Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P71099A

©2022 Pearson Education Ltd.

L:1/1/1/

P 7 1 0 9 9 A 0 1 3 6

Pearson

Leave blank

1. $\mathbf{M} = \begin{pmatrix} 3x & 7 \\ 4x+1 & 2-x \end{pmatrix}$

Find the range of values of x for which the determinant of the matrix **M** is positive.

(5)

$\det M = 3x(2 - x) - 7(4x + 1)$

$= 6x - 3x^2 - 28x - 7$

$= -3x^2 - 22x - 7$

$\det M > 0 \Rightarrow -3x^2 - 22x - 7 > 0 \Rightarrow 3x^2 + 22x + 7 < 0$

$3x^2 + 22x + 7 = 0 \Rightarrow (3x + 1)(x + 7) = 0$

$\Rightarrow x = -7$ or $x = -\frac{1}{3}$

positive x^2 coefficient, so the quadratic is < 0 between the roots

$\therefore -7 < x < -\frac{1}{3}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Leave blank

2. The complex numbers z_1 and z_2 are given by

$$z_1 = 3 + 5i \quad \text{and} \quad z_2 = -2 + 6i$$

(a) Show z_1 and z_2 on a single Argand diagram.

(2)

(b) **Without using your calculator and showing all stages of your working,**

(i) determine the value of $|z_1|$

(1)

(ii) express $\frac{z_1}{z_2}$ in the form $a + bi$, where a and b are fully simplified fractions.

(3)

(c) Hence determine the value of $\arg \frac{z_1}{z_2}$

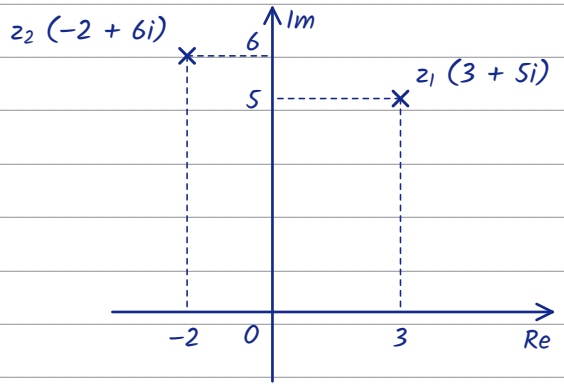
Give your answer in radians to 2 decimal places.

(2)

(a) Argand diagram:

z_1 in 1st quadrant,

z_2 in 2nd quadrant



(b)(i) $|z_1| = \sqrt{(3^2 + 5^2)} = \sqrt{34}$

(ii) $\frac{z_1}{z_2} = \frac{3 + 5i}{-2 + 6i} \times \frac{-2 - 6i}{-2 - 6i}$

$= \frac{-6 - 18i - 10i - 30i^2}{4 + 36} = \frac{24 - 28i}{40}$

$= \frac{3}{5} - \frac{7}{10} i$

(c) $\arg \frac{z_1}{z_2} = \arctan\left(\frac{-7/10}{3/5}\right) = \arctan\left(-\frac{7}{6}\right)$

Re > 0, Im < 0 so 4th quadrant

$\arg(z_1/z_2) = -0.86217\ldots = -0.86$ radians

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Leave blank

Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. The parabola C has equation $y^2 = 18x$

The point S is the focus of C

(a) Write down the coordinates of S

(1)

The point P, with $y > 0$, lies on C

The shortest distance from P to the directrix of C is 9 units.

(b) Determine the exact perimeter of the triangle OPS, where O is the origin.

Give your answer in simplest form.

(4)

(a) $y^2 = 4ax$, $4a = 18 \Rightarrow a = \frac{9}{2}$

$S\left(\frac{9}{2}, 0\right)$

(b) directrix $x = -\frac{9}{2}$

focus-directrix property: PS = distance to directrix = 9

$x_P + \frac{9}{2} = 9 \Rightarrow x_P = \frac{9}{2}$

$y_P^2 = 18 \times \frac{9}{2} = 81 \Rightarrow y_P = 9 \quad (y > 0)$

$OP = \sqrt{\left(\left(\frac{9}{2}\right)^2 + 9^2\right)} = \sqrt{\left(\frac{405}{4}\right)} = \frac{9\sqrt{5}}{2}$

$OS = \frac{9}{2}$

Perimeter $= \frac{9}{2} + 9 + \frac{9\sqrt{5}}{2}$

$= \frac{27 + 9\sqrt{5}}{2}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Leave blank

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Q3

(Total 5 marks)

Leave blank

4. The equation

$$x^4 + Ax^3 + Bx^2 + Cx + 225 = 0$$

where A, B and C are real constants, has

- a complex root 4 + 3i
- a repeated positive real root

(a) Write down the other complex root of this equation.

(1)

(b) Hence determine a quadratic factor of $x^4 + Ax^3 + Bx^2 + Cx + 225$

(2)

(c) Deduce the real root of the equation.

(2)

(d) Hence determine the value of each of the constants A, B and C

(3)

(a) $4 - 3i$

(b) $(x - (4 + 3i))(x - (4 - 3i)) = (x - 4)^2 - (3i)^2$

$= x^2 - 8x + 16 + 9 = x^2 - 8x + 25$

(c) product of roots = 225, product of complex roots = 25

$(\text{real root})^2 = \frac{225}{25} = 9$

repeated root is positive $\Rightarrow$ real root = 3

(d) $(x^2 - 8x + 25)(x - 3)^2 = (x^2 - 8x + 25)(x^2 - 6x + 9)$

$= x^4 - 6x^3 + 9x^2 - 8x^3 + 48x^2 - 72x + 25x^2 - 150x + 225$

$= x^4 - 14x^3 + 82x^2 - 222x + 225$

$A = -14, \quad B = 82, \quad C = -222$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Leave blank

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. $\mathbf{P} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

The matrix **P** represents the transformation U

(a) Give a full description of U as a single geometrical transformation.

(2)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line $y = -x$

(b) Write down the matrix **Q**

(1)

The transformation U followed by the transformation V is represented by the matrix **R**

(c) Determine the matrix **R**

(2)

The transformation W is represented by the matrix $3\mathbf{R}$

The transformation W maps a triangle T to a triangle T'

The transformation W' maps the triangle T' back to the original triangle T

(d) Determine the matrix that represents W'

(3)

(a) $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60°$

U is a rotation about O through 60° anticlockwise

(b) $Q = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(c) $R = QP = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

$= \frac{1}{2} \begin{pmatrix} -\sqrt{3} & -1 \\ -1 & \sqrt{3} \end{pmatrix}$

(d) $W' = (3R)^{-1}$, $3R = \frac{3}{2} \begin{pmatrix} -\sqrt{3} & -1 \\ -1 & \sqrt{3} \end{pmatrix}$

$\det(3R) = \left(\frac{3}{2}\right)^2 ((-\sqrt{3})(\sqrt{3}) - (-1)(-1)) = \frac{9}{4} \times (-4) = -9$

$(3R)^{-1} = \frac{1}{-9} \times \frac{3}{2} \begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

$$= -\frac{1}{6}\begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix} = \begin{pmatrix} -\sqrt{3}/6 & -1/6 \\ -1/6 & \sqrt{3}/6 \end{pmatrix}$$

Leave blank

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6. The quadratic equation

$$Ax^2 + 5x - 12 = 0$$

where A is a constant, has roots α and β

(a) Write down an expression in terms of A for

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(2)

The equation

$$4x^2 - 5x + B = 0$$

where B is a constant, has roots $\alpha - \frac{3}{\beta}$ and $\beta - \frac{3}{\alpha}$

(b) Determine the value of A

(3)

(c) Determine the value of B

(3)

(a)(i) $\alpha + \beta = -\frac{5}{A}$

(ii) $\alpha\beta = -\frac{12}{A}$

(b) sum of new roots $= \alpha + \beta - \frac{3}{\beta} - \frac{3}{\alpha} = \alpha + \beta - \frac{3(\alpha + \beta)}{\alpha\beta}$

$= -\frac{5}{A} - 3 \times \frac{-5/A}{-12/A} = -\frac{5}{A} - \frac{5}{4}$

sum of roots of $4x^2 - 5x + B = 0$ is $\frac{5}{4}$

$-\frac{5}{A} - \frac{5}{4} = \frac{5}{4} \Rightarrow \frac{5}{A} = -\frac{5}{2} \Rightarrow A = -2$

(c) product of new roots $= \left(\alpha - \frac{3}{\beta}\right)\left(\beta - \frac{3}{\alpha}\right) = \alpha\beta - 6 + \frac{9}{\alpha\beta}$

$A = -2 \Rightarrow \alpha\beta = -\frac{12}{-2} = 6$

product $= 6 - 6 + \frac{9}{6} = \frac{3}{2}$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

$\frac{B}{4} = \frac{3}{2} \Rightarrow B = 6$

Leave blank

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The rectangular hyperbola H has equation $xy = 36$

The point $P(4, 9)$ lies on H

(a) Show, using calculus, that the normal to H at P has equation

$$4x - 9y + 65 = 0$$

(4)

The normal to H at P crosses H again at the point Q

(b) Determine an equation for the tangent to H at Q, giving your answer in the form $y = mx + c$ where m and c are rational constants.

(5)

(a) $y = 36x^{-1} \Rightarrow \frac{dy}{dx} = -36x^{-2} = -\frac{36}{x^2}$

at $P(4, 9)$: $m_T = -\frac{36}{4^2} = -\frac{9}{4} \Rightarrow m_N = \frac{4}{9}$

normal: $y - 9 = \frac{4}{9}(x - 4)$

$9y - 81 = 4x - 16$

$\Rightarrow 4x - 9y + 65 = 0$ as required

(b) $y = \frac{36}{x}$: $4x - 9\left(\frac{36}{x}\right) + 65 = 0$

$4x^2 + 65x - 324 = 0$

$(4x + 81)(x - 4) = 0$

$x \neq 4 \Rightarrow x = -\frac{81}{4}$, $y = 36 \div \left(-\frac{81}{4}\right) = -\frac{16}{9}$

$Q\left(-\frac{81}{4}, -\frac{16}{9}\right)$

at Q: $\frac{dy}{dx} = -\frac{36}{(-81/4)^2} = -\frac{576}{6561} = -\frac{64}{729}$

tangent: $y + \frac{16}{9} = -\frac{64}{729}\left(x + \frac{81}{4}\right)$

$y = -\frac{64}{729}x - \frac{16}{9} - \frac{16}{9}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

$y = -\frac{64}{729}x - \frac{32}{9}$

Leave blank

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. $$f(x) = 2x^{-\frac{2}{3}} + \frac{1}{2}x - \frac{1}{3x-5} - \frac{5}{2} \qquad x \neq \frac{5}{3}$$

The table below shows values of f(x) for some values of x, with values of f(x) given to 4 decimal places where appropriate.

x	1	2	3	4	5
f(x)	0.5	−1.2401	−0.2885	0.1508	0.5834

(a) Complete the table giving the values to 4 decimal places.

(2)

The equation f(x) = 0 has exactly one positive root, α.

Using the values in the completed table and explaining your reasoning,

(b) determine an interval of width one that contains α.

(2)

(c) Hence use interval bisection twice to obtain an interval of width 0.25 that contains α.

(3)

Given also that the equation f(x) = 0 has a negative root, β, in the interval [−1, −0.5]

(d) use linear interpolation once on this interval to find an approximation for β.

Give your answer to 3 significant figures.

(3)

(a) $f(2) = -1.2401$, $f(4) = 0.1508$ (in table)

(b) $f(3) = -0.2885 < 0$ and $f(4) = 0.1508 > 0$: change of sign

and f is continuous on [3, 4] (f is not continuous on [1, 2] as $x = \frac{5}{3}$ is an asymptote, so that sign change does not give a root)

$\therefore$ α lies in the interval [3, 4]

(c) $f(3.5) = -0.06422... < 0$, $f(4) > 0 \Rightarrow \alpha \in [3.5, 4]$

$f(3.75) = 0.04359... > 0$, $f(3.5) < 0 \Rightarrow \alpha \in [3.5, 3.75]$

(d) $f(-1) = -0.875$, $f(-0.5) = 0.57864...$

$$\frac{\beta - (-1)}{0 - f(-1)} = \frac{-0.5 - \beta}{f(-0.5) - 0} \Rightarrow \frac{\beta + 1}{0.875} = \frac{-0.5 - \beta}{0.57864...}$$

$$\beta = -0.5 - \frac{0.5 \times 0.57864...}{0.57864... + 0.875}$$

$\beta = -0.69903... = -0.699$ (3 s.f.)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

Leave blank

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

(Total 10 marks)

Q8

Leave blank

9. (a) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

(b) Using the standard summation formulae, show that

$$\sum_{r=1}^{n} r(r+1)(r-1) = \frac{1}{4}n(n+A)(n+B)(n+C)$$

where A, B and C are constants to be determined.

(4)

(c) Determine the value of n for which

$$3\sum_{r=1}^{n} r(r+1)(r-1) = 17\sum_{r=n}^{2n} r^2$$

(5)

(a) $n = 1$: LHS $= \sum_{r=1}^{1} r^3 = 1$, RHS $= \frac{1}{4} \times 1^2 \times 2^2 = 1$

so the result is true for $n = 1$

Assume true for $n = k$: $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$

$\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$

$= \frac{1}{4}(k+1)^2[k^2 + 4(k+1)] = \frac{1}{4}(k+1)^2(k^2+4k+4)$

$= \frac{1}{4}(k+1)^2(k+2)^2 = \frac{1}{4}(k+1)^2((k+1)+1)^2$

which is the result with $n = k + 1$.

If the result is true for $n = k$ then it is true for $n = k + 1$.

It is true for $n = 1$, so by mathematical induction it is true for all $n \in \mathbb{N}$.

(b) $\sum_{r=1}^{n} r(r+1)(r-1) = \sum_{r=1}^{n} (r^3 - r)$

$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$

$= \frac{1}{4}n(n+1)[n(n+1) - 2] = \frac{1}{4}n(n+1)(n^2+n-2)$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 9 continued

$= \frac{1}{4}n(n+1)(n-1)(n+2)$

so $A = 1$, $B = -1$, $C = 2$

(c) $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2$

$= \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}(n-1)n(2n-1)$

$= \frac{1}{6}n[2(8n^2+6n+1) - (2n^2-3n+1)]$

$= \frac{1}{6}n(14n^2+15n+1) = \frac{1}{6}n(n+1)(14n+1)$

$3 \times \frac{1}{4}n(n+1)(n-1)(n+2) = 17 \times \frac{1}{6}n(n+1)(14n+1)$

$n \neq 0, -1$: $\frac{3}{4}(n-1)(n+2) = \frac{17}{6}(14n+1)$

$\times 12$: $9(n-1)(n+2) = 34(14n+1)$

$9n^2 + 9n - 18 = 476n + 34$

$9n^2 - 467n - 52 = 0$

$(9n+1)(n-52) = 0$

$n = -\frac{1}{9}$ (reject, n must be a positive integer) or $n = 52$

$\therefore\ n = 52$

Leave blank

Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA