Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 29 May 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \frac{2x}{3x+1} \qquad x \in \mathbb{R} \quad x \geqslant 0$$

$$g(x) = 4 - x^2 \qquad x \in \mathbb{R} \quad x \geqslant 0$$

(a) Find the value of gf(1)

(2)

(b) Find the range of f

(2)

(c) Find $f^{-1}(x)$

(2)

(d) Solve $f^{-1}(x) = f(x)$

(2)

(a) $f(1) = \frac{2(1)}{3(1)+1} = \frac{1}{2}$

$gf(1) = g\left(\frac{1}{2}\right) = 4 - \left(\frac{1}{2}\right)^2 = \frac{15}{4}$

(b) $f(0) = 0$, as $x \to \infty$, $f(x) \to \frac{2}{3}$

Range: $0 \leq f(x) < \frac{2}{3}$

(c) $y = \frac{2x}{3x+1} \Rightarrow 3xy + y = 2x \Rightarrow 3xy - 2x = -y$

$x(3y - 2) = -y$

$x = \frac{-y}{3y-2} = \frac{y}{2-3y}$

$f^{-1}(x) = \frac{x}{2-3x}, \quad 0 \leq x < \frac{2}{3}$

(d) $\frac{x}{2-3x} = \frac{2x}{3x+1}$

$x(3x+1) = 2x(2-3x)$

$3x^2 + x = 4x - 6x^2$

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Question 1 continued

$9x^2 - 3x = 0 \Rightarrow 3x(3x - 1) = 0$

$x = 0, \quad x = \frac{1}{3}$

Question 1 continued

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Question 1 continued

(Total for Question 1 is 8 marks)

2: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 7\cos x - 24\sin x$$

(a) Express $f(x)$ in the form $R\cos(x + \alpha)$ where R and α are constants, $R > 0$

and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

$$g(x) = \frac{5}{90 - 3f(2x)}$$

(b) Using the answer to part (a), find

(i) the minimum value of $g(x)$, giving your answer as a fully simplified fraction,

(ii) the smallest positive value of x for which this minimum value occurs, giving your answer to 3 decimal places.

(4)

(a) $R\cos(x + \alpha) = R\cos x \cos\alpha - R\sin x \sin\alpha$

$R\cos\alpha = 7, \quad R\sin\alpha = 24$

$R = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$

$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.28700\ldots = 1.287$

$f(x) = 25\cos(x + 1.287)$

(b)(i) $g(x) = \frac{5}{90 - 75\cos(2x + 1.287)}$

g is least when the denominator is least, i.e. $\cos(2x + 1.287) = -1$

minimum $g(x) = \frac{5}{90 - 3 \times 25 \times (-1)} = \frac{5}{165} = \frac{1}{33}$

(ii) $\cos(2x + 1.287) = -1 \Rightarrow 2x + 1.287 = \pi$

$x = \frac{\pi - 1.28700\ldots}{2} = 0.92729\ldots = 0.927$

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Question 2 continued

(Total for Question 2 is 7 marks)

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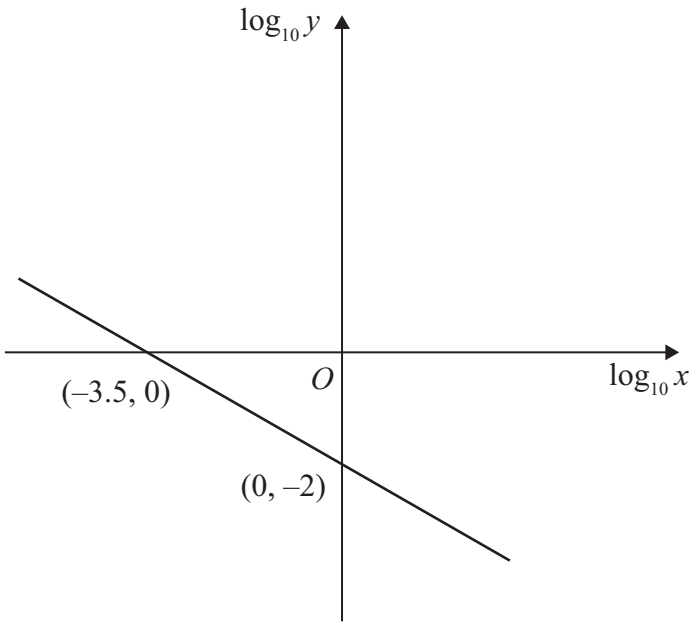


Figure 1

Figure 1 shows a linear relationship between $\log_{10} y$ and $\log_{10} x$

The line passes through the points (–3.5, 0) and (0, –2) as shown.

(a) Find an equation linking $\log_{10} y$ with $\log_{10} x$

(2)

(b) Hence, or otherwise, express y in the form px^q where p and q are rational constants.

(3)

(a) gradient $= \dfrac{-2-0}{0-(-3.5)} = -\dfrac{4}{7}$, intercept $= -2$

$\log_{10} y = -\dfrac{4}{7}\log_{10} x - 2$

(b) $\log_{10} y = \log_{10} x^{-4/7} - 2$

$\log_{10} y - \log_{10} x^{-4/7} = -2 \Rightarrow \log_{10}\left(\dfrac{y}{x^{-4/7}}\right) = -2$

$\dfrac{y}{x^{-4/7}} = 10^{-2}$

$y = \dfrac{1}{100} x^{-4/7}$ $\left(p = \dfrac{1}{100},\ q = -\dfrac{4}{7}\right)$

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Question 3 continued

(Total for Question 3 is 5 marks)

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4: The function f is defined by

$$f(x) = \frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} \qquad x > 3$$

(a) Show that

$$f(x) = \frac{7x}{x - 3} \qquad x > 3$$

(3)

(b) Hence find f′(x) giving your answer in simplest form.

(2)

(a) $x^2 + x - 12 = (x + 4)(x - 3)$

$$f(x) = \frac{49x}{(x + 4)(x - 3)} + \frac{7x}{x + 4} = \frac{49x + 7x(x - 3)}{(x + 4)(x - 3)}$$

$$= \frac{49x + 7x^2 - 21x}{(x + 4)(x - 3)} = \frac{7x^2 + 28x}{(x + 4)(x - 3)}$$

$$= \frac{7x(x + 4)}{(x + 4)(x - 3)} = \frac{7x}{x - 3}$$ as required

(b) $f'(x) = \frac{(x - 3) \times 7 - 7x \times 1}{(x - 3)^2}$

$$f'(x) = -\frac{21}{(x - 3)^2}$$

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Question 4 continued

(Total for Question 4 is 5 marks)

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5: Find

(i) $\int \sin^2 3x \, dx$

(2)

(ii) $\int x(x^2 + 4)^{\frac{3}{2}} dx$

(2)

(i) $\cos 6x = 1 - 2\sin^2 3x \Rightarrow \sin^2 3x = \frac{1}{2} - \frac{1}{2}\cos 6x$

$\int \sin^2 3x \, dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6x \right) dx$

$= \frac{1}{2}x - \frac{1}{12}\sin 6x + c$

(ii) $\frac{d}{dx}(x^2 + 4)^{5/2} = \frac{5}{2}(x^2 + 4)^{3/2} \times 2x = 5x(x^2 + 4)^{3/2}$

$\int x(x^2 + 4)^{3/2} \, dx = \frac{1}{5}(x^2 + 4)^{5/2} + c$

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Question 5 continued

(Total for Question 5 is 4 marks)

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6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ°C, of a computer processor, t minutes after the computer is switched off, is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the processor was 75°C when the computer was switched off,

(a) find the value of A.

(2)

Given also that it takes 5 minutes for the temperature of the processor to decrease from 75°C to 25°C,

(b) find the value of k, giving your answer to 3 significant figures.

(3)

At time T minutes, the temperature of the processor is decreasing at a rate of 9°C per minute.

(c) Find the value of T according to the model, giving your answer to 2 decimal places.

(3)

(a) $t = 0$, $\theta = 75$: $75 = 21 + Ae^0 = 21 + A$

$A = 54$

(b) $t = 5$, $\theta = 25$: $25 = 21 + 54e^{-5k}$

$54e^{-5k} = 4 \Rightarrow e^{-5k} = \frac{4}{54} = \frac{2}{27}$

$-5k = \ln\left(\frac{2}{27}\right) \Rightarrow k = -\frac{1}{5}\ln\left(\frac{2}{27}\right) = 0.52053\ldots$

$k = 0.521$ (3 s.f.)

(c) $\theta = 21 + 54e^{-0.52053\ldots t} \Rightarrow \frac{d\theta}{dt} = -54 \times 0.52053\ldots e^{-0.52053\ldots t}$

$\frac{d\theta}{dt} = -28.109\ldots e^{-0.52053\ldots t}$

$-28.109\ldots e^{-0.52053\ldots T} = -9$

$e^{-0.52053\ldots T} = \frac{9}{28.109\ldots} \Rightarrow -0.52053\ldots T = \ln\left(\frac{9}{28.109\ldots}\right)$

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Question 6 continued

$T = \ln\left(\frac{9}{28.109...}\right) \div (-0.52053...) = 2.18786...$

$T = 2.19$ minutes (2 d.p.)

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

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7: A continuous curve has equation

$$y = e^{-x^2} \sin 3x \qquad 0 \leqslant x \leqslant \frac{\pi}{3}$$

The curve has a stationary point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3}\arctan\left(\frac{3}{2x}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = \frac{1}{3}\arctan\left(\frac{3}{2x_n}\right) \qquad x_1 = 0.4$$

(b) find the value of

(i) x_2

(ii) x_4

giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval and a suitable function which should be stated, show that the x coordinate of P is 0.430 correct to 3 decimal places.

(2)

(a) $y = e^{-x^2}\sin 3x \Rightarrow \frac{dy}{dx} = 3e^{-x^2}\cos 3x - 2xe^{-x^2}\sin 3x$

Stationary point: $\frac{dy}{dx} = 0 \Rightarrow e^{-x^2}(3\cos 3x - 2x\sin 3x) = 0$

$e^{-x^2} \neq 0$, so $3\cos 3x = 2x\sin 3x$

$\tan 3x = \frac{\sin 3x}{\cos 3x} = \frac{3}{2x} \Rightarrow 3x = \arctan\left(\frac{3}{2x}\right)$

$x = \frac{1}{3}\arctan\left(\frac{3}{2x}\right)$ as required

(b)(i) $x_2 = \frac{1}{3}\arctan\left(\frac{3}{2 \times 0.4}\right) = 0.43673\ldots = 0.4367$

$x_3 = 0.42915\ldots,$

(ii) $x_4 = 0.43071\ldots = 0.4307$

(c) Let $f(x) = 3e^{-x^2}\cos 3x - 2xe^{-x^2}\sin 3x$

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Question 7 continued

$f(0.4295) = 3e^{-0.4295^2}\cos(1.2885) - 2(0.4295)e^{-0.4295^2}\sin(1.2885)$

$= 0.00888... > 0$

$f(0.4305) = 3e^{-0.4305^2}\cos(1.2915) - 2(0.4305)e^{-0.4305^2}\sin(1.2915)$

$= -0.000496... < 0$

Sign change and f is continuous on [0.4295, 0.4305],

so the x coordinate of P is 0.430 to 3 d.p.

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

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8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\tan 3x \equiv \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} \qquad x \neq (2n+1)\frac{\pi}{6} \qquad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 2\sec^2 3\theta - 8$$

giving your answers to 2 decimal places.

(5)

(a) $\tan 3x = \tan(2x + x) = \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$, $\quad \tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$$\tan 2x + \tan x = \frac{2\tan x}{1 - \tan^2 x} + \tan x = \frac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x} = \frac{3\tan x - \tan^3 x}{1 - \tan^2 x}$$

$$1 - \tan 2x \tan x = 1 - \frac{2\tan^2 x}{1 - \tan^2 x} = \frac{1 - \tan^2 x - 2\tan^2 x}{1 - \tan^2 x} = \frac{1 - 3\tan^2 x}{1 - \tan^2 x}$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - \tan^2 x} \div \frac{1 - 3\tan^2 x}{1 - \tan^2 x} = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$ as required

(b) Using (a), LHS = $\tan 3\theta$: $\tan 3\theta = 2\sec^2 3\theta - 8 = 2(1 + \tan^2 3\theta) - 8$

$2\tan^2 3\theta - \tan 3\theta - 6 = 0$

$(2\tan 3\theta + 3)(\tan 3\theta - 2) = 0$

$\tan 3\theta = -\frac{3}{2}$ or $\tan 3\theta = 2$

$0 < \theta < \frac{\pi}{2} \Rightarrow 0 < 3\theta < \frac{3\pi}{2}$

$\tan 3\theta = 2$: $3\theta = 1.10714\ldots, 4.24874\ldots \Rightarrow \theta = 0.36904\ldots, 1.41624\ldots$

$\tan 3\theta = -1.5$: $3\theta = \pi - 0.98279\ldots = 2.15879\ldots \Rightarrow \theta = 0.71959\ldots$

$\theta = 0.37, \quad 0.72, \quad 1.42$

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Question 8 continued

Question 8 continued

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Question 8 continued

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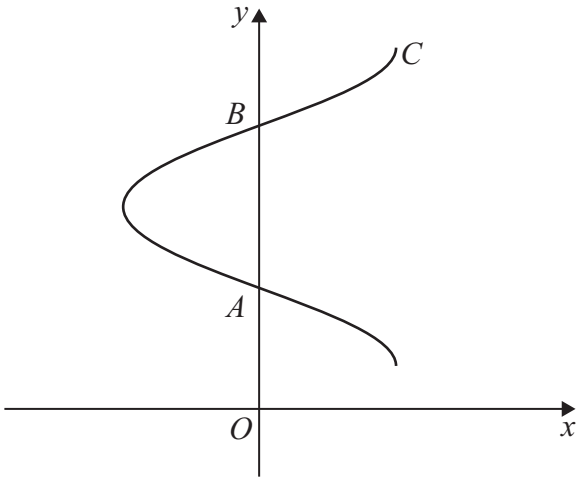


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$x = \frac{2}{3}\sin\left(3y + \frac{\pi}{4}\right) \qquad \frac{\pi}{12} < y < \frac{3\pi}{4}$$

The curve intersects the y-axis at the points A and B as shown.

(a) Find the exact value of the y coordinate of

- point A
- point B

(3)

(b) Show that

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{p - qx^2}$$

where p and q are integers to be found.

(4)

The **normal** to C at A and the **tangent** to C at B intersect at the point D.

Using

- the answer to part (b)
- the sketch of curve C in Figure 2

(c) find, in simplest form, the exact x coordinate of D.

(4)

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Question 9 continued

(a) $x = 0$: $\frac{2}{3}\sin\left(3y + \frac{\pi}{4}\right) = 0 \Rightarrow 3y + \frac{\pi}{4} = \pi$ or 2π

$3y = \frac{3\pi}{4} \Rightarrow y = \frac{\pi}{4}$; $\quad 3y = \frac{7\pi}{4} \Rightarrow y = \frac{7\pi}{12}$

A: $y = \frac{\pi}{4}$ $\quad$ B: $y = \frac{7\pi}{12}$

(b) $\frac{dx}{dy} = \frac{2}{3} \times 3\cos\left(3y + \frac{\pi}{4}\right) = 2\cos\left(3y + \frac{\pi}{4}\right)$

$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4\cos^2(3y + \pi/4)} = \frac{1}{4(1 - \sin^2(3y + \pi/4))}$

$\sin\left(3y + \frac{\pi}{4}\right) = \frac{3x}{2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4(1 - 9x^2/4)} = \frac{1}{4 - 9x^2}$

$p = 4, \quad q = 9$

(c) $x = 0$: $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4} \Rightarrow \frac{dy}{dx} = \pm\frac{1}{2}$

From the sketch, gradient at A is negative: $-\frac{1}{2}$, at B positive: $\frac{1}{2}$

Normal at A (gradient 2): $y - \frac{\pi}{4} = 2x \Rightarrow y = 2x + \frac{\pi}{4}$

Tangent at B: $y - \frac{7\pi}{12} = \frac{1}{2}x \Rightarrow y = \frac{1}{2}x + \frac{7\pi}{12}$

$2x + \frac{\pi}{4} = \frac{1}{2}x + \frac{7\pi}{12} \Rightarrow \frac{3}{2}x = \frac{7\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3}$

$x = \frac{2\pi}{9}$

Question 9 continued

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Question 9 continued

(Total for Question 9 is 11 marks)

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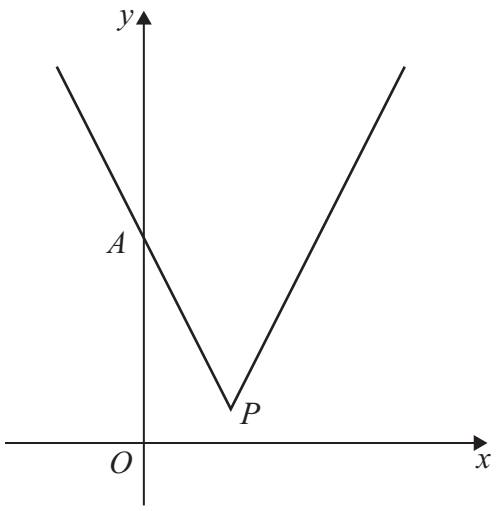


Figure 3

Figure 3 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 10| + k \qquad x \in \mathbb{R}$$

and k is a positive constant.

The graph

- cuts the y-axis at the point A
- has a vertex at the point P

(a) Find, in simplest form in terms of k,

(i) the y coordinate of A

(ii) the coordinates of P

(3)

(b) Find, in terms of k, the range of values of x which satisfy

$$|kx - 10| + k \geqslant 2k$$

(3)

Given that the line with equation $y = 3x + 1$ intersects the graph of $y = f(x)$ at 2 distinct points,

(c) find the range of values of k.

(4)

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Question 10 continued

(a)(i) $x = 0$: $y = |-10| + k = 10 + k$

(ii) vertex where $kx - 10 = 0 \Rightarrow x = \frac{10}{k}$, $y = k$

$P\left(\frac{10}{k}, k\right)$

(b) $|kx - 10| + k \geq 2k \Rightarrow |kx - 10| \geq k$

$kx - 10 \geq k \Rightarrow x \geq \frac{10 + k}{k}$ or $kx - 10 \leq -k \Rightarrow x \leq \frac{10 - k}{k}$

$x \leq \frac{10 - k}{k}$ or $x \geq \frac{10 + k}{k}$

(c) The line has gradient 3. To meet the right-hand branch (gradient k) as well as the left-hand branch we need $k > 3$

Maximum k: the line passes through the vertex P

$k = 3 \times \frac{10}{k} + 1 \Rightarrow k^2 = 30 + k \Rightarrow k^2 - k - 30 = 0$

$(k - 6)(k + 5) = 0 \Rightarrow k = 6$ $(k > 0)$, so the maximum value of k is 6

Vertex must be below the line, so $3 < k < 6$



Question 10 continued

(Total for Question 10 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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