Please check the examination details below before entering your candidate information

Candidate surname	Other names
StemUp Model Solution	

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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B:1/1/

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Pearson

1. Bhavna produces rolls of cloth. She knows that faults occur randomly in her cloth at a mean rate of 1.5 every 15 metres.

(a) Find the probability that in 15 metres of her cloth there are

(i) less than 3 faults,

(ii) at least 6 faults.

(3)

Each roll contains 100 metres of Bhavna's cloth.
She selects 15 rolls at random.

(b) Find the probability that exactly 10 of these rolls each have fewer than 13 faults.

(4)

Bhavna decides to sell her cloth in pieces.

Each piece of her cloth is 4 metres long.
The cost to make each piece is £5.00
She sells each piece of her cloth that contains no faults for £7.40
She sells each piece of her cloth that contains faults for £2.00

(c) Find the expected profit that Bhavna will make on each piece of her cloth that she sells.

(4)

(a) F = number of faults in 15 m, $F \sim Po(1.5)$

(i) $P(F < 3) = P(F \leqslant 2) = 0.8088$

(ii) $P(F \geqslant 6) = 1 - P(F \leqslant 5) = 1 - 0.9955$

$= 0.0045$

(b) R = number of faults in 100 m, $R \sim Po(10)$

$P(R < 13) = P(R \leqslant 12) = 0.7916$

X = number of rolls with fewer than 13 faults, $X \sim B(15, 0.7916)$

$P(X = 10) = {}^{15}C_{10} \times (0.7916)^{10} \times (1 - 0.7916)^5$

$= 3003 \times (0.7916)^{10} \times (0.2084)^5$

$= 0.11405\ldots = 0.114$ (3 s.f.)

(c) H = number of faults in 4 m, $H \sim Po(0.4)$

$P(H = 0) = e^{-0.4} = 0.67032\ldots$ $P(H > 0) = 1 - e^{-0.4} = 0.32967\ldots$

Profit on a piece with no faults = 7.40 − 5.00 = £2.40

Profit on a piece with faults = 2.00 − 5.00 = −£3.00

E(profit) = $2.40 \times 0.67032\ldots + (-3.00) \times 0.32967\ldots$

$= 1.60877\ldots - 0.98903\ldots$

$= 0.6197\ldots \approx$ £0.62 (62p)

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 11 marks)

2. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4} & -\frac{1}{2} \leqslant x < \frac{1}{2} \\ 2x - \frac{3}{4} & \frac{1}{2} \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a) Sketch the graph of $f(x)$

(2)

(b) By forming and solving an equation in k, show that $k = 1.25$

(4)

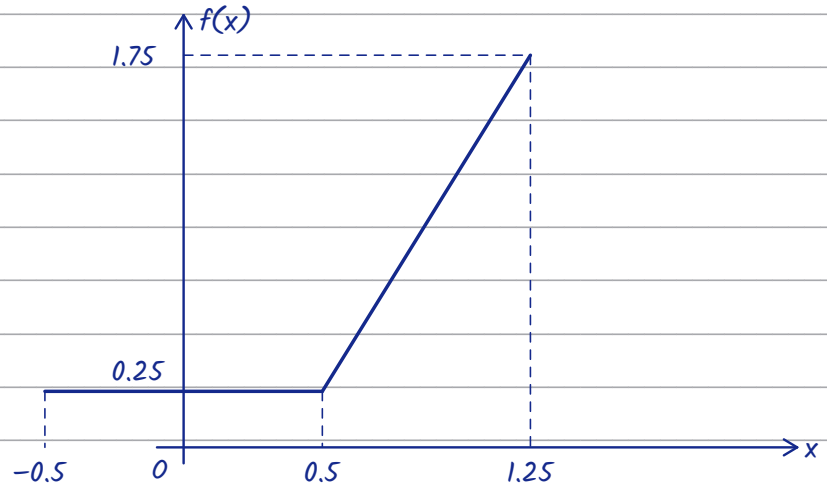
(c) Use calculus to find $E(X)$

(4)

(d) Calculate the interquartile range of X

(5)

(a)



(b) $\frac{1}{4} \times 1 + \frac{1}{2}\left(k - \frac{1}{2}\right)\left(\frac{1}{4} + 2k - \frac{3}{4}\right) = 1$

$\frac{1}{2}\left(k - \frac{1}{2}\right)\left(2k - \frac{1}{2}\right) = \frac{3}{4}$

$2k^2 - \frac{3}{2}k + \frac{1}{4} = \frac{3}{2}$ $(\times 4)$

$8k^2 - 6k - 5 = 0$

$(4k - 5)(2k + 1) = 0$

$k = 1.25$ or $k = -\frac{1}{2}$ (reject since $k > 0$)

$\therefore k = 1.25$ as required

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Question 2 continued

(c) $E(X) = \int_{-0.5}^{0.5} \frac{1}{4}x\,dx + \int_{0.5}^{1.25} \left(2x^2 - \frac{3}{4}x\right) dx$

$= \left[\frac{x^2}{8}\right]_{-0.5}^{0.5} + \left[\frac{2x^3}{3} - \frac{3x^2}{8}\right]_{0.5}^{1.25}$

$= 0 + \left(\frac{2 \times 1.25^3}{3} - \frac{3 \times 1.25^2}{8}\right) - \left(\frac{2 \times 0.5^3}{3} - \frac{3 \times 0.5^2}{8}\right)$

$= \left(\frac{125}{96} - \frac{75}{128}\right) - \left(\frac{1}{12} - \frac{3}{32}\right) = \frac{275}{384} + \frac{1}{96}$

$= \frac{93}{128} = 0.727$ (3 s.f.)

(d) For $-0.5 \leq x < 0.5$: $F(x) = \frac{1}{4}(x + 0.5)$, so $F(0.5) = 0.25$

$\therefore Q_1 = 0.5$

$0.25 + \int_{0.5}^{Q_3} \left(2x - \frac{3}{4}\right) dx = 0.75$

$0.25 + \left[x^2 - \frac{3}{4}x\right]_{0.5}^{Q_3} = 0.75$

$Q_3^2 - \frac{3}{4}Q_3 + 0.125 = 0.5 \Rightarrow Q_3^2 - \frac{3}{4}Q_3 - 0.375 = 0$

$Q_3 = \frac{0.75 \pm \sqrt{(-0.75)^2 + 4 \times 0.375}}{2} = \frac{3 \pm \sqrt{33}}{8}$

$Q_3 = 1.0930...$ (-0.343 rejected, not in $0.5 \leq x \leq 1.25$)

$IQR = Q_3 - Q_1 = 1.0930... - 0.5$

$= 0.5930... = 0.593$ (3 s.f.) $\left(= \frac{\sqrt{33} - 1}{8}\right)$

Question 2 continued

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Question 2 continued

(Total for Question 2 is 15 marks)

3. A company produces packets of sunflower seeds. Each packet contains 40 seeds. The company claims that, on average, only 35% of its sunflower seeds do not germinate.

A packet is selected at random.

(a) Using a 5% level of significance, find an appropriate critical region for a two-tailed test that the proportion of sunflower seeds that do not germinate is 0.35
You should state your hypotheses clearly and state the probability, which should be as close as possible to 2.5%, for each tail of your critical region.

(4)

(b) Write down the actual significance level of this test.

(1)

Past records suggest that 2.8% of the company's sunflower seeds grow to a height of more than 3 metres.
A random sample of 250 of the company's sunflower seeds is taken and 11 of them grow to a height of more than 3 metres.

(c) Using a suitable approximation test, at the 5% level of significance, whether or not there is evidence that the proportion of sunflower seeds that grow to a height of more than 3 metres is now greater than 2.8%
State your hypotheses clearly.

(5)

(a) X = number of seeds that do not germinate, $X \sim B(40, 0.35)$

$H_0: p = 0.35$ $H_1: p \neq 0.35$

Lower tail: $P(X \leqslant 8) = 0.0303$, $P(X \leqslant 9) = 0.0644$

Upper tail: $P(X \geqslant 21) = 1 - P(X \leqslant 20) = 1 - 0.9827 = 0.0173$

$P(X \geqslant 20) = 1 - P(X \leqslant 19) = 1 - 0.9637 = 0.0363$

Critical region: $X \leqslant 8$ or $X \geqslant 21$

(b) Actual significance level = $0.0303 + 0.0173 = 0.0476$

(c) $H_0: p = 0.028$ $H_1: p > 0.028$

Under H_0, $Y \sim B(250, 0.028)$; n large, p small, so $Y \approx Po(250 \times 0.028) = Po(7)$

$P(Y \geqslant 11) = 1 - P(Y \leqslant 10) = 1 - 0.9015$

$= 0.0985 > 0.05$

Not significant, so do not reject H_0.

There is insufficient evidence to suggest that the proportion of sunflower seeds that grow to a height of more than 3 metres is now greater than 2.8%.

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4. The probability that a person completes a particular task in less than 15 minutes is 0.4

Jeffrey selects 20 people at random and asks them to complete the task. The random variable, X, represents the number of people who complete the task in less than 15 minutes.

(a) Find $P(5 \leqslant X < 8)$

(3)

Mia takes a random sample of 140 people.
Using a normal approximation, the probability that fewer than n of these 140 people complete the task in less than 15 minutes is 0.0239 to 4 decimal places.

(b) Find the value of n
Show your working clearly.

(6)

(a) $X \sim B(20, 0.4)$

$P(5 \leq X < 8) = P(X \leq 7) - P(X \leq 4)$

$= 0.4159 - 0.0510$

$= 0.3649$

(b) $Y \sim B(140, 0.4)$: $np = 56$, $np(1-p) = 56 \times 0.6 = 33.6$

$Y \approx W \sim N(56, 33.6)$

$P(Y < n) = P(Y \leq n - 1) \approx P(W < n - 0.5) = 0.0239$

$P\left(Z < \dfrac{n - 0.5 - 56}{\sqrt{33.6}}\right) = 0.0239$ and $P(Z < -1.98) = 0.0239$

$\dfrac{n - 56.5}{\sqrt{33.6}} = -1.98$

$n = 56.5 - 1.98 \times \sqrt{33.6} = 56.5 - 11.4771\ldots$

$n = 45.0228\ldots \quad \therefore n = 45$

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Question 4 continued

(Total for Question 4 is 9 marks)

5. The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 3 \\ \frac{1}{6}(x-3)^2 & 3 \leqslant x < 4 \\ \frac{x}{3} - \frac{7}{6} & 4 \leqslant x < c \\ 1 - \frac{1}{6}(d-x)^2 & c \leqslant x < 7 \\ 1 & x \geqslant 7 \end{cases}$$

where c and d are constants.

(a) Show that $c = 6$

(4)

(b) Find $P(X > 3.5)$

(2)

(c) Find $P(X > 4.5 | 3.5 < X < 5.5)$

(3)

(a) $F(7) = 1 \Rightarrow 1 - \frac{1}{6}(d-7)^2 = 1 \Rightarrow d = 7$

At $x = c$: $\frac{c}{3} - \frac{7}{6} = 1 - \frac{1}{6}(7-c)^2$

$\times 6$: $2c - 7 = 6 - (49 - 14c + c^2)$

$2c - 7 = -43 + 14c - c^2$

$c^2 - 12c + 36 = 0$

$(c-6)^2 = 0 \Rightarrow c = 6$ as required

(b) $P(X > 3.5) = 1 - F(3.5) = 1 - \frac{1}{6}(3.5-3)^2$

$= 1 - \frac{1}{24} = \frac{23}{24}$ $(= 0.958)$

(c) $P(3.5 < X < 5.5) = F(5.5) - F(3.5) = \left(\frac{5.5}{3} - \frac{7}{6}\right) - \frac{1}{24}$

$= \frac{2}{3} - \frac{1}{24} = \frac{5}{8}$

$P(4.5 < X < 5.5) = F(5.5) - F(4.5) = \left(\frac{5.5}{3} - \frac{7}{6}\right) - \left(\frac{4.5}{3} - \frac{7}{6}\right) = \frac{1}{3}$

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Question 5 continued

$$P(X > 4.5 \mid 3.5 < X < 5.5) = \frac{P(4.5 < X < 5.5)}{P(3.5 < X < 5.5)} = \frac{1}{3} \div \frac{5}{8}$$

$$= \frac{8}{15} \quad (= 0.533)$$

Question 5 continued

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Question 5 continued

(Total for Question 5 is 9 marks)

6. A bag contains a large number of counters with one of the numbers 5, 10 or 20 written on each of them in the ratio $5:2:a$

A jar contains a large number of counters with one of the numbers 5 or 10 written on each of them in the ratio $1:3$

One counter is selected at random from the bag and then two counters are selected at random from the jar.
The random variable R represents the range of the numbers on the 3 counters.

Given that $P(R = 15) = \frac{63}{256}$

(a) by forming and solving an equation in a, show that $a = 9$

(3)

(b) find the sampling distribution of R

(6)

(a) Bag: $P(20) = \frac{a}{a+7}$. $R = 15$ needs a 20 from the bag and at least one 5 from the two jar counters

$$\frac{a}{a+7} \times \left(1 - \left(\frac{3}{4}\right)^2\right) = \frac{63}{256}$$

$$\frac{a}{a+7} \times \frac{7}{16} = \frac{63}{256} \Rightarrow \frac{a}{a+7} = \frac{9}{16}$$

$16a = 9a + 63 \Rightarrow 7a = 63 \Rightarrow a = 9$ as required

(b) Bag: $P(5) = \frac{5}{16}$, $P(10) = \frac{2}{16}$, $P(20) = \frac{9}{16}$ Jar: $P(5) = \frac{1}{4}$, $P(10) = \frac{3}{4}$

Possible ranges: $R = 0, 5, 10, 15$

$$P(R = 0) = \frac{5}{16} \times \frac{1}{4} \times \frac{1}{4} + \frac{2}{16} \times \frac{3}{4} \times \frac{3}{4} = \frac{5}{256} + \frac{18}{256} = \frac{23}{256}$$

$$P(R = 5) = 2 \times \frac{5}{16} \times \frac{1}{4} \times \frac{3}{4} + \frac{5}{16} \times \frac{3}{4} \times \frac{3}{4}$$

$$+ 2 \times \frac{2}{16} \times \frac{1}{4} \times \frac{3}{4} + \frac{2}{16} \times \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{30}{256} + \frac{45}{256} + \frac{12}{256} + \frac{2}{256} = \frac{89}{256}$$

$$P(R = 10) = \frac{9}{16} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256}$$

$$P(R = 15) = \frac{63}{256}$$

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Question 6 continued

r	0	5	10	15
$P(R = r)$	$\frac{23}{256}$	$\frac{89}{256}$	$\frac{81}{256}$	$\frac{63}{256}$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7. (i) The continuous random variable X is uniformly distributed over the interval $[a, b]$

Given that $P(5 < X < 13) = \frac{1}{5}$ and $E(X) = 9$, find $P(3X > a + b)$

(3)

(ii) The continuous random variable Y is uniformly distributed over the interval $[1, c]$

Given that $\text{Var}(Y) = 0.48$, find the exact value of $E(Y^2)$

(4)

(iii) A wire of length 20 cm is cut into 2 pieces at a random point.
The longest piece of wire is then cut into 2 pieces, equal in length, giving 3 pieces of wire altogether.

Find the probability that the length of the shortest piece of wire is less than 6 cm.

(5)

(i) $\frac{a+b}{2} = 9 \Rightarrow a + b = 18; \quad \frac{13-5}{b-a} = \frac{1}{5} \Rightarrow b - a = 40$

$a = -11, \quad b = 29$

$P(3X > a + b) = P(3X > 18) = P(X > 6) = \frac{29-6}{40} = \frac{23}{40}$

(ii) $\text{Var}(Y) = \frac{(c-1)^2}{12} = 0.48 \Rightarrow (c-1)^2 = 5.76 \Rightarrow c = 3.4$

$E(Y) = \frac{1+3.4}{2} = 2.2$

$E(Y^2) = \text{Var}(Y) + [E(Y)]^2 = 0.48 + 2.2^2$

$= 5.32 \quad \left(= \frac{133}{25}\right)$

(iii) Let W = distance of the cut from one end, $W \sim U[0, 20]$

If $W < 10$: pieces are W, $\frac{1}{2}(20 - W)$, $\frac{1}{2}(20 - W)$

Shortest piece < 6 when $W < 6$ or $\frac{1}{2}(20 - W) < 6$, i.e. $W > 8$

By symmetry, for $W > 10$: shortest < 6 when $W > 14$ or $W < 12$

So shortest piece < 6 when $W < 6$, $8 < W < 12$ or $W > 14$

$P(W < 6) = \frac{6-0}{20} = \frac{3}{10}, \quad P(W > 14) = \frac{20-14}{20} = \frac{3}{10}$

$P(8 < W < 12) = \frac{12-8}{20} = \frac{1}{5}$

$P(\text{shortest piece} < 6 \text{ cm}) = \frac{3}{10} + \frac{3}{10} + \frac{1}{5} = \frac{4}{5}$

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Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER: 75 MARKS

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