Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Thursday 21 May 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. A particle of mass 2 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{I}\,\text{N s}$, such that $\mathbf{I} = a\mathbf{i} + b\mathbf{j}$

Immediately after receiving the impulse, the particle is moving with velocity $\lambda(\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$, where λ is a constant.

Given that the magnitude of $\mathbf{I}$ is $\sqrt{40}$, find the two possible impulses.

(5)

$I = m(v - u)$

$I = 2[(\lambda i + \lambda j) - (5i + 3j)]$

$I = 2(\lambda - 5)i + 2(\lambda - 3)j$

$|I| = \sqrt{40}$: $\quad 4(\lambda - 5)^2 + 4(\lambda - 3)^2 = 40$

$(\lambda - 5)^2 + (\lambda - 3)^2 = 10$

$\lambda^2 - 10\lambda + 25 + \lambda^2 - 6\lambda + 9 = 10$

$2\lambda^2 - 16\lambda + 24 = 0 \quad \Rightarrow \quad \lambda^2 - 8\lambda + 12 = 0$

$(\lambda - 2)(\lambda - 6) = 0 \quad \Rightarrow \quad \lambda = 2 \text{ or } \lambda = 6$

$\lambda = 2$: $\quad I = 2(-3)i + 2(-1)j = -6i - 2j$

$\lambda = 6$: $\quad I = 2(1)i + 2(3)j = 2i + 6j$

$I = (-6i - 2j)\ N\ s \quad \text{or} \quad I = (2i + 6j)\ N\ s$

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Question 1 continued

Q1

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2. A truck of weight 9000 N is travelling up a hill on a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$

When the truck travels up the hill with the engine working at $3P$ watts, the truck is moving at a constant speed of $12\,\text{m s}^{-1}$

Later on, the truck travels down the hill along the same road, with the engine working at P watts. At the instant when the speed of the truck is $12\,\text{m s}^{-1}$, the acceleration of the truck is $\frac{g}{20}$

The resistance to motion of the truck from non-gravitational forces is a constant force of magnitude R newtons in all circumstances.

Find (i) the value of P,

(ii) the value of R.

(9)

Up the hill: driving force $F_1 = \frac{3P}{12} = \frac{P}{4}$, $a = 0$

$F_1 - R - 9000 \sin\theta = 0$

$\frac{3P}{12} - R - 9000 \times \frac{1}{15} = 0$

$\frac{P}{4} - R = 600 \qquad (1)$

Down the hill: driving force $F_2 = \frac{P}{12}$, mass $= \frac{9000}{9.8}$ kg

N2L: $F_2 + 9000 \sin\theta - R = ma$

$\frac{P}{12} + 9000 \times \frac{1}{15} - R = \frac{9000}{9.8} \times \frac{9.8}{20}$

$\frac{P}{12} + 600 - R = 450$

$\frac{P}{12} - R = -150 \qquad (2)$

(1) − (2):

$\frac{P}{4} - \frac{P}{12} = 600 - (-150) = 750$

$\frac{2P}{12} = 750 \Rightarrow P = 4500$

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Question 2 continued

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$R = \frac{4500}{4} - 600 = 1125 - 600 = 525$

(i) $P = 4500$

(ii) $R = 525$

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Question 2 continued

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Question 2 continued

Q2

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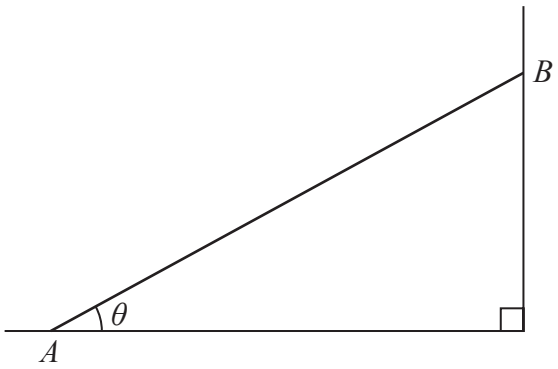


Figure 1

A uniform rod AB, of mass 25 kg and length 3 m, has end A resting on rough horizontal ground. The end B rests against a rough vertical wall.

The rod is in a vertical plane perpendicular to the wall.

The coefficient of friction between the rod and the ground is $\frac{4}{5}$

The coefficient of friction between the rod and the wall is $\frac{3}{5}$

The rod rests in limiting equilibrium.

The rod is at an angle of θ to the ground, as shown in Figure 1.

Find the exact value of $\tan\theta$.

(9)

At A: normal reaction R (up), friction F_A (towards the wall)

At B: normal reaction S (from wall), friction F_B (up the wall)

Weight 25g at the midpoint of AB. Limiting, so $F = \mu R$ at both ends:

$F_A = \frac{4}{5}R$, $F_B = \frac{3}{5}S$

Resolve (→): $S = \frac{4}{5}R$

Resolve (↑): $R + \frac{3}{5}S = 25g$

$\frac{5}{4}S + \frac{3}{5}S = 25g \Rightarrow \frac{37}{20}S = 25g \Rightarrow S = \frac{500g}{37}$

M(A): $25g \times 1.5\cos\theta = S \times 3\sin\theta + \frac{3}{5}S \times 3\cos\theta$

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Question 3 continued

$37.5g \cos\theta - \frac{9}{5} S \cos\theta = 35 \sin\theta$

$\tan\theta = \frac{37.5g - 1.85}{35} = \left(37.5g - 1.8 \times \frac{500g}{37}\right) \div \left(3 \times \frac{500g}{37}\right)$

$\tan\theta = \frac{1387.5 - 900}{1500} = \frac{487.5}{1500} = 0.325$

$\tan\theta = \frac{13}{40}$

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Question 3 continued

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Question 3 continued

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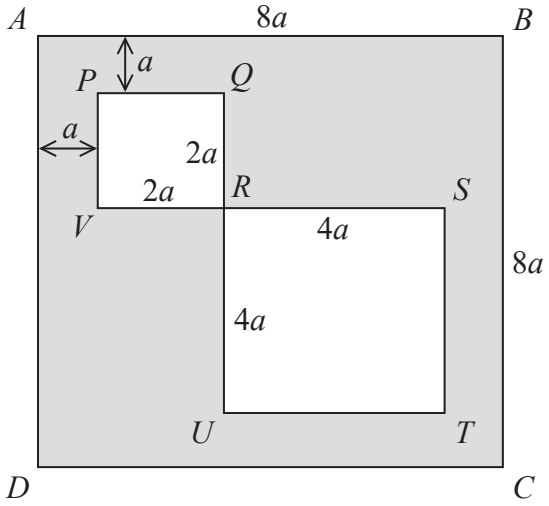


Figure 2

The uniform lamina L, shown shaded in Figure 2, is formed by removing the square $PQRV$, of side $2a$, and the square $RSTU$, of side $4a$, from a uniform square lamina $ABCD$, of side $8a$. The lines QRU and VRS are straight. The side AD is parallel to PV and the side AB is parallel to PQ. The distance between AD and PV is a and the distance between AB and PQ is a. The centre of mass of L is at the point G.

(a) Show that the distance of G from the side AD is $\frac{42}{11}a$

(5)

The mass of L is M. A particle of mass kM is attached to L at C.

The lamina, with the attached particle, is freely suspended from B and hangs in equilibrium with BC making an angle of 45° with the horizontal.

(b) Find the value of k.

(4)

(a) Masses in proportion to areas, distances from AD:

ABCD: area $64a^2$, mass ratio 64, distance $4a$

PQRV: area $4a^2$, mass ratio 4, distance $a + a = 2a$

RSTU: area $16a^2$, mass ratio 16, distance $3a + 2a = 5a$

L: mass ratio $64 - 4 - 16 = 44$, distance d

M(AD): $64 \times 4a - 4 \times 2a - 16 \times 5a = 44d$

$256a - 8a - 80a = 44d \Rightarrow 168a = 44d$

$d = \frac{168}{44}a = \frac{42}{11}a$ as required

(b) L is symmetrical about the diagonal AC, so G lies on AC.

By symmetry G is also $\frac{42}{11}a$ from AB. Let O be the centre of ABCD.

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Question 4 continued

BC at 45° to the horizontal ⇒ diagonal BD is vertical,
so the centre of mass of L + particle lies on BD (below B).

Distance of G from BD: $OG = \sqrt{2} \times \left(4a - \frac{42}{11} a \right) = \frac{2\sqrt{2}}{11} a$

Distance of C from BD: $OC = 4\sqrt{2}\, a$

Moments about B: $M \times \frac{2\sqrt{2}}{11} a = kM \times 4\sqrt{2}\, a$

$k = \frac{2}{11} \div 4 = \frac{1}{22}$

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Question 4 continued

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Question 4 continued

Q4

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5. At time t seconds ($t \geqslant 0$), a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = (3t^2 - 9t + 6)\mathbf{i} + (t^2 + t - 6)\mathbf{j}$$

(a) Find the acceleration of P when $t = 3$

(3)

When $t = 0$, P is at the fixed point O.

The particle comes to instantaneous rest at the point A.

(b) Find the distance OA.

(7)

(a) $a = \frac{dv}{dt} = (6t - 9)i + (2t + 1)j$

$t = 3$: $a = (18 - 9)i + (6 + 1)j$

$a = (9i + 7j)\ m\ s^{-2}$

(b) At rest: $v = 0$, so both components are zero:

$3t^2 - 9t + 6 = 0 \Rightarrow 3(t - 1)(t - 2) = 0 \Rightarrow t = 1$ or 2

$t^2 + t - 6 = 0 \Rightarrow (t - 2)(t + 3) = 0 \Rightarrow t = 2$ or -3

Both zero when $t = 2$

$r = \int v\,dt = \left(t^3 - \frac{9}{2}t^2 + 6t\right)i + \left(\frac{1}{3}t^3 + \frac{1}{2}t^2 - 6t\right)j + c$

$t = 0, r = 0 \Rightarrow c = 0$

$t = 2$: $r = (8 - 18 + 12)i + \left(\frac{8}{3} + 2 - 12\right)j = 2i - \frac{22}{3}j$

$OA = \sqrt{\left(2^2 + \left(\frac{22}{3}\right)^2\right)} = \sqrt{\left(\frac{520}{9}\right)} = \frac{2\sqrt{130}}{3}$

$OA = 7.6011\ldots = 7.60\ m$

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Question 5 continued

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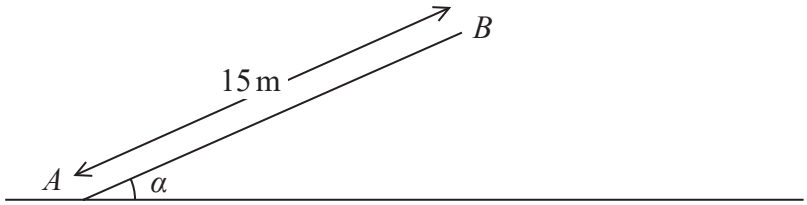


Figure 3

A rough straight ramp is fixed to horizontal ground. The ramp has length 15 m and is inclined at an angle α to the ground, where $\tan\alpha = \frac{5}{12}$. The line AB is a line of greatest slope of the ramp, where A is at the bottom of the ramp, and B is at the top of the ramp, as shown in Figure 3.

A particle P of mass 6 kg is projected up the ramp with speed 14 m s^{-1} from A in a straight line towards B. The coefficient of friction between P and the ramp is 0.25

(a) Find the work done against friction as P moves from A to B.

(3)

At the instant P reaches B, the speed of P is v m s^{-1}. After leaving the ramp at B, the particle P moves freely under gravity until it hits the horizontal ground at the point C. Immediately before hitting the ground at C, the speed of P is w m s^{-1}

(b) Use the work-energy principle to find

(i) the value of v,

(ii) the value of w.

(7)

(a) $\tan\alpha = \frac{5}{12} \Rightarrow \sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$

$R = 6g\cos\alpha = 6g \times \frac{12}{13} = \frac{72g}{13}$

$F = \mu R = 0.25 \times \frac{72g}{13} = \frac{18g}{13} = 13.569\ldots$ N

Work done against friction $= 15 \times \frac{18g}{13} = \frac{270g}{13} = 203.53\ldots$

Work done = 204 J

(b) (i) A to B: initial KE − GPE gained − WD against friction = final KE

$\frac{1}{2} \times 6 \times 14^2 - 6g \times 15 \times \frac{5}{13} - \frac{270g}{13} = \frac{1}{2} \times 6v^2$

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Question 6 continued

$588 - \frac{450g}{13} - \frac{270g}{13} = 3v^2$

$3v^2 = 45.230... \Rightarrow v^2 = 15.076...$

$v = 3.8828... = 3.88$

(ii) A to C: A and C are both on the ground, so no net change in GPE

$\frac{1}{2} \times 6 \times 14^2 - \frac{270g}{13} = \frac{1}{2} \times 6w^2$

$588 - 203.53... = 3w^2$

$w^2 = 128.15...$

$w = 11.320... = 11.3$

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Question 6 continued

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Question 6 continued

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7. Particle A of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal surface. Particle A collides directly with particle B of mass m, which is moving along the same straight line and in the same direction as A.

Immediately before the collision, the speed of B is u.

As a result of the collision, the direction of motion of B is unchanged and the kinetic energy gained by B is $\frac{48}{25}mu^2$

(a) Find the coefficient of restitution between A and B.

(8)

After the collision, B hits a smooth fixed vertical wall that is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is f.

Given that the speed of B immediately after first hitting the wall is equal to the speed of A immediately after its first collision with B,

(b) find the value of f.

(2)

(a) Speeds after collision: A: v, B: w (both in original direction)

KE gained by B:

$\frac{1}{2}mw^2 - \frac{1}{2}mu^2 = \frac{48}{25}mu^2$

$w^2 = u^2 + \frac{96}{25}u^2 = \frac{121}{25}u^2 \Rightarrow w = \frac{11}{5}u$

CLM: $3m \times 2u + m \times u = 3mv + mw$

$7mu = 3mv + \frac{11}{5}mu \Rightarrow 3v = \frac{24}{5}u \Rightarrow v = \frac{8}{5}u$

NEL: $w - v = e(2u - u)$

$\frac{11}{5}u - \frac{8}{5}u = eu$

$\frac{3}{5}u = eu \Rightarrow e = \frac{3}{5}$

(b) NEL at the wall: speed of B after impact = $fw = \frac{11}{5}fu$

This equals the speed of A: $\frac{11}{5}fu = \frac{8}{5}u$

$f = \frac{8}{11}$

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Question 7 continued

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8. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane, with* **i** *being horizontal and* **j** *being vertically upwards.*]

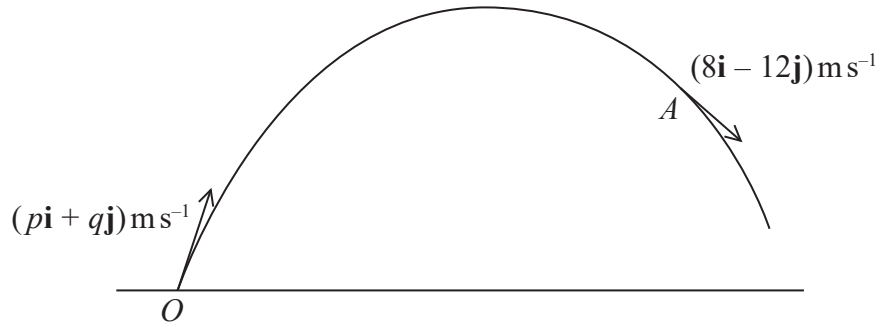


Figure 4

At time $t = 0$, a small ball is projected from a fixed point O on horizontal ground. The ball is projected from O with velocity $(p\mathbf{i} + q\mathbf{j})\,\mathrm{m\,s^{-1}}$, where p and q are positive constants. The ball moves freely under gravity.

At time $t = 3$ seconds, the ball passes through the point A with velocity $(8\mathbf{i} - 12\mathbf{j})\,\mathrm{m\,s^{-1}}$, as shown in Figure 4.

(a) Find the speed of the ball at the instant it is projected from O.

(5)

For an interval of T seconds the speed, $v\,\mathrm{m\,s^{-1}}$, of the ball is such that $v \leqslant 10$

(b) Find the value of T.

(4)

At the point B on the path of the ball, the direction of motion of the ball is perpendicular to the direction of motion of the ball at A.

(c) Find the vertical height of B above A.

(4)

(a) Horizontal velocity is constant: $p = 8$

Vertical (↑): $v = u + at \Rightarrow -12 = q - 9.8 \times 3$

$q = 29.4 - 12 = 17.4$

Speed $= \sqrt{(8^2 + 17.4^2)} = \sqrt{366.76} = 19.151\ldots$

Speed of projection $= 19.2\ \mathrm{m\ s^{-1}}$

(b) $v = 10$ when (vertical component)$^2 = 10^2 - 8^2 = 36$

vertical component $= \pm 6\ \mathrm{m\ s^{-1}}$; $v \leqslant 10$ while it goes from 6 to −6

$v = u + at$: $-6 = 6 - 9.8T$

$T = \frac{12}{9.8} = 1.2244\ldots$

$T = 1.22$

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Question 8 continued

(c) At B the velocity is $8\mathbf{i} + y\mathbf{j}$, perpendicular to $8\mathbf{i} - 12\mathbf{j}$:

$8 \times 8 + y \times (-12) = 0 \quad \Rightarrow \quad y = \frac{64}{12} = \frac{16}{3}$

Vertical motion from B to A (↑ positive, B is h above A):

$v^2 = u^2 + 2as$: $\quad (-12)^2 = \left(\frac{16}{3}\right)^2 + 2(-9.8)(-h)$

$144 = \frac{256}{9} + 19.6h$

$h = \left(144 - \frac{256}{9}\right) \div 19.6 = 5.8957\ldots$

Height of B above A = 5.90 m

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Question 8 continued

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Q8

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