Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 28 October 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference WMA14/01

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 + 5x)^{-2}$$

giving each term in simplest form.

(5)

(b) State the range of values of x for which this expansion is valid.

(1)

(c) Given that x is sufficiently small, find the values of a, b and c such that

$$\left(\frac{4}{2 + 5x}\right)^2 \approx a + bx + cx^2$$

(1)

(a) $(2 + 5x)^{-2} = 2^{-2}\left(1 + \frac{5}{2}x\right)^{-2} = \frac{1}{4}\left(1 + \frac{5}{2}x\right)^{-2}$

$\left(1 + \frac{5}{2}x\right)^{-2} = 1 + (-2)\left(\frac{5x}{2}\right) + \frac{(-2)(-3)}{2!}\left(\frac{5x}{2}\right)^2$

$+ \frac{(-2)(-3)(-4)}{3!}\left(\frac{5x}{2}\right)^3 + \ldots$

$= 1 - 5x + \frac{75}{4}x^2 - \frac{125}{2}x^3 + \ldots$

$(2 + 5x)^{-2} = \frac{1}{4}\left(1 - 5x + \frac{75}{4}x^2 - \frac{125}{2}x^3\right)$

$(2 + 5x)^{-2} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 - \frac{125}{8}x^3 + \ldots$

(b) Valid for $\left|\frac{5x}{2}\right| < 1 \Rightarrow |x| < \frac{2}{5}$

(c) $\left(\frac{4}{2 + 5x}\right)^2 = 16(2 + 5x)^{-2} \approx 16\left(\frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2\right)$

$\approx 4 - 20x + 75x^2$

$a = 4, \quad b = -20, \quad c = 75$

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Question 1 continued

(Total for Question 1 is 7 marks)

2.

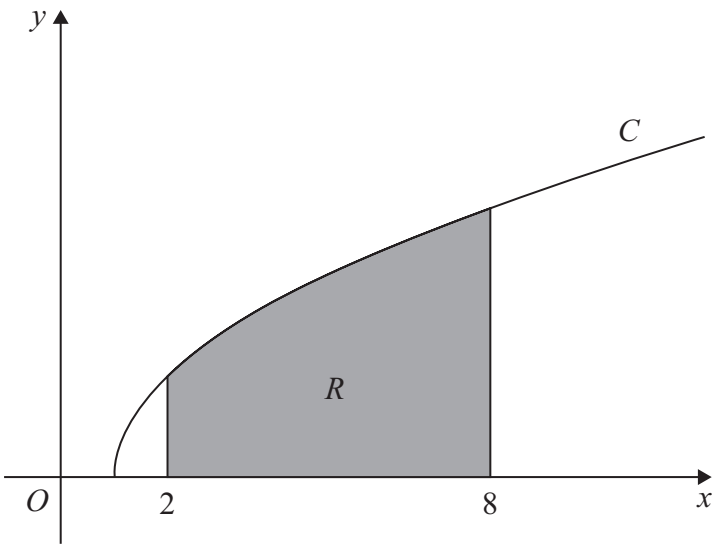


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{5}{3}\sqrt{x-1}$$

The region R, shown shaded in Figure 1, is bounded by C, the line with equation $x = 2$, the x-axis and the line with equation $x = 8$

This region is then rotated through 2π radians about the x-axis to form a solid.

Use algebraic integration to find the exact volume of this solid.

(5)

$V = \pi \int_2^8 y^2\,dx = \pi \int_2^8 \left(\frac{5}{3}\sqrt{(x-1)}\right)^2 dx = \pi \int_2^8 \frac{25(x-1)}{9}\,dx$

$= \frac{25\pi}{9}\left[\frac{x^2}{2} - x\right]_2^8$

$= \frac{25\pi}{9}\left[\left(\frac{64}{2} - 8\right) - \left(\frac{4}{2} - 2\right)\right] = \frac{25\pi}{9} \times 24$

Volume $= \frac{200\pi}{3}$

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Question 2 continued

(Total for Question 2 is 5 marks)

3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \qquad y = \frac{5}{t+2} \qquad t \geqslant 0$$

(a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax+b}{cx+d} \qquad e < x \leqslant f$$

and a, b, c, d, e and f are constants to be found.

(5)

(b) State the range of g.

(2)

(a) $x = \frac{t+15}{t+4} \Rightarrow xt + 4x = t + 15 \Rightarrow t(x-1) = 15 - 4x \Rightarrow t = \frac{15-4x}{x-1}$

$y = \frac{5}{t+2} = \frac{5}{(15-4x)/(x-1) + 2}$

$y = \frac{5(x-1)}{15 - 4x + 2(x-1)} = \frac{5(x-1)}{13-2x}$

$g(x) = \frac{5x-5}{13-2x}$ $\quad (a = 5,\ b = -5,\ c = -2,\ d = 13)$

$t = 0 : x = \frac{15}{4}$; as $t \to \infty$, $x = \frac{1 + 15/t}{1 + 4/t} \to 1$

$1 < x \leqslant \frac{15}{4}$ $\qquad e = 1,\ f = \frac{15}{4}$

(b) $t = 0 : y = \frac{5}{2}$; as $t \to \infty$, $y \to 0$, and y decreases as t increases

Range: $0 < g(x) \leqslant \frac{5}{2}$

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$4x^2 + y^2 - 2xy = 24x$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of x and y.

(5)

The point P lies on C.

Given that

- the gradient of C at P is 2
- P has coordinates (a, b) where $a > 0$ and $b > 0$

(b) find the value of a and the value of b.

(5)

(a) $8x + 2y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 24$

$(2y - 2x)\frac{dy}{dx} = 24 - 8x + 2y$

$\frac{dy}{dx} = \frac{24 - 8x + 2y}{2y - 2x} = \frac{12 - 4x + y}{y - x}$

(b) $\frac{12 - 4x + y}{y - x} = 2 \Rightarrow 12 - 4x + y = 2y - 2x \Rightarrow y = 12 - 2x$

$4x^2 + (12 - 2x)^2 - 2x(12 - 2x) = 24x$

$4x^2 + 144 - 48x + 4x^2 - 24x + 4x^2 = 24x$

$12x^2 - 96x + 144 = 0 \Rightarrow x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0 \Rightarrow x = 2, y = 8$ or $x = 6, y = 0$

$b > 0$, so P is $(2, 8)$

$a = 2, \quad b = 8$

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 10 marks)

5. The line l_1 has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$ where λ is a scalar parameter

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ \beta \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}$$ where β is a constant and μ is a scalar parameter

Given that l_1 and l_2 intersect at the point P

(i) find the value of β

(ii) find the coordinates of P

(6)

(i) i : $2 + \lambda = -1 + 2\mu$ (1)

j : $-1 + 4\lambda = \beta - \mu$ (2)

k : $3 + 2\lambda = 6 + 7\mu$ (3)

(1): $\lambda = 2\mu - 3$; into (3): $3 + 4\mu - 6 = 6 + 7\mu \Rightarrow -3\mu = 9$

$\mu = -3$, $\lambda = 2(-3) - 3 = -9$

(2): $-1 + 4(-9) = \beta - (-3) \Rightarrow -37 = \beta + 3$

$\beta = -40$

(ii) $\lambda = -9$: $r = (2 - 9)\,i + (-1 - 36)\,j + (3 - 18)\,k$

$= -7i - 37j - 15k$

$P\,(-7, -37, -15)$

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Question 5 continued

(Total for Question 5 is 6 marks)

6. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Use the substitution $u = 3 + \cos\theta$ to show that

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{\sqrt{3 + \cos\theta}} \, \mathrm{d}\theta = a\sqrt{3} + b$$

where a and b are constants to be found.

(7)

$u = 3 + \cos\theta \Rightarrow \frac{du}{d\theta} = -\sin\theta$, $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos\theta = u - 3$

$\theta = 0 : u = 3 + 1 = 4$; $\theta = \pi/2 : u = 3 + 0 = 3$

$$\int \frac{\sin 2\theta}{\sqrt{(3 + \cos\theta)}} \, d\theta = \int \frac{2\sin\theta\cos\theta}{\sqrt{u}} \times \frac{du}{-\sin\theta} = \int -\frac{2(u-3)}{\sqrt{u}} \, du$$

$$= \int_4^3 -\left(2u^{1/2} - 6u^{-1/2}\right) du = \left[-\frac{4}{3}u^{3/2} + 12u^{1/2}\right]_4^3$$

$$= \left(-\frac{4}{3}(3\sqrt{3}) + 12\sqrt{3}\right) - \left(-\frac{4}{3}(8) + 12(2)\right)$$

$$= (-4\sqrt{3} + 12\sqrt{3}) - \left(-\frac{32}{3} + 24\right) = 8\sqrt{3} - \frac{40}{3}$$

$a = 8$, $b = -\frac{40}{3}$

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Question 6 continued

(Total for Question 6 is 7 marks)

7.

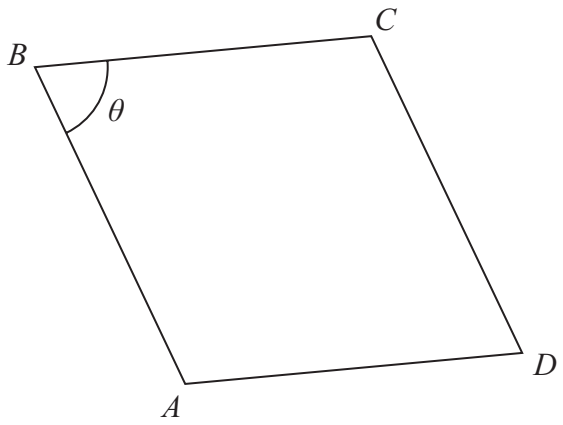


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\overrightarrow{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90°$

(a) find the exact value of $\cos\theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

(a) $\overrightarrow{BA} = -6i + 2j - 3k$, $\overrightarrow{BC} = -4i + 3j - 5k$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = (-6)(-4) + (2)(3) + (-3)(-5) = 24 + 6 + 15 = 45$

$|\overrightarrow{BA}| = \sqrt{(36 + 4 + 9)} = 7$, $|\overrightarrow{BC}| = \sqrt{(16 + 9 + 25)} = \sqrt{50} = 5\sqrt{2}$

$\cos\theta = \frac{45}{7 \times 5\sqrt{2}} = \frac{9}{7\sqrt{2}} = \frac{9\sqrt{2}}{14}$

(b) $\sin\theta = \sqrt{\left(1 - \left(\frac{9\sqrt{2}}{14}\right)^2\right)} = \sqrt{\left(1 - \frac{162}{196}\right)} = \sqrt{\left(\frac{34}{196}\right)} = \frac{\sqrt{34}}{14}$

Area $= |\overrightarrow{BA}||\overrightarrow{BC}| \sin\theta = 7 \times 5\sqrt{2} \times \frac{\sqrt{34}}{14} = \frac{5\sqrt{68}}{2} = \frac{5 \times 2\sqrt{17}}{2}$

Area $= 5\sqrt{17}$, $k = 17$

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Question 7 continued

(Total for Question 7 is 6 marks)

8. A container initially contains 6 litres of water.

Water starts to flow into the container.

The rate at which the water flows into the container is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 12te^{-t}$$

where V litres is the volume of water in the container, t minutes after water starts flowing into the container.

(a) Solve the differential equation and use the initial condition to find an equation for V in terms of t

(5)

Given that the capacity of the container is 20 litres,

(b) determine, according to the model, whether or not the container will ever become full, giving a reason for your answer.

(1)

(a) $V = \int 12te^{-t}\,dt$: $u = 12t$, $\frac{dv}{dt} = e^{-t} \Rightarrow \frac{du}{dt} = 12$, $v = -e^{-t}$

$V = -12te^{-t} - \int -12e^{-t}\,dt = -12te^{-t} + \int 12e^{-t}\,dt$

$V = -12te^{-t} - 12e^{-t} + c$

$t = 0$, $V = 6$: $6 = 0 - 12 + c \Rightarrow c = 18$

$V = 18 - 12te^{-t} - 12e^{-t}$

(b) As $t \to \infty$, $te^{-t} \to 0$ and $e^{-t} \to 0$, so $V \to 18$

No, the container never becomes full: the upper limit of the volume is 18 litres, which is less than 20 litres.

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Question 8 continued

(Total for Question 8 is 6 marks)

9.

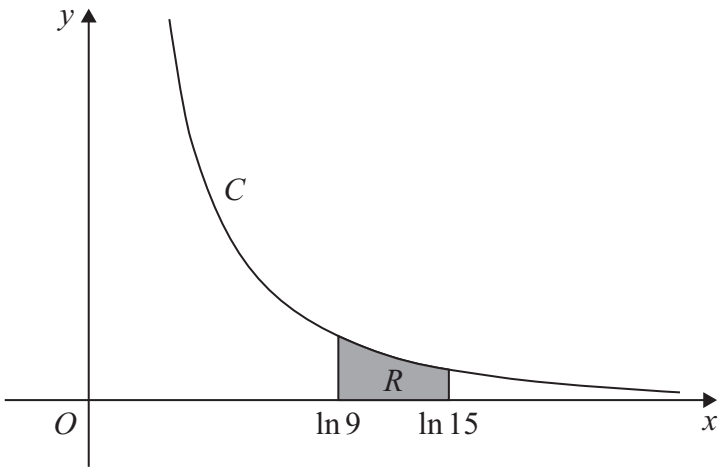


Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t + 1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P.

(6)

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = \ln 9$, the x-axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t + 1)(2t + 5)}\,\mathrm{d}t$$

where a, b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R.

Write your answer in the form $\frac{1}{3}\ln\alpha$, where α is a rational number.

(9)

(a) $\frac{dx}{dt} = \frac{2}{2t + 5}$, $\frac{dy}{dt} = -\frac{1}{(t + 1)^2}$

$\frac{dy}{dx} = -\frac{1}{(t + 1)^2} \div \frac{2}{2t + 5} = -\frac{2t + 5}{2(t + 1)^2}$

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$-\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 2t+5 = 8(t+1)^2 = 8t^2+16t+8$

$8t^2+14t+3 = 0 \Rightarrow (4t+1)(2t+3) = 0$

$t = -\frac{1}{4}$ or $t = -\frac{3}{2}$ (rejected as $t > -1$)

$t = -\frac{1}{4}$: $y = \frac{1}{-1/4+1} = \frac{1}{3/4} = \frac{4}{3}$

y coordinate of P is $\frac{4}{3}$

(b)(i) $x = \ln 9$: $2t+5 = 9 \Rightarrow t = 2$; $x = \ln 15$: $2t+5 = 15 \Rightarrow t = 5$

Area $= \int y\,dx = \int_2^5 y\frac{dx}{dt}\,dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5}\,dt$

Area $= \int_2^5 \frac{2}{(t+1)(2t+5)}\,dt$, $a = 2$, $b = 5$, $k = 2$

(ii) $\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{t+1} + \frac{B}{2t+5} \Rightarrow 2 \equiv A(2t+5) + B(t+1)$

$t = -1$: $2 = 3A \Rightarrow A = \frac{2}{3}$; $t = -\frac{5}{2}$: $2 = -\frac{3}{2}B \Rightarrow B = -\frac{4}{3}$

$\int_2^5 \left(\frac{2/3}{t+1} - \frac{4/3}{2t+5}\right) dt = \left[\frac{2}{3}\ln(t+1) - \frac{2}{3}\ln(2t+5)\right]_2^5$

$= \left(\frac{2}{3}\ln 6 - \frac{2}{3}\ln 15\right) - \left(\frac{2}{3}\ln 3 - \frac{2}{3}\ln 9\right)$

$= \frac{2}{3}\ln\left(\frac{6 \times 9}{15 \times 3}\right) = \frac{2}{3}\ln\frac{6}{5} = \frac{1}{3}\ln\frac{36}{25}$

Area of R $= \frac{1}{3}\ln\frac{36}{25}$

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 15 marks)

10. (a) Statement X: "If x^2 is an odd square number, then x is an odd number."

A student was asked to prove Statement X using proof by contradiction.

The student begins by writing the following.

> Assume that x^2 is an odd square number and x is an even number.
>
> If x is an even number it can be expressed in the form $x = 2m \quad m \in \mathbb{N}$

Complete the proof.

(2)

(b) Use proof by contradiction to show that

"There are no integers a and b such that $a^2 - 4b = 27$"

You may assume Statement X without further justification.

(4)

(a) $x = 2m \Rightarrow x^2 = (2m)^2 = 4m^2 = 2(2m^2)$

so x^2 is even, which contradicts x^2 being odd.

Hence if x^2 is an odd square number, x is an odd number.

(b) Assume there exist integers a and b such that $a^2 - 4b = 27$.

Then $a^2 = 27 + 4b = 2(2b + 13) + 1$, which is odd.

By Statement X, a is odd, so $a = 2m + 1$ for some integer m.

$(2m + 1)^2 = 27 + 4b \Rightarrow 4m^2 + 4m + 1 = 27 + 4b$

$4b = 4m^2 + 4m - 26 \Rightarrow b = m^2 + m - \frac{13}{2}$

$m^2 + m$ is an integer, so $m^2 + m - \frac{13}{2}$ is not an integer,

so b is not an integer. This is a contradiction.

Hence there are no integers a and b such that $a^2 - 4b = 27$.

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Question 10 continued

Question 10 continued

(Total for Question 10 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS

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