Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 5 June 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference **WMA14/01A**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Find, by algebraic integration,

$$\int_1^3 2x \ln(3x)\,dx$$

giving the answer in the form $A\ln 3 + B$ where A and B are integers.

(5)

$u = \ln(3x) \Rightarrow \frac{du}{dx} = \frac{3}{3x} = \frac{1}{x}$; $\frac{dv}{dx} = 2x \Rightarrow v = x^2$

$\int 2x \ln(3x)\,dx = x^2 \ln(3x) - \int x^2 \times \frac{1}{x}\,dx$

$= x^2 \ln(3x) - \int x\,dx$

$= x^2 \ln(3x) - \frac{x^2}{2}$

$\int_1^3 2x \ln(3x)\,dx = \left[x^2 \ln(3x) - \frac{x^2}{2} \right]_1^3$

$= \left(9 \ln 9 - \frac{9}{2} \right) - \left(\ln 3 - \frac{1}{2} \right)$

$= 18 \ln 3 - \ln 3 - 4$ (since $\ln 9 = 2 \ln 3$)

$= 17 \ln 3 - 4$

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Question 1 continued

(Total for Question 1 is 5 marks)

2:

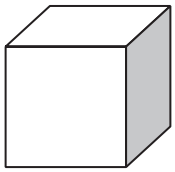


Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the volume of the cube is V cm^3

Given that the volume of the cube is increasing at a constant rate of $5\,\text{cm}^3\text{s}^{-1}$

(a) find $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ giving your answer in terms of x.

(3)

(b) Hence find the value of $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ when $V = 64$

(2)

(a) $V = x^3 \Rightarrow \dfrac{dV}{dx} = 3x^2$

$\dfrac{dx}{dt} = \dfrac{dx}{dV} \times \dfrac{dV}{dt} = \dfrac{1}{3x^2} \times 5$

$\dfrac{dx}{dt} = \dfrac{5}{3x^2}$

(b) $V = 64 \Rightarrow x^3 = 64 \Rightarrow x = 4$

$\dfrac{dx}{dt} = \dfrac{5}{3 \times 4^2} = \dfrac{5}{48}$ cm s^{-1}

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Question 2 continued

(Total for Question 2 is 5 marks)

3: Given that $y = 0$ at $x = 5$ solve the differential equation

$$8y\frac{dy}{dx} = xe^{-y^2} \qquad y \geqslant 0 \quad x \geqslant 5$$

giving your answer in the form $y^2 = f(x)$

(5)

$8y\frac{dy}{dx} = xe^{-y^2} \Rightarrow \int 8y\, e^{y^2}\, dy = \int x\, dx$

$4e^{y^2} = \frac{x^2}{2} + c$

$y = 0, x = 5: \quad 4e^0 = \frac{25}{2} + c \Rightarrow 4 = \frac{25}{2} + c \Rightarrow c = -\frac{17}{2}$

$4e^{y^2} = \frac{x^2}{2} - \frac{17}{2} \Rightarrow e^{y^2} = \frac{x^2 - 17}{8}$

$y^2 = \ln\left(\frac{x^2 - 17}{8}\right)$

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Question 3 continued

(Total for Question 3 is 5 marks)

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4: (a) Find, in ascending powers of x, the first 4 terms of the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}$$

giving each term in simplest form.

(5)

(b) State the range of values of x for which this expansion is valid.

(1)

Using the expansion from part (a),

(c) (i) state the first 4 terms of the binomial series expansion of $(4 - 5x)^{\frac{1}{2}}$

(ii) show that, if x is sufficiently small,

$$(4 + 5x)^{\frac{1}{2}} + (4 - 5x)^{\frac{1}{2}} \approx a + bx^2$$

where a and b are constants to be found.

(3)

(a) $(4 + 5x)^{½} = 4^{½}\left(1 + \frac{5}{4}x\right)^{½} = 2\left(1 + \frac{5}{4}x\right)^{½}$

$\left(1 + \frac{5}{4}x\right)^{½} = 1 + \left(\frac{1}{2}\right)\left(\frac{5}{4}x\right) + \frac{(½)(-½)}{2!}\left(\frac{5}{4}x\right)^2$

$+ \frac{(½)(-½)(-3/2)}{3!}\left(\frac{5}{4}x\right)^3 + \ldots$

$= 1 + \frac{5}{8}x - \frac{25}{128}x^2 + \frac{125}{1024}x^3 + \ldots$

$(4 + 5x)^{½} = 2 + \frac{5}{4}x - \frac{25}{64}x^2 + \frac{125}{512}x^3 + \ldots$

(b) $\left|\frac{5x}{4}\right| < 1 \Rightarrow |x| < \frac{4}{5}$ i.e. $-\frac{4}{5} < x < \frac{4}{5}$

(c)(i) Replace x with $-x$:

$(4 - 5x)^{½} = 2 - \frac{5}{4}x - \frac{25}{64}x^2 - \frac{125}{512}x^3 + \ldots$

(ii) $(4 + 5x)^{½} + (4 - 5x)^{½} \approx \left(2 + \frac{5}{4}x - \frac{25}{64}x^2\right) + \left(2 - \frac{5}{4}x - \frac{25}{64}x^2\right)$

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Question 4 continued

$\approx 4 - \frac{25}{32}x^2$ (x and x^3 terms cancel)

$a = 4, \quad b = -\frac{25}{32}$

Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

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5: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$2x + 4y^2 = 6 + 3e^{xy}$$

The point $P\left(\frac{9}{2}, 0\right)$ lies on C.

Find an equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

$2x + 4y^2 = 6 + 3e^{xy}$

$2 + 8y\frac{dy}{dx} = 3\left(y + x\frac{dy}{dx}\right)e^{xy}$

At $P\left(\frac{9}{2}, 0\right)$: $2 + 0 = 3\left(0 + \frac{9}{2}\frac{dy}{dx}\right)e^0$

$2 = \frac{27}{2}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{4}{27}$

Tangent: $y - 0 = \frac{4}{27}\left(x - \frac{9}{2}\right)$

$54y = 8x - 36 \Rightarrow 27y = 4x - 18$

$4x - 27y - 18 = 0$

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Question 5 continued

(Total for Question 5 is 6 marks)

6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = 8\cos t + 2 \qquad y = 6\sin t - 3 \qquad 0 < t \leqslant 2\pi$$

The point P lies on C where $t = \frac{\pi}{3}$

(a) Use parametric differentiation to find the exact gradient of the **tangent** to C at P.

(3)

The **normal** to C at P intersects the x-axis at the point R.

(b) Find, using algebra, the exact coordinates of R.

(6)

(c) Find a Cartesian equation of C. The equation must not involve trigonometric expressions.

(2)

(a) $\frac{dx}{dt} = -8\sin t$, $\frac{dy}{dt} = 6\cos t$

$\frac{dy}{dx} = \frac{6\cos t}{-8\sin t} = -\frac{3}{4}\cot t$

$t = \frac{\pi}{3}$: $\frac{dy}{dx} = -\frac{3}{4} \times \frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{4}$

(b) $t = \frac{\pi}{3}$: $x = 8 \times \frac{1}{2} + 2 = 6$, $y = 6 \times \frac{\sqrt{3}}{2} - 3 = 3\sqrt{3} - 3$

$P(6, 3\sqrt{3} - 3)$

Gradient of normal $= -1 \div \left(-\frac{\sqrt{3}}{4}\right) = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

Normal: $y - (3\sqrt{3} - 3) = \frac{4}{\sqrt{3}}(x - 6)$

$y = 0$: $-(3\sqrt{3} - 3) = \frac{4}{\sqrt{3}}(x - 6)$

$x - 6 = -\frac{\sqrt{3}(3\sqrt{3} - 3)}{4} = -\frac{9 - 3\sqrt{3}}{4}$

$x = 6 - \frac{9}{4} + \frac{3\sqrt{3}}{4} = \frac{15 + 3\sqrt{3}}{4}$

$R\left(\frac{15 + 3\sqrt{3}}{4}, 0\right)$

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Question 6 continued

(c) $\cos t = \frac{x-2}{8}$, $\sin t = \frac{y+3}{6}$

$\cos^2 t + \sin^2 t = 1$:

$\frac{(x-2)^2}{64} + \frac{(y+3)^2}{36} = 1$



Question 6 continued

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Question 6 continued

(Total for Question 6 is 11 marks)

7: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Show that the substitution $u = e^x - \frac{1}{2}$ transforms

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2}\,dx$$

to

$$\int_a^b \frac{12u + 5}{u^2(2u + 1)}\,du$$

where a and b are constants to be found.

(4)

(b) Find an expression for $\frac{12u + 5}{u^2(2u + 1)}$ in the form

$$\frac{A}{u^2} + \frac{B}{u} + \frac{C}{(2u + 1)}$$

where A, B and C are integers.

(3)

(c) Hence show that

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2}\,dx = p + \ln q$$

where p and q are constants to be found.

(4)

(a) $u = e^x - \frac{1}{2} \Rightarrow \frac{du}{dx} = e^x = u + \frac{1}{2} \Rightarrow dx = \frac{du}{u + \frac{1}{2}} = \frac{2\,du}{2u + 1}$

$e^x = u + \frac{1}{2}$: $24e^x - 2 = 24u + 12 - 2 = 24u + 10$, $2e^x - 1 = 2u$

$\int \frac{24e^x - 2}{(2e^x - 1)^2}\,dx = \int \frac{24u + 10}{(2u)^2} \times \frac{2}{2u + 1}\,du$

$= \int \frac{48u + 20}{4u^2(2u + 1)}\,du = \int \frac{12u + 5}{u^2(2u + 1)}\,du$

$x = \ln 3$: $u = 3 - \frac{1}{2} = \frac{5}{2}$; $x = \ln 8$: $u = 8 - \frac{1}{2} = \frac{15}{2}$

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Question 7 continued

$a = \frac{5}{2}$, $b = \frac{15}{2}$

(b) $12u + 5 \equiv A(2u + 1) + Bu(2u + 1) + Cu^2$

$u = 0$: $5 = A \Rightarrow A = 5$

$u = -\frac{1}{2}$: $-6 + 5 = \frac{1}{4}C \Rightarrow C = -4$

u^2 : $0 = 2B + C \Rightarrow B = 2$

$$\frac{12u + 5}{u^2(2u + 1)} = \frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u + 1)}$$

(c) $\int_{5/2}^{15/2} \left(\frac{5}{u^2} + \frac{2}{u} - \frac{4}{2u + 1} \right) du = \left[-\frac{5}{u} + 2 \ln u - 2 \ln(2u + 1) \right]_{5/2}^{15/2}$

$= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln 16) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln 6)$

$= \left(-\frac{2}{3} + 2 \ln \frac{15}{32} \right) - \left(-2 + 2 \ln \frac{5}{12} \right)$

$= \frac{4}{3} + 2 \ln \left(\frac{15}{32} \times \frac{12}{5} \right) = \frac{4}{3} + 2 \ln \frac{9}{8}$

$= \frac{4}{3} + \ln \left(\frac{81}{64} \right)$ $p = \frac{4}{3}$, $q = \frac{81}{64}$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

8: Relative to a fixed origin O,

- the point A has position vector $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
- the point B has position vector $3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$

Given that the line l passes through A and B,

(a) (i) find $\overrightarrow{AB}$

(ii) find a vector equation for l.

(3)

The point C has position vector $p\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, where p is a constant.

Given that $\overrightarrow{CA}$ is perpendicular to $\overrightarrow{CB}$,

(b) find the possible values of p.

(4)

The points A, B and C form a triangle ABC.

Given also that $p > 0$

(c) find the area of triangle ABC, giving your answer to 3 significant figures.

(4)

(a)(i) $AB = b - a = (3i + 4j - 6k) - (2i - j + 5k)$

$AB = i + 5j - 11k$

(ii) $r = 2i - j + 5k + \lambda(i + 5j - 11k)$

(b) $CA = a - c = (2 - p)i - 5j + 6k$, $CB = b - c = (3 - p)i + 0j - 5k$

$CA \cdot CB = 0$: $(2 - p)(3 - p) + (-5)(0) + (6)(-5) = 0$

$6 - 5p + p^2 - 30 = 0 \Rightarrow p^2 - 5p - 24 = 0$

$(p - 8)(p + 3) = 0 \Rightarrow p = 8$ or $p = -3$

(c) $p = 8$: $CA = -6i - 5j + 6k$, $CB = -5i + 0j - 5k$

$|CA| = \sqrt{(36 + 25 + 36)} = \sqrt{97}$, $|CB| = \sqrt{(25 + 0 + 25)} = \sqrt{50}$

Angle $ACB = 90°$, so

Area $= \frac{1}{2} \times |CA| \times |CB| = \frac{1}{2} \times \sqrt{97} \times \sqrt{50} = 34.8209...$

Area $= 34.8$ (3 s.f.)

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Question 8 continued

Question 8 continued

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Question 8 continued

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9: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

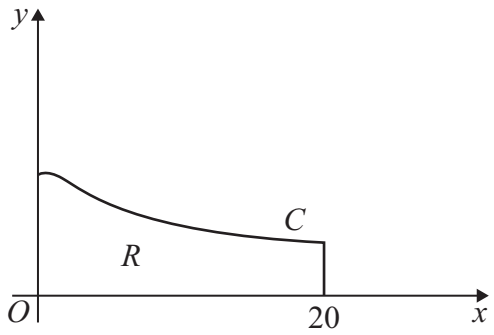


Figure 2

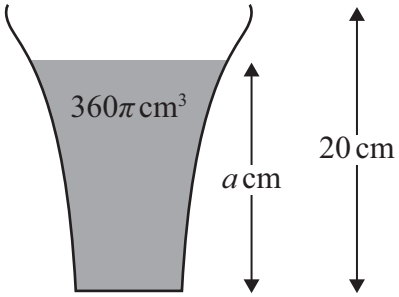


Figure 3

The curve C has equation

$$y = 12\left(\frac{2x+3}{x^2+3x+5}\right)^{\frac{1}{2}}$$

The finite region, R, shown in Figure 2, is bounded by the curve C, the line with equation $x = 20$, the x-axis and the y-axis.

The region R is rotated through 2π radians about the x-axis to form a solid, S.

(a) Use algebraic integration to find the exact volume of S.

Give your answer in the form $k\ln\lambda$ where k and λ are constants.

(4)

Figure 3 shows a hollow vase with identical shape to solid S.

The vase is made of glass, which has negligible thickness.

When the vase is full of water,

volume of water in the vase = volume of S

When the vase contains $360\pi\,\text{cm}^3$ of water, the depth of the water is a cm, as shown in Figure 3.

(b) Use algebra to find the value of a, giving your answer to one decimal place.

(3)

(a) $V = \pi\int_0^{20} y^2\,dx = \pi\int_0^{20} 144\left(\frac{2x+3}{x^2+3x+5}\right)dx$

$= 144\pi\left[\ln(x^2+3x+5)\right]_0^{20}$

$= 144\pi(\ln 465 - \ln 5) = 144\pi\ln\frac{465}{5}$

$V = 144\pi\ln 93 \quad \text{cm}^3$

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Question 9 continued

(b) Vase is S with $x = 20$ at the base, so water fills $20 - a \leqslant x \leqslant 20$

$$144\pi \left[\ln(x^2 + 3x + 5) \right]_{20-a}^{20} = 360\pi \quad \text{(let } p = 20 - a\text{)}$$

$$\ln 465 - \ln(p^2 + 3p + 5) = \frac{360}{144} = \frac{5}{2}$$

$$p^2 + 3p + 5 = 465e^{-5/2} = 38.1695\ldots$$

$$p^2 + 3p - 33.1695\ldots = 0$$

$$p = \frac{-3 + \sqrt{(9 + 4 \times 33.1695\ldots)}}{2} = 4.4514\ldots \quad (p > 0)$$

$$a = 20 - 4.4514\ldots = 15.5485\ldots$$

$$a = 15.5$$

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 7 marks)

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10: [*In this question you may assume that if n is an integer and n^2 is even, then n is even.*]

Given that p and q are integers, use proof by contradiction to show that

$$p^2 - 4q + 2 \neq 0$$

(5)

Assume there exist integers p and q such that $p^2 - 4q + 2 = 0$

Then $p^2 = 4q - 2 = 2(2q - 1)$, so p^2 is even

Hence p is even, so let $p = 2k$ where k is an integer

$(2k)^2 = 4q - 2 \quad \Rightarrow \quad 4k^2 = 4q - 2$

$\Rightarrow \quad 2k^2 = 2q - 1$

$2k^2$ is even but $2q - 1$ is odd, so $2k^2 = 2q - 1$ is not possible

This is a contradiction, so the assumption is false.

Hence for integers p and q, $p^2 - 4q + 2 \neq 0$

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Question 10 continued

Question 10 continued

(Total for Question 10 is 5 marks)

TOTAL FOR PAPER IS 75 MARKS

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