Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 19 October 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. Sam is a telephone sales representative.

For each call to a customer

- Sam either makes a sale or does not make a sale
- sales are made independently

Past records show that, for each call to a customer, the probability that Sam makes a sale is 0.2

(a) Find the probability that Sam makes

(i) exactly 2 sales in 14 calls,

(ii) more than 3 sales in 25 calls.

(4)

Sam makes n calls each day.

(b) Find the minimum value of n

(i) so that the expected number of sales each day is at least 6

(2)

(ii) so that the probability of at least 1 sale in a randomly selected day exceeds 0.95

(4)

(a)(i) $X \sim B(14, 0.2)$

$P(X = 2) = {}^{14}C_2 (0.2)^2 (0.8)^{12}$

$= 91 \times 0.04 \times 0.068719... = 0.250138... = 0.2501$

(ii) $X \sim B(25, 0.2)$

$P(X > 3) = 1 - P(X \leq 3)$

$= 1 - 0.2340$

$= 0.7660$

(b)(i) $E(\text{sales}) = np = 0.2n \geq 6$

$n \geq \frac{6}{0.2} = 30$

minimum $n = 30$

(ii) $Y \sim B(n, 0.2)$, require $P(Y \geq 1) > 0.95$

$P(Y \geq 1) = 1 - P(Y = 0) > 0.95$

$\Rightarrow P(Y = 0) = {}^{n}C_0 (0.2)^0 (0.8)^n = 0.8^n < 0.05$

$n \ln 0.8 < \ln 0.05 \Rightarrow n > \frac{\ln 0.05}{\ln 0.8} = 13.425...$

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Question 1 continued

check: $0.8^{13} = 0.05497... > 0.05$, $0.8^{14} = 0.04398... < 0.05$

minimum $n = 14$

(Total for Question 1 is 10 marks)

2. The continuous random variable X has probability density function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x) = \begin{cases} ax^3 & 0 \leqslant x \leqslant 4 \\ bx + c & 4 < x \leqslant d \\ 0 & \text{otherwise} \end{cases}$$

where a, b, c and d are constants such that

- $bx + c = ax^3$ at $x = 4$
- $bx + c$ is a straight line segment with end coordinates $(4, 64a)$ and $(d, 0)$

(a) State the mode of X

(1)

Given that the mode of X is equal to the median of X

(b) use algebraic integration to show that $a = \dfrac{1}{128}$

(2)

(c) Find the value of d

(2)

(d) Hence find the value of b and the value of c

(3)

(a) Mode = 4

(b) Median = mode = 4 $\Rightarrow$ $P(X \leq 4) = 0.5$

$$\int_0^4 ax^3\,dx = \left[\frac{ax^4}{4}\right]_0^4 = \frac{1}{2}$$

$$\frac{256a}{4} = 64a = \frac{1}{2}$$

$$a = \frac{1}{128} \quad \text{as required}$$

(c) At $x = 4$, $f(4) = 64a = 64 \times \frac{1}{128} = \frac{1}{2}$

Area of triangle from $x = 4$ to $x = d$ must be 0.5:

$$\frac{1}{2} \times (d - 4) \times \frac{1}{2} = \frac{1}{2}$$

$$d - 4 = 2$$

$$d = 6$$

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Question 2 continued

(d) b = gradient $= \frac{0 - 0.5}{6 - 4} = -\frac{1}{4}$

Line passes through $(6, 0)$:

$0 = -\frac{1}{4} \times 6 + c \quad \Rightarrow \quad c = \frac{3}{2}$

$b = -\frac{1}{4}, \quad c = \frac{3}{2}$

Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

3. Every morning Navtej travels from home to work. Navtej leaves home at a random time between 08:00 and 08:15

- It always takes Navtej 3 minutes to walk to the bus stop
- Buses run every 15 minutes and Navtej catches the first bus that arrives
- Once Navtej has caught the bus it always takes a further 29 minutes for Navtej to reach work

The total time, T minutes, for Navtej's journey from home to work is modelled by a continuous uniform distribution over the interval $[\alpha, \beta]$

(a) (i) Show that $\alpha = 32$

(ii) Show that $\beta = 47$

(2)

(b) State fully the probability density function for this distribution.

(2)

(c) Find the value of

(i) E(T)

(ii) Var(T)

(3)

(d) Find the probability that the time for Navtej's journey is within 5 minutes of 35 minutes.

(2)

(a)(i) Shortest time: a bus arrives just as Navtej reaches the stop (wait 0)

$\alpha = 3 + 0 + 29 = 32$

(ii) Longest time: Navtej just misses a bus and waits 15 minutes

$\beta = 3 + 15 + 29 = 47$

(b) $f(t) = \frac{1}{15}$ for $32 \leq t \leq 47$

$f(t) = 0$ otherwise

(c)(i) $E(T) = \frac{32 + 47}{2} = 39.5$

(ii) $\text{Var}(T) = \frac{(47 - 32)^2}{12} = \frac{225}{12} = \frac{75}{4} = 18.75$

(d) Within 5 minutes of 35: $P(30 < T < 40) = P(32 < T < 40)$

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Question 3 continued

$= \frac{1}{15} \times (40 - 32) = \frac{8}{15}$ $(= 0.5333...)$

(Total for Question 3 is 9 marks)

4. A manufacturer makes t-shirts in 3 sizes, small, medium and large.

20% of the t-shirts made by the manufacturer are small and sell for £10

30% of the t-shirts made by the manufacturer are medium and sell for £12

The rest of the t-shirts made by the manufacturer are large and sell for £15

(a) Find the mean value of the t-shirts made by the manufacturer.

(2)

A random sample of 3 t-shirts made by the manufacturer is taken.

(b) List all the possible combinations of the individual selling prices of these 3 t-shirts.

(2)

(c) Find the sampling distribution of the **median** selling price of these 3 t-shirts.

(6)

(a) Large: $P(£15) = 1 - 0.2 - 0.3 = 0.5$

Mean $= 0.2 \times 10 + 0.3 \times 12 + 0.5 \times 15$

$= 2 + 3.6 + 7.5 = £13.10$

(b) (10, 10, 10) (12, 12, 12) (15, 15, 15)

(10, 10, 12) (10, 10, 15) (12, 12, 10)

(12, 12, 15) (15, 15, 10) (15, 15, 12)

(10, 12, 15)

(c) $P(10) = 0.2$, $P(12) = 0.3$, $P(15) = 0.5$

Median M can be 10, 12 or 15

M = 10 from (10,10,10), (10,10,12)×3, (10,10,15)×3:

$P(M = 10) = 0.2^3 + 3(0.2^2)(0.3) + 3(0.2^2)(0.5)$

$= 0.008 + 0.036 + 0.06 = 0.104$

M = 12 from (12,12,12), (12,12,10)×3, (12,12,15)×3, (10,12,15)×6:

$P(M = 12) = 0.3^3 + 3(0.3^2)(0.2) + 3(0.3^2)(0.5) + 6(0.2)(0.3)(0.5)$

$= 0.027 + 0.054 + 0.135 + 0.18 = 0.396$

M = 15 from (15,15,15), (15,15,10)×3, (15,15,12)×3:

$P(M = 15) = 0.5^3 + 3(0.5^2)(0.2) + 3(0.5^2)(0.3)$

$= 0.125 + 0.15 + 0.225 = 0.5$

check: $0.104 + 0.396 + 0.5 = 1$

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Question 4 continued

Sampling distribution of the median:

m	10	12	15
$P(M = m)$	$\frac{13}{125}$	$\frac{99}{250}$	$\frac{1}{2}$

i.e. $P(M = 10) = 0.104$, $P(M = 12) = 0.396$, $P(M = 15) = 0.5$

Question 4 continued

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Question 4 continued

(Total for Question 4 is 10 marks)

5. A supermarket receives complaints at a mean rate of 6 per week.

(a) State one assumption necessary, in order for a Poisson distribution to be used to model the number of complaints received by the supermarket.

(1)

(b) Find the probability that, in a given week, there are

(i) fewer than 3 complaints received by the supermarket,

(ii) at least 6 complaints received by the supermarket.

(3)

In a randomly selected week, the supermarket received 12 complaints.

(c) Test, at the 5% level of significance, whether or not there is evidence that the mean number of complaints is greater than 6 per week.
State your hypotheses clearly.

(5)

Following changes made by the supermarket, it received 26 complaints over a 6-week period.

(d) Use a suitable approximation to test whether or not there is evidence that, following the changes, the mean number of complaints received is less than 6 per week.
You should state your hypotheses clearly and use a 5% significance level.

(7)

(a) Complaints are received independently of each other (and at a constant average rate).

(b)(i) $X \sim Po(6)$

$P(X < 3) = P(X \leq 2) = 0.0620$

(ii) $P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.4457$

$= 0.5543$

(c) $H_0 : \lambda = 6$ $H_1 : \lambda > 6$

Under H_0, $X \sim Po(6)$

$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9799$

$= 0.0201$

$0.0201 < 0.05$, so the result is significant: reject H_0

There is sufficient evidence to suggest that the mean number of complaints received is greater than 6 per week.

(d) $H_0 : \lambda = 6$ $H_1 : \lambda < 6$ (per week)

Over 6 weeks, under H_0: $Y \sim Po(36)$

λ large so approximate by $W \sim N(36, 36)$

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Question 5 continued

$P(Y \leqslant 26) \approx P(W < 26.5) = P\left(Z < \frac{26.5 - 36}{6}\right)$

$= P(Z < -1.5833...)$

$= 0.05667...\ = 0.0567$

$0.0567 > 0.05$, so the result is not significant: do not reject H_0

There is insufficient evidence to suggest that, following the changes, the mean number of complaints received is less than 6 per week.

Question 5 continued

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Question 5 continued

(Total for Question 5 is 16 marks)

6. The continuous random variable Y has cumulative distribution function given by

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{21}y^2 & 0 \leqslant y \leqslant k \\ \frac{2}{15}\left(6y - \frac{y^2}{2}\right) - \frac{7}{5} & k < y \leqslant 6 \\ 1 & y > 6 \end{cases}$$

(a) Find $P\left(Y < \frac{1}{4}k \mid Y < k\right)$

(2)

(b) Find the value of k

(4)

(c) Use algebraic calculus to find $E(Y)$

(6)

(a) $P\left(Y < \frac{1}{4}k \mid Y < k\right) = \frac{F(k/4)}{F(k)}$

$= \frac{(1/21)(k/4)^2}{(1/21)k^2} = \frac{1}{16}$

(b) F is continuous at $y = k$:

$\frac{1}{21}k^2 = \frac{2}{15}\left(6k - \frac{k^2}{2}\right) - \frac{7}{5}$

$\frac{1}{21}k^2 = -\frac{1}{15}k^2 + \frac{4}{5}k - \frac{7}{5}$

$\times 105: \quad 5k^2 = -7k^2 + 84k - 147$

$12k^2 - 84k + 147 = 0 \quad \Rightarrow \quad 4k^2 - 28k + 49 = 0$

$(2k - 7)^2 = 0$

$k = \frac{7}{2} = 3.5$

(c) $f(y) = \frac{d}{dy}F(y)$: $\quad f(y) = \frac{2}{21}y \qquad 0 \leq y \leq 3.5$

$f(y) = \frac{2}{15}(6 - y) \qquad 3.5 < y \leq 6, \qquad 0 \text{ otherwise}$

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Question 6 continued

$$E(Y) = \int_0^{3.5} \frac{2}{21} y^2 \, dy + \int_{3.5}^{6} \frac{2}{15} (6y - y^2) \, dy$$

$$= \frac{2}{21} \left[\frac{y^3}{3} \right]_0^{3.5} + \frac{2}{15} \left[3y^2 - \frac{y^3}{3} \right]_{3.5}^{6}$$

$$= \frac{2}{21} \left(\frac{343}{24} \right) + \frac{2}{15} \left(36 - \frac{539}{24} \right)$$

$$= \frac{49}{36} + \frac{2}{15} \times \frac{325}{24} = \frac{49}{36} + \frac{65}{36}$$

$$= \frac{19}{6} = 3.1666\ldots = 3.17 \text{ (3 s.f.)}$$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 12 marks)

7. The discrete random variable X is given by

$$X \sim \mathrm{B}(n, p)$$

The value of n and the value of p are such that X can be approximated by a normal random variable Y where

$$Y \sim \mathrm{N}(\mu, \sigma^2)$$

Given that when using a normal approximation

$$\mathrm{P}(X < 86) = 0.2266 \quad \text{and} \quad \mathrm{P}(X > 97) = 0.1056$$

(a) show that $\sigma = 6$

(7)

(b) Hence find the value of n and the value of p

(3)

(a) Using a continuity correction:

$P(X < 86) \approx P(Y < 85.5) = 0.2266 \Rightarrow z = -0.75$

$P(X > 97) \approx P(Y > 97.5) = 0.1056 \Rightarrow z = 1.25$

$\frac{85.5 - \mu}{\sigma} = -0.75 \Rightarrow 85.5 - \mu = -0.75\sigma$ (1)

$\frac{97.5 - \mu}{\sigma} = 1.25 \Rightarrow 97.5 - \mu = 1.25\sigma$ (2)

(2) − (1): $12 = 2\sigma$

$\sigma = 6$ as required (and $\mu = 85.5 + 0.75 \times 6 = 90$)

(b) $np = \mu = 90$ and $np(1 - p) = \sigma^2 = 36$

$1 - p = \frac{36}{90} = 0.4 \Rightarrow p = 0.6$

$n = \frac{90}{0.6} = 150$

$n = 150, \quad p = 0.6$

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Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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