mathrm{i} \quad z_3 = p + \mathrm{i}$$

where p is a real number.

(a) Find $\dfrac{z_2 z_3}{z_1}$ in the form $a + b\mathrm{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(3)

Given that $\left|\dfrac{z_2 z_3}{z_1}\right| = 2\sqrt{5}$

(b) find the possible values of p.

(4)

(a)

$$\frac{z_2 z_3}{z_1} = \frac{(p - i)(p + i)}{2 - i} = \frac{p^2 + 1}{2 - i} \times \frac{2 + i}{2 + i}$$

$$= \frac{(p^2 + 1)(2 + i)}{4 - i^2} = \frac{(p^2 + 1)(2 + i)}{5}$$

$$= \frac{2(p^2 + 1)}{5} + \frac{p^2 + 1}{5} i$$

(b)

$$\left|\frac{z_2 z_3}{z_1}\right|^2 = \frac{4(p^2 + 1)^2}{25} + \frac{(p^2 + 1)^2}{25} = \frac{(p^2 + 1)^2}{5}$$

$$\frac{(p^2 + 1)^2}{5} = (2\sqrt{5})^2 = 20$$

$(p^2 + 1)^2 = 100 \Rightarrow p^2 + 1 = 10 \quad (p^2 + 1 > 0)$

$p^2 = 9$

$p = 3$ or $p = -3$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Leave blank

Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Leave blank

3. The triangle T has vertices $A(2,1)$, $B(2,3)$ and $C(0,1)$.

The triangle T' is the image of T under the transformation represented by the matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) Find the coordinates of the vertices of T'

(2)

(b) Describe fully the transformation represented by **P**

(2)

The 2×2 matrix **Q** represents a reflection in the x-axis and the 2×2 matrix **R** represents a rotation through 90° anticlockwise about the origin.

(c) Write down the matrix **Q** and the matrix **R**

(2)

(d) Find the matrix **RQ**

(2)

(e) Give a full geometrical description of the single transformation represented by the answer to part (d).

(2)

(a)

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 \\ -2 & -2 & 0 \end{pmatrix}$

Vertices of T': $(1, -2)$, $(3, -2)$, $(1, 0)$

(b)

Rotation through 90° clockwise about the origin O

(equivalently 270° anticlockwise about (0, 0))

(c)

$Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(d)

$RQ = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(e)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Reflection in the line y = x

Leave blank

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

(Total 10 marks)

Q3

Leave blank

4. A rectangular hyperbola H has equation $xy = 25$

The point $P\left(5t, \frac{5}{t}\right)$, $t \neq 0$, is a general point on H.

(a) Show that the equation of the tangent to H at P is $t^2y + x = 10t$

(4)

The distinct points Q and R lie on H. The tangent to H at the point Q and the tangent to H at the point R meet at the point $(15, -5)$.

(b) Find the coordinates of the points Q and R.

(4)

(a)

$x = 5t, \quad y = \frac{5}{t} \quad \Rightarrow \quad \frac{dx}{dt} = 5, \quad \frac{dy}{dt} = -\frac{5}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = -\frac{5}{t^2} \times \frac{1}{5} = -\frac{1}{t^2}$

Tangent at P: $y - \frac{5}{t} = -\frac{1}{t^2}(x - 5t)$

$\times t^2$: $t^2y - 5t = -x + 5t$

$t^2y + x = 10t$ as required

(b)

Substitute $(15, -5)$: $t^2(-5) + 15 = 10t$

$5t^2 + 10t - 15 = 0 \Rightarrow t^2 + 2t - 3 = 0$

$(t + 3)(t - 1) = 0 \Rightarrow t = -3$ or $t = 1$

$t = -3$: $x = -15, \quad y = \frac{5}{-3} = -\frac{5}{3}$

$t = 1$: $x = 5, \quad y = 5$

Q and R are $\left(-15, -\frac{5}{3}\right)$ and $(5, 5)$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Leave blank

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. $f(x) = (9x^2 + d)\left(x^2 - 8x + (10d + 1)\right)$

where d is a positive constant.

(a) Find the four roots of $f(x)$ giving your answers in terms of d.

(3)

Given $d = 4$

(b) Express these four roots in the form $a + ib$, where $a, b \in \mathbb{R}$.

(2)

(c) Show these four roots on a single Argand diagram.

(2)

(a)

$9x^2 + d = 0 \Rightarrow x^2 = -\frac{d}{9} \Rightarrow x = \pm \frac{i\sqrt{d}}{3}$

$x^2 - 8x + (10d + 1) = 0 \Rightarrow x = \frac{8 \pm \sqrt{(64 - 4(10d + 1))}}{2} = \frac{8 \pm \sqrt{(60 - 40d)}}{2}$

$x = 4 \pm \sqrt{(15 - 10d)}$

Roots: $x = \frac{i\sqrt{d}}{3}$, $x = -\frac{i\sqrt{d}}{3}$, $x = 4 + \sqrt{(15 - 10d)}$, $x = 4 - \sqrt{(15 - 10d)}$

(b)

$d = 4$: $x = \pm \frac{2}{3}i$ and $x = 4 \pm \sqrt{(-25)} = 4 \pm 5i$

Roots: $0 + \frac{2}{3}i$, $0 - \frac{2}{3}i$, $4 + 5i$, $4 - 5i$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

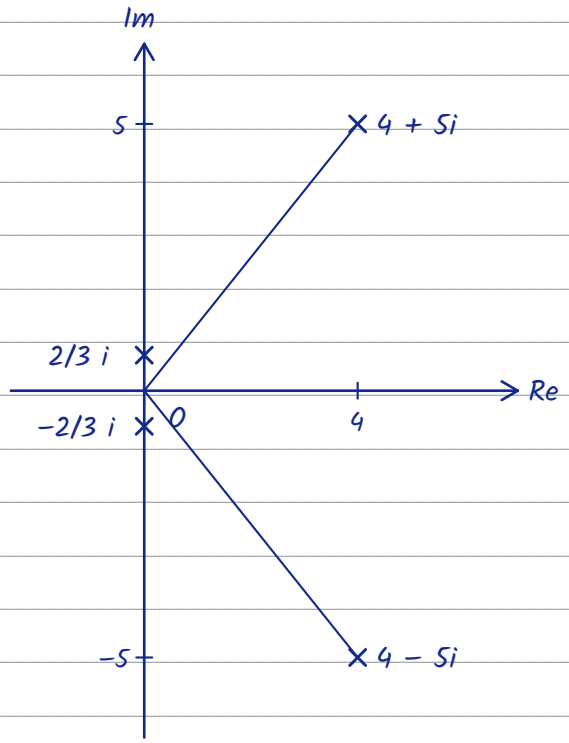
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

(c)



Leave blank

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6. The parabola C has Cartesian equation $y^2 = 8x$

The point $P(2p^2, 4p)$ and the point $Q(2q^2, 4q)$, where $p, q \neq 0$, $p \neq q$, are points on C.

(a) Show that an equation of the normal to C at P is

$$y + px = 2p^3 + 4p$$

(5)

(b) Write down an equation of the normal to C at Q

(1)

The normal to C at P and the normal to C at Q meet at the point N

(c) Show that N has coordinates

$$\left(2(p^2 + pq + q^2 + 2), -2pq(p + q)\right)$$

(5)

The line ON, where O is the origin, is perpendicular to the line PQ

(d) Find the value of $(p + q)^2 - 3pq$

(5)

(a)

$x = 2p^2$, $y = 4p$ $\Rightarrow$ $\frac{dx}{dp} = 4p$, $\frac{dy}{dp} = 4$

$\frac{dy}{dx} = \frac{4}{4p} = \frac{1}{p}$

Gradient of normal at P $= -p$

$y - 4p = -p(x - 2p^2)$

$y - 4p = -px + 2p^3$

$y + px = 2p^3 + 4p$ as required

(b)

$y + qx = 2q^3 + 4q$

(c)

Subtract: $px - qx = 2p^3 - 2q^3 + 4p - 4q$

$(p - q)x = 2(p - q)(p^2 + pq + q^2) + 4(p - q)$

$p \neq q$, divide by $(p - q)$: $x = 2(p^2 + pq + q^2) + 4 = 2(p^2 + pq + q^2 + 2)$

$y = 2p^3 + 4p - px = 2p^3 + 4p - 2p^3 - 2p^2q - 2pq^2 - 4p$

$y = -2p^2q - 2pq^2 = -2pq(p + q)$

N is $(2(p^2 + pq + q^2 + 2), -2pq(p + q))$ as required

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

(d)

Gradient $PQ = \frac{4q - 4p}{2q^2 - 2p^2} = \frac{4(q - p)}{2(q - p)(q + p)} = \frac{2}{p + q}$

Gradient $ON = \frac{-2pq(p + q)}{2(p^2 + pq + q^2 + 2)} = \frac{-pq(p + q)}{p^2 + pq + q^2 + 2}$

Perpendicular: $\frac{2}{p + q} \times \frac{-pq(p + q)}{p^2 + pq + q^2 + 2} = -1$

$-2pq = -(p^2 + pq + q^2 + 2)$

$p^2 + pq + q^2 + 2 = 2pq \Rightarrow p^2 - pq + q^2 = -2$

$(p + q)^2 - 3pq = p^2 + 2pq + q^2 - 3pq = p^2 - pq + q^2$

$(p + q)^2 - 3pq = -2$

Leave blank

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

(Total 16 marks)

Q6

Leave blank

7. (a) Prove by induction that for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

(5)

(b) Hence show that

$$\sum_{r=1}^{n} (r^2 + 2) = \frac{n}{6}(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

(c) Using your answers to part (b), find the value of

$$\sum_{r=10}^{25} (r^2 + 2)$$

(2)

(a)

$n = 1$: LHS $= 1^2 = 1$, RHS $= \frac{1}{6} \times 2 \times 3 = 1$, so true for $n = 1$

Assume true for $n = k$: $\sum_{r=1}^{k} r^2 = \frac{k}{6}(k+1)(2k+1)$

$n = k + 1$: $\sum_{r=1}^{k+1} r^2 = \frac{k}{6}(k+1)(2k+1) + (k+1)^2$

$= \frac{k+1}{6}[k(2k+1) + 6(k+1)] = \frac{k+1}{6}(2k^2 + 7k + 6)$

$= \frac{k+1}{6}(k+2)(2k+3) = \frac{k+1}{6}((k+1)+1)(2(k+1)+1)$

This is the result with $n = k + 1$.

True for $n = 1$, and if true for $n = k$ then true for $n = k + 1$,

therefore true for all $n \in \mathbb{N}$ by mathematical induction.

(b)

$\sum_{r=1}^{n} (r^2 + 2) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} 2 = \frac{n}{6}(n+1)(2n+1) + 2n$

$= \frac{n}{6}[(n+1)(2n+1) + 12] = \frac{n}{6}(2n^2 + 3n + 1 + 12)$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

$= \frac{n}{6}(2n^2 + 3n + 13)$

$a = 2, \quad b = 3, \quad c = 13$

(c)

$$\sum_{r=10}^{25}(r^2 + 2) = \sum_{r=1}^{25}(r^2 + 2) - \sum_{r=1}^{9}(r^2 + 2)$$

$$= \frac{25}{6}(2 \times 25^2 + 3 \times 25 + 13) - \frac{9}{6}(2 \times 9^2 + 3 \times 9 + 13)$$

$$= \frac{25}{6} \times 1338 - \frac{9}{6} \times 202 = 5575 - 303$$

$$= 5272$$

Leave blank

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8. Prove by induction that $4^{n+2} + 5^{2n+1}$ is divisible by 21 for all positive integers n.

(6)

Let $f(n) = 4^{n+2} + 5^{2n+1}$

$n = 1$: $f(1) = 4^3 + 5^3 = 64 + 125 = 189 = 21 \times 9$, so true for $n = 1$

Assume true for $n = k$, i.e. $f(k) = 4^{k+2} + 5^{2k+1}$ is divisible by 21

$f(k + 1) - f(k) = 4^{k+3} + 5^{2k+3} - (4^{k+2} + 5^{2k+1})$

$= 4 \times 4^{k+2} + 25 \times 5^{2k+1} - 4^{k+2} - 5^{2k+1}$

$= 3 \times 4^{k+2} + 24 \times 5^{2k+1}$

$= 3(4^{k+2} + 5^{2k+1}) + 21 \times 5^{2k+1}$

$= 3f(k) + 21 \times 5^{2k+1}$

$\therefore f(k + 1) = 4f(k) + 21 \times 5^{2k+1}$

$f(k)$ is divisible by 21 and $21 \times 5^{2k+1}$ is divisible by 21,

so $f(k + 1)$ is divisible by 21.

If the result is true for $n = k$, then it is true for $n = k + 1$.

As it is true for $n = 1$, the result is true for all positive integers n

by mathematical induction.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA