Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 29 May 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01A**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1: The point $P(6, -2)$ lies on the continuous curve with equation $y = f(x), x \in \mathbb{R}$.

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = 2f(3x)$

(2)

(b) $y = f(x - 2) + 8$

(2)

(c) $y = f^{-1}(x)$

(1)

(a) x-coordinate ÷ 3 : 6 ÷ 3 = 2 ; y-coordinate × 2 : 2 × (−2) = −4

P is mapped to (2, −4)

(b) x-coordinate + 2 : 6 + 2 = 8 ; y-coordinate + 8 : −2 + 8 = 6

P is mapped to (8, 6)

(c) Reflection in the line y = x (swap coordinates)

P is mapped to (−2, 6)

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Question 1 continued

(Total for Question 1 is 5 marks)

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2: $f(x) = 4x^3 + 2x^2 - 12$

The equation $f(x) = 0$ has a single root α.

(a) Show that α lies in the interval [1, 2]

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\frac{k}{2x+1}}$$

where k is a constant to be found.

(2)

(c) Using the iteration formula

$$x_{n+1} = \sqrt{\frac{k}{2x_n+1}}$$

with $x_1 = 1$ and the value of k found in part (b), find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α.

(3)

(a) $f(1) = 4 \times 1^3 + 2 \times 1^2 - 12 = -6 < 0$

$f(2) = 4 \times 2^3 + 2 \times 2^2 - 12 = 28 > 0$

Change of sign and f is continuous, so α lies in [1, 2]

(b) $4x^3 + 2x^2 - 12 = 0 \Rightarrow x^2(4x + 2) = 12$

$x^2 = \frac{12}{4x+2} = \frac{6}{2x+1} \Rightarrow x = \sqrt{\frac{6}{2x+1}}$ $\quad k = 6$

(c) $x_1 = 1$, $x_2 = \sqrt{\frac{6}{2 \times 1 + 1}} = \sqrt{2} = 1.41421...$

$x_3 = \sqrt{\frac{6}{2 \times 1.41421... + 1}} = 1.251887...$

(i) $x_3 = 1.2519$

$x_4 = 1.308601...$, $x_5 = 1.287920...$, $x_6 = 1.295348...$, $x_7 = 1.292665...$

$x_8 = 1.293632...$, $x_9 = 1.293283...$, $x_{10} = 1.293409...$, $x_{11} = 1.293364...$

$x_{12} = 1.293380...$, $x_{13} = 1.293374...$

(ii) $\alpha = 1.2934$

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Question 2 continued

(Total for Question 2 is 7 marks)

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3: The curve C has equation

$$x = \frac{4y^2 - 3}{2y + 1} \qquad y \neq -\frac{1}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of y, giving the answer in simplest form.

(4)

A point P lies on C.

Given that

- the gradient of the tangent to C at P is $\frac{1}{3}$
- the point P lies above the x-axis

(b) find the coordinates of P.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(a) $x = \frac{4y^2 - 3}{2y + 1}$ (quotient rule)

$\frac{dx}{dy} = \frac{8y(2y + 1) - 2(4y^2 - 3)}{(2y + 1)^2} = \frac{16y^2 + 8y - 8y^2 + 6}{(2y + 1)^2} = \frac{8y^2 + 8y + 6}{(2y + 1)^2}$

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} \Rightarrow \frac{dy}{dx} = \frac{(2y + 1)^2}{8y^2 + 8y + 6}$

(b) $\frac{(2y + 1)^2}{8y^2 + 8y + 6} = \frac{1}{3} \Rightarrow 3(4y^2 + 4y + 1) = 8y^2 + 8y + 6$

$12y^2 + 12y + 3 = 8y^2 + 8y + 6 \Rightarrow 4y^2 + 4y - 3 = 0$

$(2y - 1)(2y + 3) = 0 \Rightarrow y = \frac{1}{2}$ or $y = -\frac{3}{2}$

P lies above the x-axis so $y = \frac{1}{2}$

$x = \frac{4 \times (1/2)^2 - 3}{2 \times (1/2) + 1} = \frac{1 - 3}{2} = -1$

P $\left(-1, \frac{1}{2}\right)$

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

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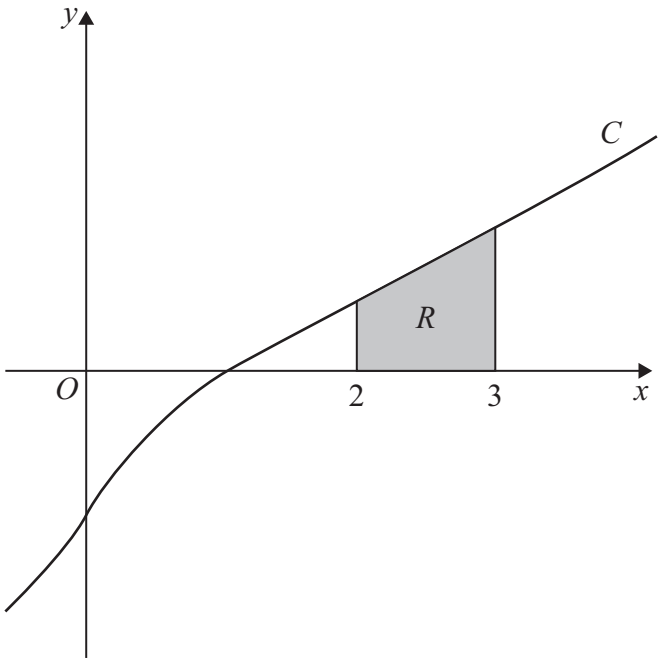


Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1}$$

Given that

$$\frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1} \equiv Ax + B + \frac{Cx + D}{x^2 + 1}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

The finite region R, shown shaded in Figure 1, is bounded by C, the x-axis and the lines with equations $x = 2$ and $x = 3$

(b) Find the exact area of region R using algebraic integration.
Give the answer in the form $\alpha + \ln\beta$ where α and β are constants to be found.

(4)

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Question 4 continued

(a)(i)

$$\begin{array}{rrrrr} & & & 3x & -5 \\ x^2+0x+1 &) & 3x^3 & -5x^2 & +7x & -5 \\ & & 3x^3 & +0x^2 & +3x & \\ \hline & & & -5x^2 & +4x & -5 \\ & & & -5x^2 & +0x & -5 \\ \hline & & & & 4x & +0 \end{array}$$

Quotient $3x-5$, remainder $4x+0$

$$\frac{3x^3-5x^2+7x-5}{x^2+1} \equiv 3x-5+\frac{4x}{x^2+1}$$

$A=3$, $B=-5$, $C=4$

(ii) The remainder is $4x+0$, which has no constant term, so $D=0$

(b) $$\text{Area} = \int_2^3 \left(3x-5+\frac{4x}{x^2+1}\right) dx = \left[\frac{3x^2}{2}-5x+2\ln(x^2+1)\right]_2^3$$

$$= \left(\frac{3\times 3^2}{2}-5\times 3+2\ln(3^2+1)\right)-\left(\frac{3\times 2^2}{2}-5\times 2+2\ln(2^2+1)\right)$$

$$= \left(\frac{27}{2}-15+2\ln 10\right)-\left(6-10+2\ln 5\right) = -\frac{3}{2}+4+2\ln 10-2\ln 5$$

$$= \frac{5}{2}+2\ln\left(\frac{10}{5}\right) = \frac{5}{2}+2\ln 2 = \frac{5}{2}+\ln 4$$

Area $= \frac{5}{2}+\ln 4$ $\quad \alpha = \frac{5}{2}$, $\beta = 4$

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

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5: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 5\cos 2x - 12\sin 2x \qquad -\pi < x < \pi$$

The point A with x coordinate $\frac{\pi}{3}$ lies on C.

(a) Use algebraic differentiation to find the gradient of the tangent to C at A.
Give the answer in simplest form.

(3)

(b) Express $5\cos 2x - 12\sin 2x$ in the form

$$R\cos(2x + \alpha)$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Find the exact value of R and find the value of α to 3 decimal places.

(3)

A point P lies on C.

Given that the gradient of the tangent to C at P is 6

(c) find the greatest possible value for the x coordinate of P.
Give the answer to 2 decimal places.

(4)

(a) $y = 5\cos 2x - 12\sin 2x \Rightarrow \frac{dy}{dx} = -10\sin 2x - 24\cos 2x$

At $x = \frac{\pi}{3}$: $\frac{dy}{dx} = -10\sin\left(\frac{2\pi}{3}\right) - 24\cos\left(\frac{2\pi}{3}\right) = -10 \times \frac{\sqrt{3}}{2} - 24 \times \left(-\frac{1}{2}\right)$

Gradient $= 12 - 5\sqrt{3}$

(b) $R\cos(2x + \alpha) = R\cos 2x \cos\alpha - R\sin 2x \sin\alpha$

$R\cos\alpha = 5, \quad R\sin\alpha = 12$

$R = \sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$

$\tan\alpha = \frac{12}{5} \Rightarrow \alpha = 1.176005\ldots = 1.176$

$5\cos 2x - 12\sin 2x = 13\cos(2x + 1.176)$

(c) $y = 13\cos(2x + 1.176) \Rightarrow \frac{dy}{dx} = -26\sin(2x + 1.176)$

$-26\sin(2x + 1.176) = 6 \Rightarrow \sin(2x + 1.176) = -\frac{3}{13}$

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Question 5 continued

$-\pi < x < \pi \quad \Rightarrow \quad -5.107 < 2x + 1.176 < 7.459$

$2x + 1.176 = -0.23287\ldots , \quad \pi + 0.23287\ldots = 3.37446\ldots , \quad 2\pi - 0.23287\ldots = 6.05032\ldots$

Greatest: $2x + 1.17601 = 6.05032\ldots \quad \Rightarrow \quad x = \dfrac{6.05032\ldots - 1.17601\ldots}{2} = 2.43716\ldots$

Greatest x coordinate = 2.44

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

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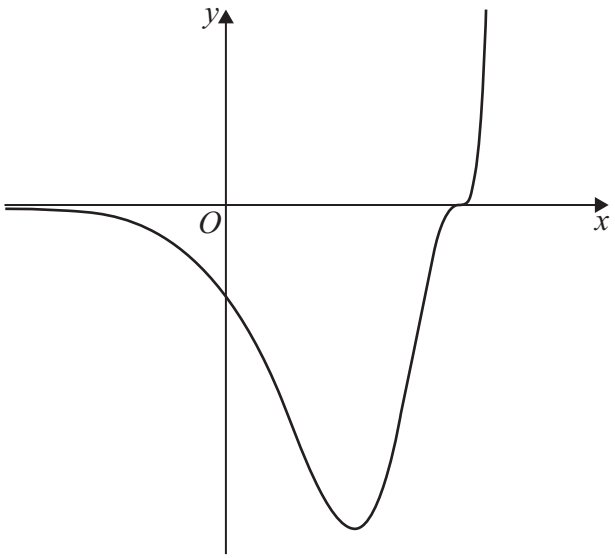


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (2x - 3)^3 e^{4x-2}$$

(a) Show that

$$f'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P, Q and n are constants to be found.

(4)

(b) Hence find

(i) the x coordinates of the stationary points on the curve with equation $y = f(x)$,

(ii) the range of the function g defined by

$$g(x) = 8(2x - 3)^3 e^{4x-2} \qquad x \leqslant \frac{3}{2}$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(a) $f(x) = (2x - 3)^3 e^{4x-2}$ (product rule and chain rule)

$f'(x) = 3(2x - 3)^2 \times 2 \times e^{4x-2} + (2x - 3)^3 \times 4e^{4x-2}$

$= 6(2x - 3)^2 e^{4x-2} + 4(2x - 3)^3 e^{4x-2}$

$= 2(2x - 3)^2 e^{4x-2}[3 + 2(2x - 3)]$

$= 2(4x - 3)(2x - 3)^2 e^{4x-2}$

$P = 4$, $Q = -3$, $n = 2$

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Question 6 continued

(b)(i) $f'(x) = 0$: $e^{4x-2} > 0$, so $4x - 3 = 0$ or $2x - 3 = 0$

$x = \frac{3}{4}$ and $x = \frac{3}{2}$

(ii) $g(x) = 8f(x)$. The minimum of f for $x \leq \frac{3}{2}$ is at $x = \frac{3}{4}$:

$f\left(\frac{3}{4}\right) = \left(\frac{3}{2} - 3\right)^3 e^{3-2} = \left(-\frac{3}{2}\right)^3 e = -\frac{27}{8}e$

$g\left(\frac{3}{4}\right) = 8 \times \left(-\frac{27}{8}e\right) = -27e$

$g\left(\frac{3}{2}\right) = 0$, and $g(x) \to 0$ as $x \to -\infty$

Range: $-27e \leq g(x) \leq 0$

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

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7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Write $\sin 4\theta$ in the form

$$\sin\theta\cos\theta(P + Q\sin^n\theta)$$

where P, Q and n are constants to be found.

(3)

(b) Use the result from part (a) to show that, for $x \neq \frac{k\pi}{2}$ where $k \in \mathbb{Z}$, the equation

$$\sec x \sin 4x = 5\sin^3 x \cot x$$

can be written in the form

$$4\sec^2 x - 5\tan x - 8\tan^2 x = 0$$

(3)

(c) Use the result from part (b) to solve, for $0 < x < \pi, x \neq \frac{\pi}{2}$, the equation

$$\sec x \sin 4x = 5\sin^3 x \cot x$$

giving the answers in radians to 3 significant figures.

(4)

(a) $\sin 4\theta = 2\sin 2\theta \cos 2\theta$

$= 2(2\sin\theta\cos\theta)(1 - 2\sin^2\theta)$

$= 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$

$= \sin\theta\cos\theta(4 - 8\sin^2\theta)$

$P = 4, \quad Q = -8, \quad n = 2$

(b) $\sec x \sin 4x = 5\sin^3 x \cot x$

$\frac{1}{\cos x} \times \sin x \cos x(4 - 8\sin^2 x) = 5\sin^3 x \times \frac{\cos x}{\sin x}$

$\sin x(4 - 8\sin^2 x) = 5\sin^2 x \cos x$

$\div \sin x: \quad 4 - 8\sin^2 x = 5\sin x \cos x$

$\div \cos^2 x: \quad \frac{4}{\cos^2 x} - \frac{8\sin^2 x}{\cos^2 x} = \frac{5\sin x}{\cos x} \Rightarrow 4\sec^2 x - 8\tan^2 x = 5\tan x$

$\therefore \quad 4\sec^2 x - 5\tan x - 8\tan^2 x = 0$

(c) $\sec^2 x = 1 + \tan^2 x: \quad 4(1 + \tan^2 x) - 5\tan x - 8\tan^2 x = 0$

$4\tan^2 x + 5\tan x - 4 = 0$

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Question 7 continued

$\tan x = \frac{-5 \pm \sqrt{(5^2 - 4 \times 4 \times (-4))}}{2 \times 4} = \frac{-5 \pm \sqrt{89}}{8}$

$\tan x = 0.554247...$ or $\tan x = -1.804247...$

$x = \arctan(0.554247...) = 0.506098...$

$x = \pi + \arctan(-1.804247...) = \pi - 1.064698... = 2.076894...$

$x = 0.506$, $x = 2.08$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

8: The functions f and g are defined by

$$f(x) = \ln(2x - 3) \qquad x \in \mathbb{R} \quad x > 2$$

$$g(x) = \frac{3x - 2a}{x - a} \qquad x \in \mathbb{R} \quad x \neq a$$

where a is a positive constant.

(a) Find the exact value of $fg(2a)$

(2)

(b) Find f^{-1}

(3)

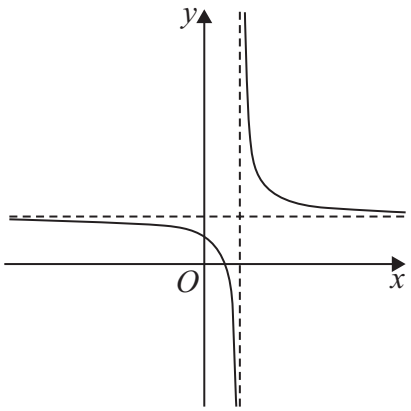


Figure 3

Figure 3 shows a sketch of the curve with equation $y = g(x)$

(c) Sketch a graph of the curve with equation $y = |g(x)|$

Indicate clearly on your sketch the equation of the **horizontal** asymptote.

(3)

The equation $|g(x)| = 10$ has two solutions.

Given that the larger solution is 8

(d) (i) find the value of a,

(ii) find the value of the smaller solution.

(4)

(a) $g(2a) = \frac{3 \times 2a - 2a}{2a - a} = \frac{4a}{a} = 4$

$fg(2a) = f(4) = \ln(2 \times 4 - 3) = \ln 5$

(b) $y = \ln(2x - 3) \Rightarrow e^y = 2x - 3 \Rightarrow x = \frac{e^y + 3}{2}$

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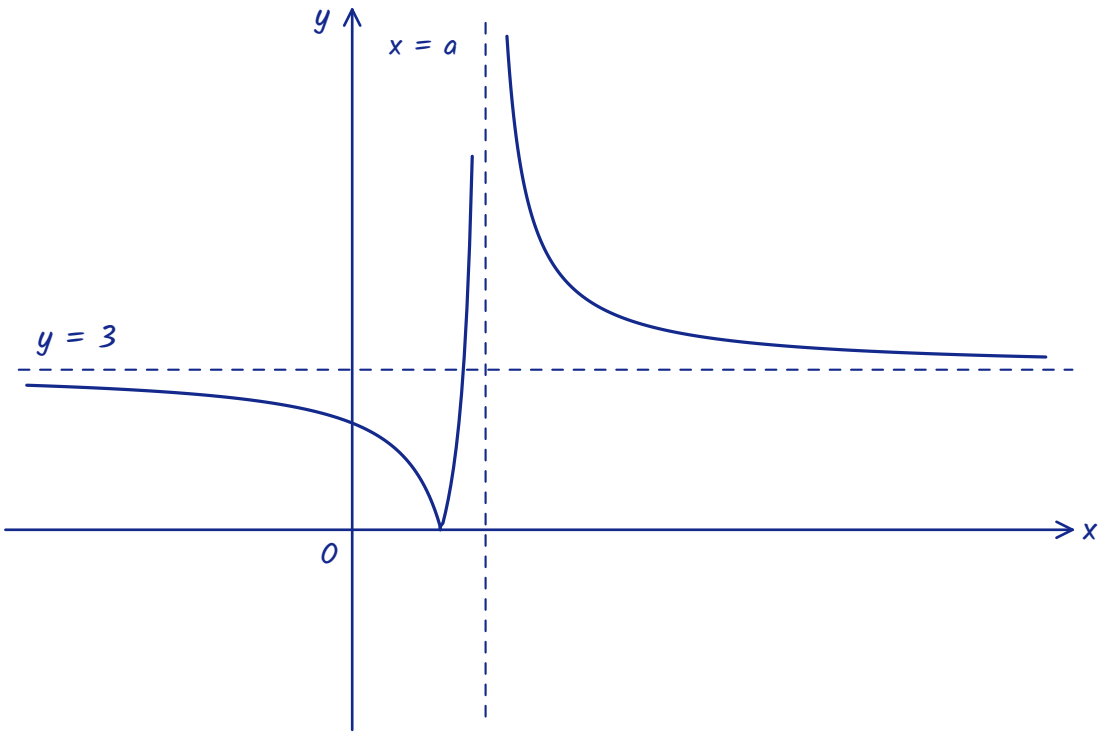
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Question 8 continued

$f^{-1}(x) = \frac{e^x + 3}{2}$, $x > 0$

(c)



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Question 8 continued

(d)(i) Larger solution $x = 8$: $g(8) = 10$

$$\frac{3 \times 8 - 2a}{8 - a} = 10 \Rightarrow 24 - 2a = 80 - 10a \Rightarrow 8a = 56$$

$a = 7$

(ii) Smaller solution: $g(x) = -10$

$$\frac{3x - 2 \times 7}{x - 7} = -10 \Rightarrow 3x - 14 = -10x + 70 \Rightarrow 13x = 84$$

$x = \frac{84}{13}$

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Question 8 continued

(Total for Question 8 is 12 marks)

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9: The amount of an antibiotic, x milligrams, in the bloodstream of a horse, t hours after the antibiotic had been administered, is given by the formula

$$x = D\,e^{-0.2t}$$

where D milligrams is the dose of the antibiotic given to the horse.

A dose of 30 mg of the antibiotic is given to the horse.

(a) Find the amount of the antibiotic in the bloodstream of the horse 8 hours after the dose is given. Give your answer in mg to 2 decimal places.

(2)

A second dose of 20 mg is given to the horse 8 hours after the first dose.

(b) Show that the amount of the antibiotic in the bloodstream of the horse, 2 hours after the second dose is given, is 17.5 mg to one decimal place.

(1)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the amount of the antibiotic in the bloodstream is 10 mg.

(c) Find the value of T, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(a) $x = 30e^{-0.2 \times 8} = 30e^{-1.6} = 6.056895...$

$x = 6.06$ mg

(b) $x = 30e^{-0.2 \times 10} + 20e^{-0.2 \times 2} = 4.060058... + 13.406400...$

$= 17.466459... = 17.5$ mg (1 d.p.)

(c) $30e^{-0.2(T+8)} + 20e^{-0.2T} = 10$

$30e^{-1.6}e^{-0.2T} + 20e^{-0.2T} = 10 \Rightarrow (3e^{-1.6} + 2)e^{-0.2T} = 1$

$e^{-0.2T} = \frac{1}{3e^{-1.6} + 2}$

$-0.2T = -\ln(3e^{-1.6} + 2) \Rightarrow T = 5\ln(3e^{-1.6} + 2) = 4.788486...$

$T = 4.79$

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Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

TOTAL FOR PAPER IS 75 MARKS

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