Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Friday 8 January 2021

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. (a) Show that the equation $4x - 2\sin x - 1 = 0$, where x is in radians, has a root α in the interval [0.2, 0.6]

(2)

(b) Starting with the interval [0.2, 0.6], use interval bisection twice to find an interval of width 0.1 in which α lies.

(3)

(a) $f(x) = 4x - 2\sin x - 1$

$f(0.2) = 0.8 - 2\sin 0.2 - 1 = -0.5973...$

$f(0.6) = 2.4 - 2\sin 0.6 - 1 = 0.2707...$

f is continuous and there is a change of sign in [0.2, 0.6]

$\therefore$ the equation has a root α in the interval [0.2, 0.6]

(b) $f(0.4) = 1.6 - 2\sin 0.4 - 1 = -0.1788... < 0$

$\Rightarrow 0.4 \leq \alpha \leq 0.6$

$f(0.5) = 2 - 2\sin 0.5 - 1 = 0.04114... > 0$

$\Rightarrow 0.4 \leq \alpha \leq 0.5$

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Question 1 continued

Q1

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2. Given that $x = \frac{3}{8} + \frac{\sqrt{71}}{8}\mathrm{i}$ is a root of the equation

$$4x^3 - 19x^2 + px + q = 0$$

(a) write down the other complex root of the equation.

(1)

Given that $x = 4$ is also a root of the equation,

(b) find the value of p and the value of q.

(4)

(a) $x = \frac{3}{8} - \frac{\sqrt{71}}{8}i$

(b) $\left(x - \frac{3}{8} - \frac{\sqrt{71}}{8}i\right)\left(x - \frac{3}{8} + \frac{\sqrt{71}}{8}i\right) = \left(x - \frac{3}{8}\right)^2 + \frac{71}{64}$

$= x^2 - \frac{3}{4}x + \frac{9}{64} + \frac{71}{64} = x^2 - \frac{3}{4}x + \frac{5}{4}$

$4x^3 - 19x^2 + px + q = 4\left(x^2 - \frac{3}{4}x + \frac{5}{4}\right)(x - 4)$

$= (4x^2 - 3x + 5)(x - 4)$

$= 4x^3 - 16x^2 - 3x^2 + 12x + 5x - 20$

$= 4x^3 - 19x^2 + 17x - 20$

$p = 17, \quad q = -20$

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Question 2 continued

Q2

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3. The matrix **M** is defined by

$$\mathbf{M} = \begin{pmatrix} k+5 & -2 \\ -3 & k \end{pmatrix}$$

(a) Determine the values of k for which **M** is singular.

(2)

Given that **M** is non-singular,

(b) find $\mathbf{M}^{-1}$ in terms of k.

(2)

(a) M singular $\Rightarrow$ det M = 0

$k(k+5) - (-2)(-3) = 0$

$k^2 + 5k - 6 = 0$

$(k-1)(k+6) = 0$

$k = 1$ or $k = -6$

(b) $M^{-1} = \frac{1}{k^2+5k-6}\begin{pmatrix} k & 2 \\ 3 & k+5 \end{pmatrix}$

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Question 3 continued

Q3

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4. The equation $2x^2 + 5x + 7 = 0$ has roots α and β

Without solving the equation

(a) determine the exact value of $\alpha^3 + \beta^3$

(3)

(b) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha^2}{\beta} \text{ and } \frac{\beta^2}{\alpha}$$

(5)

(a) $\alpha + \beta = -\frac{5}{2}$, $\quad \alpha\beta = \frac{7}{2}$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= \left(-\frac{5}{2}\right)^3 - 3\left(\frac{7}{2}\right)\left(-\frac{5}{2}\right)$

$= -\frac{125}{8} + \frac{105}{4} = \frac{85}{8}$

(b) Sum $= \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{85}{8} \div \frac{7}{2} = \frac{85}{8} \times \frac{2}{7} = \frac{85}{28}$

Product $= \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = \frac{7}{2}$

$x^2 - \frac{85}{28}x + \frac{7}{2} = 0$

$28x^2 - 85x + 98 = 0$

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Question 4 continued

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Question 4 continued

Q4

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5. (a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n}(r+1)(r+5) = \frac{n}{6}(n+7)(2n+7)$$

for all positive integers n.

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n}(r+1)(r+5) = \frac{7n}{6}(n+1)(an+b)$$

where a and b are integers to be determined.

(2)

(a) $\sum_{r=1}^{n}(r+1)(r+5) = \sum_{r=1}^{n}(r^2+6r+5)$

$= \sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r + 5n$

$= \frac{n}{6}(n+1)(2n+1) + 6 \times \frac{n}{2}(n+1) + 5n$

$= \frac{n}{6}[(n+1)(2n+1) + 18(n+1) + 30]$

$= \frac{n}{6}(2n^2 + 3n + 1 + 18n + 18 + 30)$

$= \frac{n}{6}(2n^2 + 21n + 49)$

$= \frac{n}{6}(n+7)(2n+7)$ as required

(b) $\sum_{r=n+1}^{2n}(r+1)(r+5) = \sum_{r=1}^{2n}(r+1)(r+5) - \sum_{r=1}^{n}(r+1)(r+5)$

$= \frac{2n}{6}(2n+7)(4n+7) - \frac{n}{6}(n+7)(2n+7)$

$= \frac{n}{6}(2n+7)[2(4n+7) - (n+7)]$

$= \frac{n}{6}(2n+7)(7n+7)$

$= \frac{7n}{6}(n+1)(2n+7)$

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Question 5 continued

$a = 2, \quad b = 7$

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Question 5 continued

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Question 5 continued

Q5

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6. The complex number z is defined by

$$z = -\lambda + 3\mathrm{i} \quad \text{where } \lambda \text{ is a positive real constant}$$

Given that the modulus of z is 5

(a) write down the value of λ

(1)

(b) determine the argument of z, giving your answer in radians to one decimal place.

(2)

In part (c) you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

(c) Express in the form $a + \mathrm{i}b$ where a and b are real,

(i) $\dfrac{z + 3\mathrm{i}}{2 - 4\mathrm{i}}$

(ii) z^2

(5)

(d) Show on a single Argand diagram the points A, B, C and D that represent the complex numbers

$$z,\ z^*,\ \frac{z + 3\mathrm{i}}{2 - 4\mathrm{i}} \text{ and } z^2$$

(3)

(a) $|z| = \sqrt{(\lambda^2 + 9)} = 5 \Rightarrow \lambda^2 = 16 \Rightarrow \lambda = 4$

(b) $z = -4 + 3i$ (second quadrant), $\arctan\frac{3}{4} = 0.6435...$

$\arg z = \pi - 0.6435... = 2.498...$

$\arg z = 2.5$ (rad)

(c)(i) $\dfrac{z + 3i}{2 - 4i} = \dfrac{-4 + 6i}{2 - 4i} \times \dfrac{2 + 4i}{2 + 4i}$

$= \dfrac{-8 - 16i + 12i + 24i^2}{4 + 16} = \dfrac{-8 - 4i - 24}{20} = \dfrac{-32 - 4i}{20}$

$= -\dfrac{8}{5} - \dfrac{1}{5}i$

(ii) $z^2 = (-4 + 3i)^2 = 16 - 24i + 9i^2$

$= 16 - 24i - 9 = 7 - 24i$

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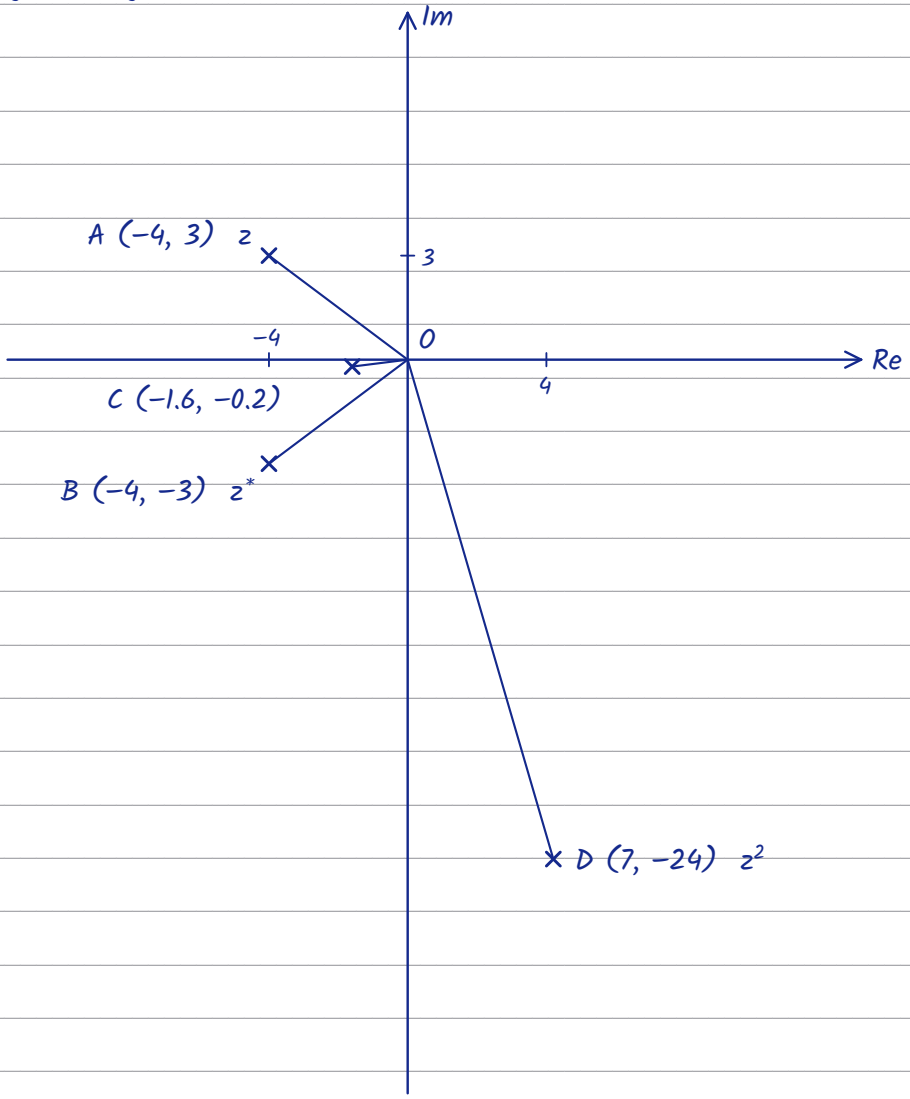
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Question 6 continued

(d) Argand diagram (D not to scale):



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Question 6 continued

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Question 6 continued

Q6

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7. The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}$$

The transformation represented by **A** maps triangle T onto triangle T'

Given that the area of triangle T is 23 cm^2

(a) determine the area of triangle T'

(2)

The point P has coordinates $(3p + 2, 2p - 1)$ where p is a constant. The transformation represented by **A** maps P onto the point P' with coordinates $(17, -18)$

(b) Determine the value of p.

(2)

Given that

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(c) describe fully the single geometrical transformation represented by matrix **B**

(2)

The transformation represented by matrix **A** followed by the transformation represented by matrix **C** is equivalent to the transformation represented by matrix **B**

(d) Determine **C**

(3)

(a) $\det A = 4 \times 2 - (-5)(-3) = 8 - 15 = -7$

Area $T' = |-7| \times 23 = 161$ cm^2

(b) $\begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 3p+2 \\ 2p-1 \end{pmatrix} = \begin{pmatrix} 17 \\ -18 \end{pmatrix}$

$4(3p + 2) - 5(2p - 1) = 17$

$12p + 8 - 10p + 5 = 17 \Rightarrow 2p + 13 = 17$

$p = 2$

(check: $-3(8) + 2(3) = -18$ ✓)

(c) Rotation through 90° clockwise about the origin (0, 0)

(d) $CA = B \Rightarrow C = BA^{-1}$

$A^{-1} = -\frac{1}{7}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$

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Question 7 continued

$$C = -\frac{1}{7}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$$

$$C = -\frac{1}{7}\begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix} = \begin{pmatrix} -3/7 & -4/7 \\ 2/7 & 5/7 \end{pmatrix}$$

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Question 7 continued

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Question 7 continued

Q7

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8. The hyperbola H has Cartesian equation $xy = 25$

The parabola P has parametric equations $x = 10t^2$, $y = 20t$

The hyperbola H intersects the parabola P at the point A

(a) Use algebra to determine the coordinates of A

(3)

The point B with coordinates $(10, 20)$ lies on P

(b) Find an equation for the normal to P at B

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined.

(5)

(c) Use algebra to determine, in simplest form, the exact coordinates of the points where this normal intersects the hyperbola H

(6)

(a) $xy = 25 \Rightarrow 10t^2 \times 20t = 25$

$200t^3 = 25 \Rightarrow t^3 = \frac{1}{8} \Rightarrow t = \frac{1}{2}$

$x = 10\left(\frac{1}{2}\right)^2 = \frac{5}{2}, \quad y = 20\left(\frac{1}{2}\right) = 10$

$A\left(\frac{5}{2}, 10\right)$

(b) $\frac{dx}{dt} = 20t, \quad \frac{dy}{dt} = 20 \Rightarrow \frac{dy}{dx} = \frac{20}{20t} = \frac{1}{t}$

At B (10, 20): $t = 1$, so gradient of tangent $= 1$

Gradient of normal $= -1$

$y - 20 = -(x - 10)$

$x + y - 30 = 0$

(c) $xy = 25$ and $y = 30 - x$

$x(30 - x) = 25$

$x^2 - 30x + 25 = 0$

$x = \frac{30 \pm \sqrt{(900 - 100)}}{2} = \frac{30 \pm \sqrt{800}}{2} = 15 \pm \sqrt{200} = 15 \pm 10\sqrt{2}$

$x = 15 + 10\sqrt{2} \Rightarrow y = 30 - x = 15 - 10\sqrt{2}$

$x = 15 - 10\sqrt{2} \Rightarrow y = 30 - x = 15 + 10\sqrt{2}$

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Question 8 continued

$(15 + 10\sqrt{2}, 15 - 10\sqrt{2})$ and $(15 - 10\sqrt{2}, 15 + 10\sqrt{2})$

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Question 8 continued

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Question 8 continued

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9. (i) A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{3}(2u_n - 1) \qquad u_1 = 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3\left(\frac{2}{3}\right)^n - 1$$

(6)

(ii) $f(n) = 2^{n+2} + 3^{2n+1}$

Prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is a multiple of 7

(6)

(i) $n = 1$: $u_1 = 3\left(\frac{2}{3}\right)^1 - 1 = 2 - 1 = 1$ ✓ so true for $n = 1$

Assume true for $n = k$: $u_k = 3\left(\frac{2}{3}\right)^k - 1$

$u_{k+1} = \frac{1}{3}(2u_k - 1) = \frac{1}{3}\left(2\left[3\left(\frac{2}{3}\right)^k - 1\right] - 1\right)$

$= \frac{1}{3}\left(6\left(\frac{2}{3}\right)^k - 3\right) = 2\left(\frac{2}{3}\right)^k - 1$

$= 3 \times \frac{2}{3}\left(\frac{2}{3}\right)^k - 1 = 3\left(\frac{2}{3}\right)^{k+1} - 1$

$\therefore$ if true for $n = k$, then true for $n = k + 1$.

As the result is true for $n = 1$, it is true for all $n \in \mathbb{Z}^+$ by mathematical induction.

(ii) $n = 1$: $f(1) = 2^3 + 3^3 = 8 + 27 = 35 = 7 \times 5$

so $f(1)$ is a multiple of 7, true for $n = 1$

Assume true for $n = k$: $f(k) = 2^{k+2} + 3^{2k+1}$ is a multiple of 7

$f(k + 1) = 2^{k+3} + 3^{2k+3}$

$= 2 \times 2^{k+2} + 9 \times 3^{2k+1}$

$= 2(2^{k+2} + 3^{2k+1}) + 7 \times 3^{2k+1}$

$= 2f(k) + 7 \times 3^{2k+1}$

Both $2f(k)$ and $7 \times 3^{2k+1}$ are multiples of 7, so $f(k + 1)$ is a multiple of 7.

$\therefore$ if true for $n = k$, then true for $n = k + 1$.

As the result is true for $n = 1$, it is true for all $n \in \mathbb{Z}^+$ by mathematical induction.

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

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TOTAL FOR PAPER: 75 MARKS

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