Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 12 June 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either two significant figures or three significant figures.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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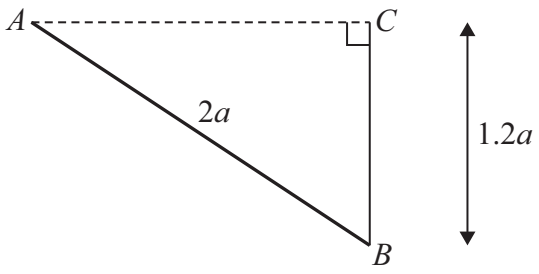


Figure 1

A uniform rod AB has mass m and length $2a$.
The end A is freely hinged to a fixed point.
A light elastic string has modulus of elasticity $2mg$ and natural length L.
One end of the elastic string is attached to the end B of the rod.
The other end of the elastic string is attached to a fixed point C, where AC is horizontal.
The rod rests in equilibrium with the elastic string taut and vertical and $BC = 1.2a$, as shown in Figure 1.

(a) Find, in terms of m and g, the tension in the elastic string.

(3)

(b) Find L in terms of a.

(4)

(a) $AC = \sqrt{((2a)^2 - (1.2a)^2)} = 1.6a$

$M(A)$: $mg \times 0.8a = T \times 1.6a$

$T = \frac{1}{2}mg$

(b) Hooke's law: $T = \frac{2mg(1.2a - L)}{L}$

$\frac{1}{2}mg = \frac{2mg(1.2a - L)}{L}$

$L = 4(1.2a - L) = 4.8a - 4L$

$5L = 4.8a$

$L = 0.96a \quad (= \frac{24}{25}a)$

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Question 1 continued

(Total for Question 1 is 7 marks)

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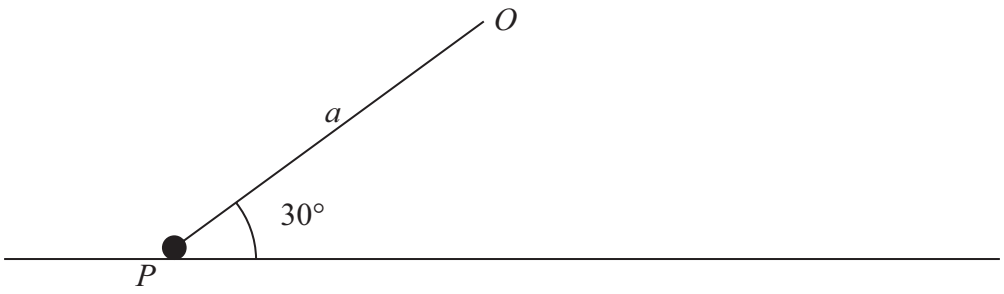


Figure 2

One end of a light inextensible string of length a is attached to a fixed point O which lies above a smooth horizontal table.
The other end is attached to a particle P of mass m.

The particle P moves on the table in a horizontal circle with constant angular speed $\sqrt{\frac{g}{2a}}$ and with the string taut.

The string makes a constant angle of 30° with the table, as shown in Figure 2.

Find, in terms of m and g, the magnitude of the force exerted on P by the table.

(7)

Let R be the normal reaction of the table and T the tension in the string.

Resolving vertically: $R + T \sin 30° = mg$

Radius of circle: $r = a \cos 30°$

N2L horizontally (towards centre): $T \cos 30° = mr\omega^2$

$T \cos 30° = m(a \cos 30°)\frac{g}{2a}$

$T = \frac{mg}{2}$

$R = mg - \frac{mg}{2} \times \frac{1}{2} = mg - \frac{mg}{4}$

Force exerted by the table: $R = \frac{3mg}{4}$

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Question 2 continued

(Total for Question 2 is 7 marks)

3: A particle P moves along the x-axis.

At time t seconds, where $t \geqslant 1$

- the displacement of P from O is x metres in the positive x direction, where $x < \frac{1}{2}$
- the velocity of P is v m s^{-1} in the positive x direction
- the acceleration of P is a m s^{-2} in the positive x direction

Given that

$$a = 4x - 2$$

and that when $t = 1$, $x = 0$ and $v = 1$

(a) find v in terms of x,

(4)

(b) find v in terms of t.

(5)

(a) $v\frac{dv}{dx} = 4x - 2$

$\int v\,dv = \int (4x - 2)\,dx$

$\frac{1}{2}v^2 = 2x^2 - 2x + C$

$x = 0, v = 1$: $\frac{1}{2} = C$

$\frac{1}{2}v^2 = 2x^2 - 2x + \frac{1}{2} \Rightarrow v^2 = 4x^2 - 4x + 1 = (1 - 2x)^2$

$v = 1 - 2x$ (positive root since $v = 1$ when $x = 0$, and $x < \frac{1}{2}$)

(b) $\frac{dx}{dt} = 1 - 2x$

$\int \frac{1}{1 - 2x}\,dx = \int dt$

$-\frac{1}{2}\ln(1 - 2x) = t + C$

$t = 1, x = 0$: $-\frac{1}{2}\ln 1 = 1 + C \Rightarrow C = -1$

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Question 3 continued

$-\frac{1}{2}\ln(1-2x) = t - 1$

Since $v = 1 - 2x$: $-\frac{1}{2}\ln v = t - 1 \Rightarrow \ln v = 2 - 2t$

$v = e^{2-2t}$

(Total for Question 3 is 9 marks)

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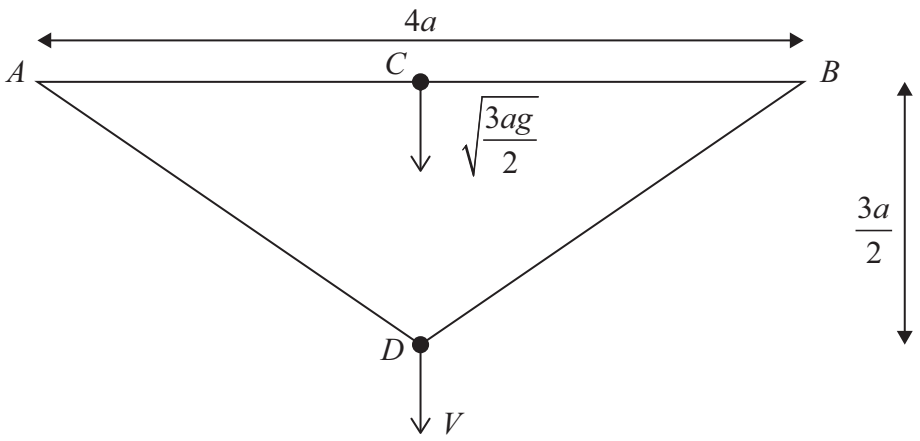


Figure 3

A light elastic string has natural length $3a$ and modulus of elasticity mg.
One end of the string is attached to the point A and the other end is attached to the point B, where AB is horizontal and $AB = 4a$.

A particle P of mass m is attached to the midpoint of the string.
The particle P is held at rest at C, the midpoint of AB.

The particle P is then projected **vertically downwards from C** with speed $\sqrt{\frac{3ag}{2}}$

At the instant when P reaches the point D, where $CD = \frac{3a}{2}$, the speed of P is V, as shown in Figure 3.

(a) Show that the elastic energy stored in the string increases by $\frac{1}{2}mga$, as P moves downwards from C to D.

(3)

Air resistance is modelled as a constant force of magnitude $\frac{1}{5}mg$

Using the model and the work-energy principle,

(b) find V in terms of a and g.

(5)

(a) $AD = BD = \sqrt{((2a)^2 + (\frac{3a}{2})^2)} = \frac{5a}{2} = 2.5a$

Whole string: natural length $3a$, extension at $C = 4a - 3a = a$,
extension at $D = 5a - 3a = 2a$

EPE at $C = \frac{mg(a)^2}{2(3a)} = \frac{mga}{6}$; EPE at $D = \frac{mg(2a)^2}{2(3a)} = \frac{2mga}{3}$

Increase in EPE $= \frac{2mga}{3} - \frac{mga}{6} = \frac{1}{2}mga$ as required

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Question 4 continued

(b) Loss in GPE from C to D = $mg \times \frac{3a}{2}$

Work done against air resistance = $\frac{1}{5} mg \times \frac{3a}{2} = \frac{3mga}{10}$

Work-energy principle:

$$\frac{1}{2}mV^2 = \frac{1}{2}m\left(\frac{3ag}{2}\right) + mg\left(\frac{3a}{2}\right) - \frac{1}{2}mga - \frac{1}{5}mg\left(\frac{3a}{2}\right)$$

$$\frac{1}{2}V^2 = \frac{3ag}{4} + \frac{3ag}{2} - \frac{ag}{2} - \frac{3ag}{10} = \frac{29ag}{20}$$

$$V^2 = \frac{29ag}{10}$$

$$V = \sqrt{\frac{29ag}{10}}$$



Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

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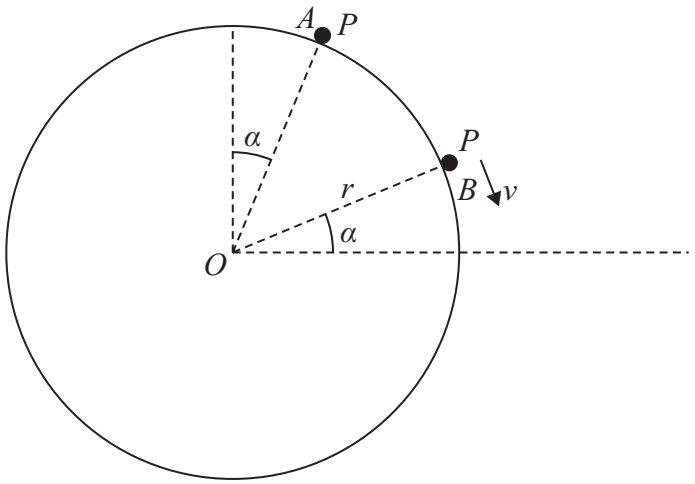


Figure 4

A fixed solid sphere has centre O and radius r.

A particle P of mass m is held at the point A on the smooth outer surface of the sphere, where OA makes an angle α, where $\alpha < 45°$, with the upward vertical.

The particle is released from rest and leaves the surface of the sphere at the point B, where OB makes an angle α with the horizontal, with speed v, as shown in Figure 4.

Air resistance is assumed to be negligible.

(a) Show that $v^2 = 2gr(\cos\alpha - \sin\alpha)$

(3)

(b) Show that $\tan\alpha = \frac{2}{3}$

(4)

At the instant when P crosses the horizontal through O, P is moving at an angle θ to the horizontal.

(c) Show that $\cos\theta = \frac{2}{\sqrt{39}}$

(7)

(a) Energy from A to B (height of A above O = r cos α, of B above O = r sin α):

$\frac{1}{2}mv^2 = mgr\cos\alpha - mgr\sin\alpha$

$v^2 = 2gr(\cos\alpha - \sin\alpha)$ as required

(b) At B, P leaves the surface so R = 0.

N2L towards O: $mg\sin\alpha - R = mv^2/r$ with $R = 0$

$mg\sin\alpha = \frac{mv^2}{r} \Rightarrow v^2 = gr\sin\alpha$

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Question 5 continued

Eliminate v^2: $gr\sin\alpha = 2gr(\cos\alpha - \sin\alpha)$

$\sin\alpha = 2\cos\alpha - 2\sin\alpha \Rightarrow 3\sin\alpha = 2\cos\alpha$

$\tan\alpha = \frac{2}{3}$ as required

(c) $\tan\alpha = \frac{2}{3} \Rightarrow \sin\alpha = \frac{2}{\sqrt{13}}$, $\cos\alpha = \frac{3}{\sqrt{13}}$

$v^2 = gr\sin\alpha = \frac{2gr}{\sqrt{13}}$

After leaving B, P moves freely under gravity, so the horizontal component of velocity is constant. At B the velocity is perpendicular to OB, so

horizontal component $= v\sin\alpha$; $(v\sin\alpha)^2 = \frac{2gr}{\sqrt{13}} \times \frac{4}{13} = \frac{8gr}{13\sqrt{13}}$

Let W be the speed of P at C, where P crosses the horizontal through O.

Energy from A to C (P falls $r\cos\alpha$):

$\frac{1}{2}mW^2 = mgr\cos\alpha \Rightarrow W^2 = 2gr \times \frac{3}{\sqrt{13}} = \frac{6gr}{\sqrt{13}}$

$\cos\theta = \frac{\text{horizontal component}}{\text{speed}} = \frac{v\sin\alpha}{W}$

$\cos^2\theta = \frac{8gr}{13\sqrt{13}} \div \frac{6gr}{\sqrt{13}} = \frac{8}{78} = \frac{4}{39}$

$\cos\theta = \frac{2}{\sqrt{39}}$ as required



Question 5 continued

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Question 5 continued

(Total for Question 5 is 14 marks)

6: (a) Show, using algebraic integration, that the centre of mass of a uniform **solid** hemisphere H of radius a is a distance $\frac{3}{8}a$ from O, the centre of its plane face.

[*You may assume that the volume of the hemisphere is* $\frac{2}{3}\pi a^3$]

(5)

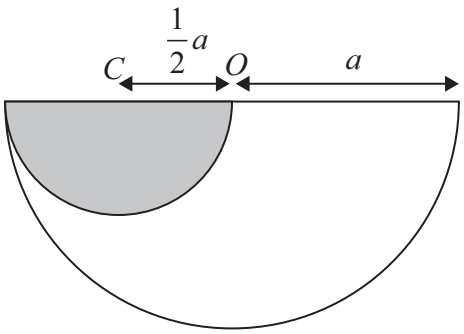


Figure 5

A uniform **solid** S is formed by removing a smaller **solid** hemisphere of radius $\frac{1}{2}a$ from H such that

- the plane face of the smaller hemisphere has centre C and is part of the plane face of H
- $OC = \frac{1}{2}a$

Figure 5 shows a cross section of S, where S is the **unshaded** part.

(b) Show that the centre of mass of S is $\frac{45}{112}a$ from the line through O and C.

(4)

The solid S rests in equilibrium with its curved surface in contact with a rough horizontal plane, as shown in Figure 6.

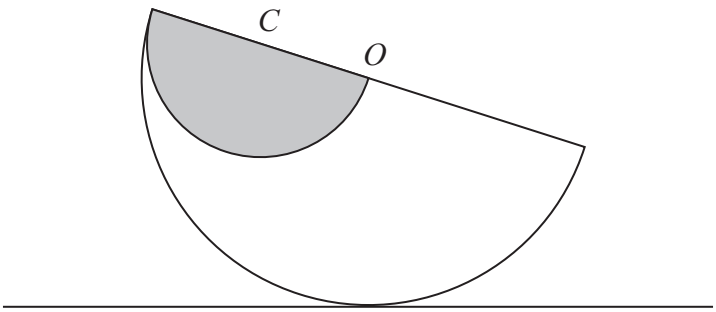


Figure 6

The angle between CO and the horizontal is θ.

(c) Find the exact value of $\tan\theta$.

(5)

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Question 6 continued

(a) Take O as origin, x-axis along the axis of symmetry: $y^2 = a^2 - x^2$

$$\pi\int_0^a xy^2\,dx = \pi\int_0^a x(a^2 - x^2)\,dx = \pi\int_0^a (a^2x - x^3)\,dx$$

$$= \pi\left[\frac{a^2x^2}{2} - \frac{x^4}{4}\right]_0^a = \pi\left(\frac{a^4}{2} - \frac{a^4}{4}\right) = \frac{\pi a^4}{4}$$

$$\bar{x} = \frac{\pi\int xy^2\,dx}{(2/3)\pi a^3} = \frac{\pi a^4/4}{2\pi a^3/3} = \frac{a^4}{4} \times \frac{3}{2a^3}$$

$\bar{x} = \frac{3}{8}a$ from O, as required

(b) Masses are proportional to volumes, i.e. to (radius)^3:

	H	small hemisphere	S
Mass ratio:	8	1	7
Distance from OC:	$\frac{3}{8}a$	$\frac{3}{8} \times \frac{a}{2} = \frac{3}{16}a$	$\bar{y}$

Moments about OC: $8 \times \frac{3a}{8} - 1 \times \frac{3a}{16} = 7\bar{y}$

$$3a - \frac{3a}{16} = \frac{45a}{16} = 7\bar{y}$$

$\bar{y} = \frac{45}{112}a$ from the line through O and C, as required

(c) Moments about the axis through O perpendicular to OC (distance along OC, measured from O away from C):

$$8 \times 0 - 1 \times \left(-\frac{a}{2}\right) = 7\bar{x} \quad \Rightarrow \quad \bar{x} = \frac{a}{14}$$

So G is a/14 from the axis of symmetry, on the opposite side to C.
The curved surface is part of a sphere centre O, so the normal at the point of contact passes through O. In equilibrium G is vertically below O, so OG is vertical and the angle between OG and the axis of symmetry is θ.

$$\tan\theta = \frac{\bar{x}}{\bar{y}} = \frac{a/14}{45a/112} = \frac{112}{14 \times 45}$$

$\tan\theta = \frac{8}{45}$



Question 6 continued

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Question 6 continued

(Total for Question 6 is 14 marks)



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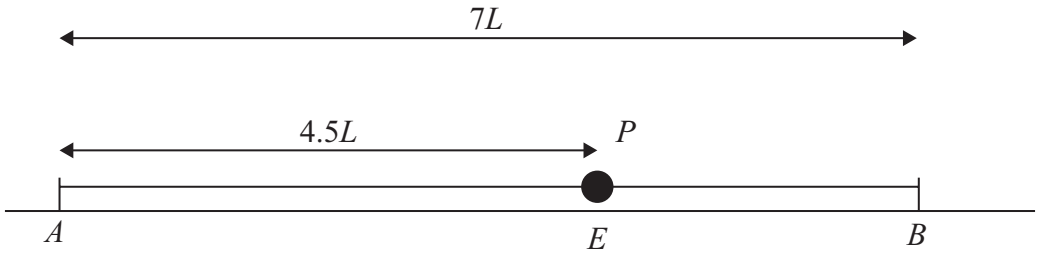


Figure 7

A particle P of mass m lies at rest on a smooth horizontal table.

One end of a light elastic string, of natural length $3L$ and modulus of elasticity mg, is attached to P. The other end is attached to a point A on the table.

One end of a second light elastic string, of natural length $2L$ and modulus of elasticity $2mg$, is also attached to P. The other end is attached to a point B on the table where $AB = 7L$.

The particle P rests in equilibrium on the table at the point E, where AEB is a straight line and $AE = 4.5L$, as shown in Figure 7.

The particle P is now held at the point C on AB, where $AC = 5L$, and released.

(a) Show that P moves with simple harmonic motion with centre E and period $\pi\sqrt{\frac{3L}{g}}$

(7)

(b) Find, in terms of L and g, the maximum speed of P.

(3)

(c) Find, in terms of L and g, the exact amount of time, in any one oscillation, for which the speed of P is less than or equal to $\sqrt{\frac{gL}{12}}$

(6)

(a) At E: AP extension = $4.5L - 3L = 1.5L$, BP extension = $2.5L - 2L = 0.5L$

$T_{AP} = \frac{mg(1.5L)}{3L} = \frac{mg}{2}$, $T_{BP} = \frac{2mg(0.5L)}{2L} = \frac{mg}{2}$ (equal, so E is equilibrium)

Let x be the displacement of P from E towards B, at a general position.

$T_{AP} = \frac{mg(1.5L + x)}{3L}$, $T_{BP} = \frac{2mg(0.5L - x)}{2L}$

N2L (positive towards B): $T_{BP} - T_{AP} = m\ddot{x}$

$\frac{2mg}{2L}(0.5L - x) - \frac{mg}{3L}(1.5L + x) = m\ddot{x}$

$\frac{mg}{2} - \frac{mgx}{L} - \frac{mg}{2} - \frac{mgx}{3L} = m\ddot{x}$

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Question 7 continued

$\ddot{x} = -\frac{4g}{3L}x$, which is of the form $\ddot{x} = -\omega^2 x$

$\therefore$ P moves with simple harmonic motion with centre E.

$\omega^2 = \frac{4g}{3L} \Rightarrow \omega = 2\sqrt{\frac{g}{3L}}$

Period $= \frac{2\pi}{\omega} = 2\pi \times \frac{1}{2}\sqrt{\frac{3L}{g}} = \pi\sqrt{\frac{3L}{g}}$ as required

(Amplitude EC = 0.5L, so the extension of BP, 0.5L − x, is never negative.)

(b) Amplitude = AC − AE = 5L − 4.5L = 0.5L

$v_{max} = a\omega = 0.5L \times \sqrt{\frac{4g}{3L}} = \sqrt{\frac{L^2 \times 4g}{4 \times 3L}}$

$v_{max} = \sqrt{\frac{gL}{3}}$

(c) $v^2 = \omega^2(a^2 - x^2)$: $\frac{gL}{12} = \frac{4g}{3L}\left(\frac{L^2}{4} - x^2\right)$

$\frac{L^2}{16} = \frac{L^2}{4} - x^2 \Rightarrow x^2 = \frac{3L^2}{16} \Rightarrow x = \frac{\sqrt{3}}{4}L$

Speed $\leq \sqrt{(gL/12)}$ when P is between $x = \sqrt{3}L/4$ and an end of the motion.

Using $x = a\cos\omega t$ ($t = 0$ at C): $\frac{\sqrt{3}}{4}L = \frac{L}{2}\cos\omega t$

$\cos\omega t = \frac{\sqrt{3}}{2} \Rightarrow \omega t = \frac{\pi}{6} \Rightarrow t = \frac{\pi}{6\omega} = \frac{\pi}{6} \times \frac{1}{2}\sqrt{\frac{3L}{g}}$

This occurs four times in each oscillation (near each end, on each side):

Total time $= 4 \times \frac{\pi}{12}\sqrt{\frac{3L}{g}} = \frac{\pi}{3}\sqrt{\frac{3L}{g}}$

$= \pi\sqrt{\frac{L}{3g}}$ seconds



Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS

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