Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Thursday 11 June 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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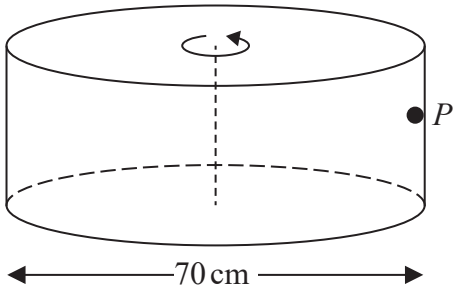


Figure 1

Figure 1 shows a hollow cylinder with diameter 70 cm. The cylinder is rotating at a constant angular speed about its axis, which is vertical. As the cylinder rotates, a particle P remains in contact with the same point on the rough inside surface of the cylinder. The particle is moving in a horizontal circle of diameter 70 cm.

The cylinder makes 2 complete revolutions every second.

The coefficient of friction between P and the cylinder is μ.

Find the range of possible values of μ.

(6)

$\omega = \frac{2\pi}{0.5} = 4\pi$ rad s^{-1}, $r = 0.35$ m

Let R be the normal reaction (towards the axis) and F the friction.

$R(\rightarrow$, towards axis$)$: $R = mr\omega^2 = m \times 0.35 \times (4\pi)^2 = \frac{28m\pi^2}{5}$

$R(\uparrow)$: $F = mg$

No slipping: $F \leq \mu R$

$mg \leq \mu \times \frac{28m\pi^2}{5} \Rightarrow \mu \geq \frac{5g}{28\pi^2} = 0.17731\ldots$

Range of possible values: $\mu \geq 0.177$ (3 s.f.)

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Question 1 continued

Q1

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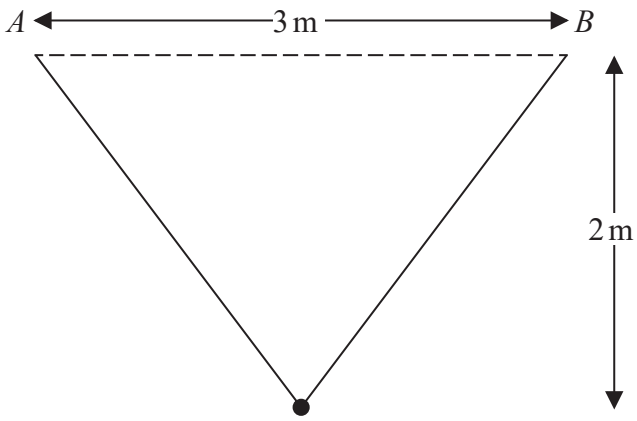


Figure 2

A smooth bead of weight 12 N is threaded onto a light elastic string of natural length 3 m. The points A and B are on a horizontal ceiling, with $AB = 3$ m. One end of the string is attached to A and the other end of the string is attached to B.

The bead hangs freely in equilibrium, 2 m below the ceiling, as shown in Figure 2.

(a) Find the tension in the string.

(4)

(b) Show that the modulus of elasticity of the string is 11.25 N.

(2)

The bead is now pulled down to a point vertically below its equilibrium position and released from rest.

(c) Find the elastic energy stored in the string at the instant when the bead is moving at its maximum speed.

(2)

(a) Each half of the string: horizontal 1.5 m, vertical 2 m, length 2.5 m

Angle θ with the vertical: $\cos\theta = \frac{2}{2.5} = \frac{4}{5}$

Tension T is the same throughout (smooth bead).

$R(\uparrow)$: $2T\cos\theta = 12 \Rightarrow 2T \times \frac{4}{5} = 12$

$T = \frac{60}{8} = 7.5$ N

(b) Stretched length $= 2 \times 2.5 = 5$ m, extension $= 5 - 3 = 2$ m

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Question 2 continued

Hooke: $T = \frac{\lambda x}{l} \Rightarrow 7.5 = \frac{\lambda \times 2}{3}$

$\lambda = 7.5 \times 1.5 = 11.25$ N (as required)

(c) Maximum speed occurs at the equilibrium position, extension 2 m.

$EPE = \frac{\lambda x^2}{2l} = \frac{11.25 \times 2^2}{2 \times 3} = 7.5$ J

Q2

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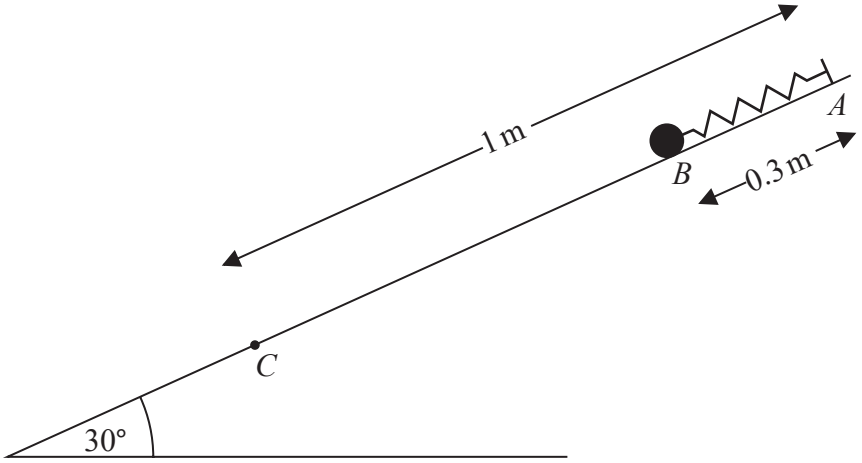


Figure 3

A particle P of mass 2 kg is attached to one end of a light elastic spring, of natural length 0.8 m and modulus of elasticity 12 N. The other end of the spring is attached to a fixed point A on a rough plane. The plane is inclined at 30° to the horizontal. Initially P is held at rest on the plane at the point B, where B is below A, with $AB = 0.3$ m and AB lies along a line of greatest slope of the plane. The point C lies on the plane with $AC = 1$ m, as shown in Figure 3.

The coefficient of friction between P and the plane is 0.3

After being released P passes through the point C.

Find the speed of P at the instant it passes through C.

(7)

At B the spring is compressed by $0.8 - 0.3 = 0.5$ m;

at C it is extended by $1 - 0.8 = 0.2$ m. Distance BC = 0.7 m.

Loss of GPE $= 2g \times 0.7 \sin 30° = 6.86$ J

$R = 2g \cos 30°$, $F = 0.3R = 0.3 \times 2g \cos 30° = 5.0922\ldots$ N

Work done against friction $= 2g \cos 30° \times 0.3 \times 0.7 = 3.5645\ldots$ J

EPE at B $= \frac{12 \times 0.5^2}{2 \times 0.8} = 1.875$ J, EPE at C $= \frac{12 \times 0.2^2}{2 \times 0.8} = 0.3$ J

Work-energy from B to C:

$$\frac{12(0.5)^2}{2 \times 0.8} + 6.86 = \frac{12(0.2)^2}{2 \times 0.8} + 0.7 \times 5.0922\ldots + \frac{1}{2} \times 2v^2$$

$$1.875 + 6.86 = 0.3 + 3.5645\ldots + v^2$$

$v^2 = 4.8705\ldots \Rightarrow v = 2.2069\ldots$

Speed of P at C $= 2.21$ m s^{-1} (3 s.f.)

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Question 3 continued

Q3

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4. (a) Use algebraic integration to show that the centre of mass of a uniform solid hemisphere of radius a is a distance $\frac{3}{8}a$ from the centre of its plane face.

[*You may assume that the volume of a sphere of radius r is* $\frac{4}{3}\pi r^3$]

(5)

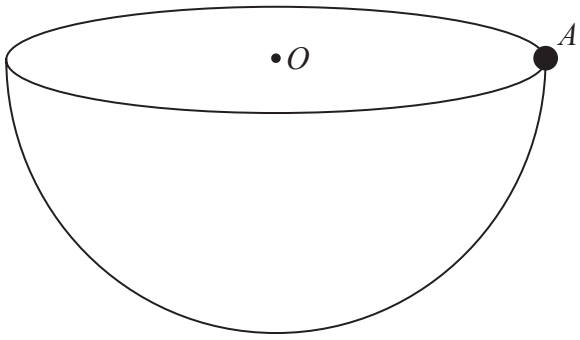


Figure 4

A uniform solid hemisphere has mass m and radius a. A particle of mass km is attached to a point A on the circumference of the plane face of the hemisphere to form the loaded solid S. The centre of the plane face of the hemisphere is the point O, as shown in Figure 4.

The loaded solid S is placed on a horizontal plane. The curved surface of S is in contact with the plane and S rests in equilibrium with OA making an angle α with the horizontal, where $\tan\alpha = \sqrt{3}$

(b) Find the exact value of k.

(5)

(a) Use thin discs perpendicular to the axis of symmetry Oy, centre O on the plane face: disc at distance y from O has radius x, $x^2 = a^2 - y^2$.

$M\bar{y} = \rho\pi\int_0^a x^2 y\,dy = \rho\pi\int_0^a y(a^2 - y^2)\,dy$

$= \rho\pi\left[\frac{a^2y^2}{2} - \frac{y^4}{4}\right]_0^a = \rho\pi\left(\frac{a^4}{2} - \frac{a^4}{4}\right) = \rho\pi\frac{a^4}{4}$

$M = \rho \times \frac{1}{2} \times \frac{4}{3}\pi a^3 = \rho\frac{2\pi a^3}{3}$

$\bar{y} = \frac{\rho\pi a^4/4}{\rho 2\pi a^3/3} = \frac{a^4}{4} \times \frac{3}{2a^3} = \frac{3a}{8}$

So the centre of mass is $\frac{3}{8}a$ from the centre of the plane face.

(b) S rests on its curved surface, so the vertical through the point of

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Question 4 continued

contact passes through O; the centre of mass G of S lies on this vertical.

Axes from O: x along OA, y along the axis into the hemisphere.

$(k + 1)m\,\bar{x} = kma \quad \Rightarrow \quad \bar{x} = \dfrac{ka}{k + 1}$

$(k + 1)m\,\bar{y} = m \times \dfrac{3}{8}a \quad \Rightarrow \quad \bar{y} = \dfrac{3a}{8(k + 1)}$

OG is vertical and OA makes angle α with the horizontal, so

$\tan\alpha = \dfrac{\bar{x}}{\bar{y}} = \dfrac{ka}{k + 1} \times \dfrac{8(k + 1)}{3a} = \dfrac{8k}{3}$

$\dfrac{8k}{3} = \sqrt{3} \quad \Rightarrow \quad k = \dfrac{3\sqrt{3}}{8}$

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Question 4 continued

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Question 4 continued

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5. A particle P of mass 0.5 kg moves along the positive x-axis in the positive x direction.

At time t seconds, $t \geqslant 1$, P is x metres from the origin O and is moving with speed v m s^{-1}.

The resultant force acting on P has magnitude $\frac{2}{x^3}$ N and is directed towards O.

When $t = 1$, $x = 1$ and $v = 3$

Show that

(a) $v^2 = \frac{4}{x^2} + 5$ **(5)**

(b) $t = \frac{a + \sqrt{bx^2 + c}}{d}$, where a, b, c and d are integers to be found. **(7)**

(a) Force towards O: $0.5v\frac{dv}{dx} = -\frac{2}{x^3}$

$\int v\,dv = \int -\frac{4}{x^3}\,dx$

$\frac{v^2}{2} = \frac{2}{x^2} + k \Rightarrow v^2 = \frac{4}{x^2} + 2k$

$x = 1, v = 3$: $9 = 4 + 2k \Rightarrow 2k = 5$

$\therefore v^2 = \frac{4}{x^2} + 5$

(b) $v = \frac{dx}{dt} = \sqrt{4/x^2 + 5} = \frac{\sqrt{(5x^2 + 4)}}{x}$ $(v > 0, x > 0)$

$\int \frac{x}{\sqrt{(5x^2 + 4)}}\,dx = \int dt$

$\frac{1}{5}(5x^2 + 4)^{\frac{1}{2}} = t + k_1$

$t = 1, x = 1$: $\frac{1}{5} \times \sqrt{9} = 1 + k_1 \Rightarrow k_1 = \frac{3}{5} - 1 = -\frac{2}{5}$

$t = \frac{1}{5}\sqrt{(5x^2 + 4)} + \frac{2}{5} = \frac{2 + \sqrt{(5x^2 + 4)}}{5}$

$a = 2,\ b = 5,\ c = 4,\ d = 5$

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Question 5 continued

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6. A light elastic string has natural length a and modulus of elasticity $\frac{3}{4}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. Particle P hangs freely in equilibrium at the point O, vertically below A.

(a) Find the distance OA.

(2)

The particle P is now pulled vertically down to a point B, where $AB = 3a$, and released from rest.

(b) Show that, throughout the subsequent motion, P performs **only** simple harmonic motion, justifying your answer.

(6)

The point C is vertically below A, where $AC = 2a$.

Find, in terms of a and g,

(c) the speed of P at the instant that it passes through C,

(3)

(d) the time taken for P to move directly from B to C.

(4)

(a) Equilibrium, extension e: $\frac{\frac{3}{4}mg\,e}{a} = mg \Rightarrow e = \frac{4}{3}a$

$OA = a + \frac{4}{3}a = \frac{7}{3}a$

(b) Let x be the displacement of P below O (downwards positive).

$m\ddot{x} = mg - T = mg - \frac{\frac{3}{4}mg(x + 4a/3)}{a}$

$m\ddot{x} = mg - \frac{3mgx}{4a} - mg = -\frac{3mgx}{4a}$

$\ddot{x} = -\frac{3g}{4a}x$, of the form $\ddot{x} = -\omega^2 x$ with $\omega^2 = \frac{3g}{4a}$

so P moves with SHM about O while the string is taut.

Amplitude: $OB = 3a - \frac{7}{3}a = \frac{2}{3}a$ (released from rest at B)

Highest point is $\frac{2}{3}a$ above O, and $\frac{2}{3}a < \frac{4}{3}a$ (the equilibrium extension),

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so at the highest point the extension is $\frac{2}{3}a > 0$: the string never becomes slack, therefore P performs only SHM throughout the motion.

(c) C: $AC = 2a$, so C is $\frac{7}{3}a - 2a = \frac{a}{3}$ above O, $x = -\frac{a}{3}$

$v^2 = \omega^2(A^2 - x^2) = \frac{3g}{4a}\left(\left(\frac{2a}{3}\right)^2 - \left(\frac{a}{3}\right)^2\right) = \frac{3g}{4a} \times \frac{a^2}{3}$

$v^2 = \frac{ag}{4} \Rightarrow v = \frac{\sqrt{(ag)}}{2}$

(d) Measuring t from B: $x = \frac{2a}{3}\cos \omega t$, at C $x = -\frac{a}{3}$

$-\frac{a}{3} = \frac{2a}{3}\cos \omega t \Rightarrow \cos \omega t = -\frac{1}{2} \Rightarrow \omega t = \frac{2\pi}{3}$

$t\sqrt{3g/4a} = \frac{2\pi}{3}$

$t = \frac{2\pi}{3}\sqrt{4a/3g} = \frac{4\pi}{3}\sqrt{a/3g}$

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Question 6 continued

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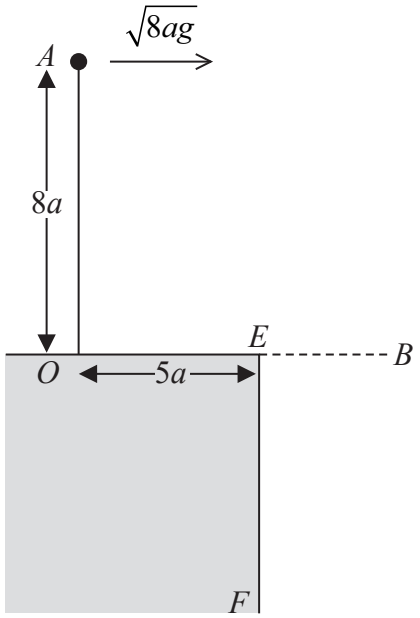


Figure 5

A particle of mass m is attached to one end of a light inextensible string of length $8a$. The other end of the string is fixed to the point O on the smooth horizontal surface of a desk. The point E is on the edge of the desk, where $OE = 5a$ and OE is perpendicular to the edge of the desk. The particle is held at the point A, vertically above O, with the string taut.

The particle is projected horizontally from A with speed $\sqrt{8ag}$ in the direction OE, as shown in Figure 5.

When the particle is above the level of OE the particle is moving in a vertical circle with radius $8a$.

Given that, when the string makes an angle θ with the upward vertical through O, the tension in the string is T,

(a) show that $T = 3mg(1 - \cos\theta)$

(7)

At the instant when the string is horizontal, the particle passes through the point B.

(b) Find the instantaneous change in the tension in the string as the particle passes through B.

(3)

The particle hits the vertical side EF of the desk and rebounds. As a result of the impact, the particle loses one third of the kinetic energy it had immediately before the impact.

In the subsequent motion the string becomes slack when it makes an angle α with the upward vertical through O.

(c) Show that $\cos\alpha = \frac{7}{12}$

(7)

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Question 7 continued

(a) Let v be the speed when the string makes angle θ with the upward vertical.

Energy (height of A is 8a above O, particle is 8a cos θ above O):

$\frac{1}{2}m(8ag) + mg(8a) = \frac{1}{2}mv^2 + mg(8a\cos\theta)$

$v^2 = 8ag + 16ag - 16ag\cos\theta = 24ag - 16ag\cos\theta$

Radially (towards O): $T + mg\cos\theta = \frac{mv^2}{8a}$

$T + mg\cos\theta = \frac{m(24ag - 16ag\cos\theta)}{8a} = 3mg - 2mg\cos\theta$

$T = 3mg - 3mg\cos\theta = 3mg(1 - \cos\theta)$ (as required)

(b) At B, θ = 90°: $v_B^2 = 24ag - 0 = 24ag$

Just before B (radius 8a): $T_1 = \frac{m(24ag)}{8a} = 3mg$

After B the string turns about the edge E, so the radius is 8a − 5a = 3a.

Just after B (radius 3a): $T_2 = \frac{m(24ag)}{3a} = 8mg$

Change in tension = 8mg − 3mg = 5mg (an increase)

(c) The particle hits EF at the lowest point, 3a below E, i.e. 11a below A.

$\frac{1}{2}mv_1^2 = \frac{1}{2}m(8ag) + mg(11a) \Rightarrow v_1^2 = 30ag$

After impact, KE = $\frac{2}{3} \times \frac{1}{2}m(30ag)$: $v_2^2 = 20ag$

Rising back through B, the particle then moves on the circle radius 8a.

At angle α with the upward vertical (3a + 8a cos α above the wall point):

$\frac{1}{2}m(20ag) - mg(3a) = \frac{1}{2}mv^2 + mg(8a\cos\alpha)$

$v^2 = 20ag - 6ag - 16ag\cos\alpha = 14ag - 16ag\cos\alpha$

String slack: $T = 0$, so radially $mg\cos\alpha = \frac{mv^2}{8a}$

$mg\cos\alpha = \frac{m(14ag - 16ag\cos\alpha)}{8a} = \frac{7mg}{4} - 2mg\cos\alpha$

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Question 7 continued

$3 \cos \alpha = \frac{7}{4} \Rightarrow \cos \alpha = \frac{7}{12}$ (as required)

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TOTAL FOR PAPER = 75 MARKS

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