Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Q:1/1/

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1. The independent random variables W and X have the following distributions.

$$W \sim \text{Po}(4) \qquad X \sim \text{B}(3, 0.8)$$

(a) Write down the value of the variance of W

(1)

(b) Determine the mode of X
Show your working clearly.

(2)

One observation from each distribution is recorded as W_1 and X_1 respectively.

(c) Find $\text{P}(W_1 = 2 \text{ and } X_1 = 2)$

(3)

(d) Find $\text{P}(X_1 < W_1)$

(4)

(a) $\text{Var}(W) = 4$

(b) $X \sim B(3, 0.8)$

$P(X = 0) = 0.2^3 = 0.008$

$P(X = 1) = 3 \times 0.8 \times 0.2^2 = 0.096$

$P(X = 2) = 3 \times 0.2 \times 0.8^2 = \frac{48}{125} = 0.384$

$P(X = 3) = 0.8^3 = \frac{64}{125} = 0.512$

0.512 is the largest probability, so the mode of X is 3

(c) $P(W_1 = 2) = \frac{e^{-4} \times 4^2}{2!} = 0.14652...$ $\quad P(X_1 = 2) = 0.384$

$P(W_1 = 2 \text{ and } X_1 = 2) = \frac{e^{-4} \times 4^2}{2} \times (3 \times 0.2 \times 0.8^2)$

$= 0.14652... \times 0.384 = 0.0562656... = 0.0563$

(d) $X_1 < W_1$: $X_1 = 0$ and $W_1 > 0$, $\quad X_1 = 1$ and $W_1 > 1$,

$X_1 = 2$ and $W_1 > 2$, $\quad X_1 = 3$ and $W_1 > 3$

$P(X_1 < W_1) = 0.008 \times (1 - 0.0183) + 0.096 \times (1 - 0.0916)$

$+ 0.384 \times (1 - 0.2381) + 0.512 \times (1 - 0.4335)$

$= 0.008 \times 0.9817 + 0.096 \times 0.9084 + 0.384 \times 0.7619 + 0.512 \times 0.5665$

$= 0.0078536 + 0.0872064 + 0.2925696 + 0.290048$

$= 0.677677... = 0.678$

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 10 marks)

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2. The time, in minutes, spent waiting for a call to a call centre to be answered is modelled by the random variable T with probability density function

$$f(t) = \begin{cases} \frac{1}{192}(t^3 - 48t + 128) & 0 \leqslant t \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find, in minutes and seconds, the mean waiting time.

(3)

(b) Show that $P(1 < T < 3) = \frac{7}{16}$

(3)

A supervisor randomly selects 256 calls to the call centre.

(c) Use a suitable approximation to find the probability that more than 125 of these calls take between 1 and 3 minutes to be answered.

(5)

(a) $E(T) = \int_0^4 \frac{1}{192} t(t^3 - 48t + 128)\, dt = \frac{1}{192} \int_0^4 (t^4 - 48t^2 + 128t)\, dt$

$= \frac{1}{192}\left[\frac{t^5}{5} - 16t^3 + 64t^2\right]_0^4 = \frac{1}{192}\left(\frac{4^5}{5} - 16(4^3) + 64(4^2) - 0\right)$

$= \frac{204.8}{192} = \frac{16}{15}$ minutes = 1 minute 4 seconds

(b) $P(1 < T < 3) = \int_1^3 \frac{1}{192}(t^3 - 48t + 128)\, dt = \frac{1}{192}\left[\frac{t^4}{4} - 24t^2 + 128t\right]_1^3$

$= \frac{1}{192}\left[\left(\frac{3^4}{4} - 24(3^2) + 128(3)\right) - \left(\frac{1^4}{4} - 24(1^2) + 128(1)\right)\right]$

$= \frac{1}{192}(188.25 - 104.25) = \frac{84}{192} = \frac{7}{16}$ as required

(c) C = number of calls taking 1 to 3 minutes, $C \sim B\left(256, \frac{7}{16}\right)$

$np = 256 \times \frac{7}{16} = 112, \quad np(1-p) = 112 \times \frac{9}{16} = 63$

Normal approximation: $C \approx Y \sim N(112, 63)$

$P(C > 125) \approx P(Y > 125.5) = P\left(Z > \frac{125.5 - 112}{\sqrt{63}}\right)$

$= P(Z > 1.7008\ldots) = P(Z > 1.70) = 1 - 0.9554$

$= 0.0446$

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 11 marks)

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3. A point is to be randomly plotted on the x-axis, where the units are measured in cm.

The random variable R represents the x coordinate of the point on the x-axis and R is uniformly distributed over the interval $[-5, 19]$

A negative value indicates that the point is to the left of the origin and a positive value indicates that the point is to the right of the origin.

(a) Find the exact probability that the point is plotted to the right of the origin.

(1)

(b) Find the exact probability that the point is plotted more than 3.5 cm away from the origin.

(2)

(c) Sketch the cumulative distribution function of R

(2)

Three independent points with x coordinates R_1, R_2 and R_3 are plotted on the x-axis.

(d) Find the exact probability that

(i) all three points are more than 10 cm from the origin

(3)

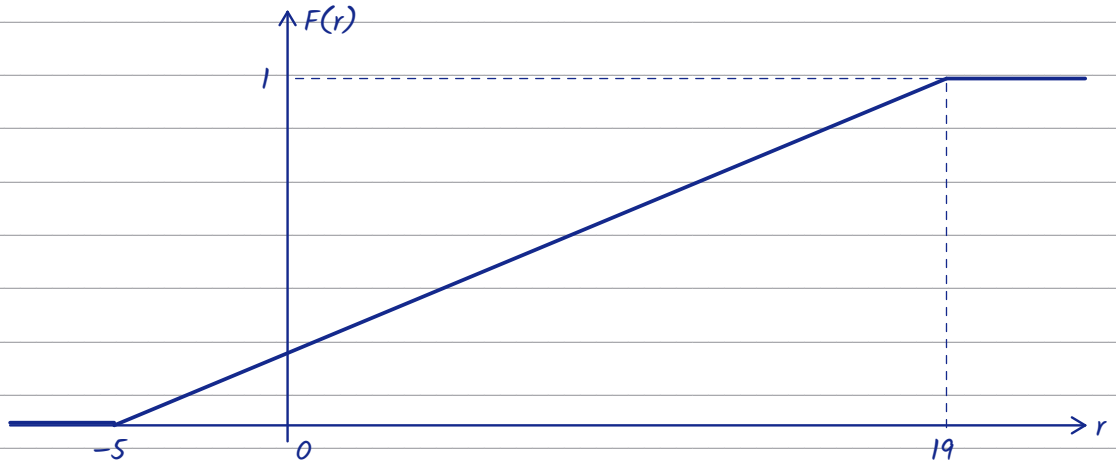
(ii) the point furthest from the origin is more than 10 cm from the origin.

(2)

(a) $P(R > 0) = \frac{19 - 0}{19 - (-5)} = \frac{19}{24}$

(b) $P(|R| > 3.5) = \frac{-3.5 - (-5)}{19 - (-5)} + \frac{19 - 3.5}{19 - (-5)} = \frac{1.5}{24} + \frac{15.5}{24} = \frac{17}{24}$

(c)



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Question 3 continued

(d)(i) $P(R > 10) = \frac{19 - 10}{19 - (-5)} = \frac{9}{24} = 0.375$ ($R < -10$ is impossible)

$P(\text{all three} > 10 \text{ cm}) = \left(\frac{9}{24}\right)^3 = \left(\frac{3}{8}\right)^3 = \frac{27}{512}$

(ii) $P(R < 10) = \frac{15}{24} = \frac{5}{8}$ (point within 10 cm of the origin)

$P(\text{furthest} > 10 \text{ cm}) = 1 - P(\text{all three within 10 cm}) = 1 - \left(\frac{5}{8}\right)^3$

$= 1 - \frac{125}{512} = \frac{387}{512}$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4. Past evidence shows that 7% of pears grown by a farmer are unfit for sale.

This season it is believed that the proportion of pears that are unfit for sale has decreased. To test this belief a random sample of n pears is taken. The random variable Y represents the number of pears in the sample that are unfit for sale.

(a) Find the smallest value of n such that $Y = 0$ lies in the critical region for this test at a 5% level of significance.

(3)

In the past, 8% of the pears grown by the farmer weigh more than 180 g. This season the farmer believes the proportion of pears weighing more than 180 g has changed. She takes a random sample of 75 pears and finds that 11 of them weigh more than 180 g.

(b) Test, using a suitable approximation, whether there is evidence of a change in the proportion of pears weighing more than 180 g.
You should use a 5% level of significance and state your hypotheses clearly.

(6)

(a) H_0: $p = 0.07$ H_1: $p < 0.07$, $Y \sim B(n, 0.07)$

Need $P(Y = 0) < 0.05$: $(1 - 0.07)^n < 0.05$

$0.93^n < 0.05 \Rightarrow n \log(0.93) < \log(0.05)$

$n > \dfrac{\log(0.05)}{\log(0.93)} = 41.28...$ $(0.93^{41} = 0.0510,\ 0.93^{42} = 0.0474)$

Smallest $n = 42$

(b) H_0: $p = 0.08$ H_1: $p \neq 0.08$

X = number of pears in the sample weighing more than 180 g

Under H_0, $X \sim B(75, 0.08)$; n large, p small, so $X \approx Po(75 \times 0.08) = Po(6)$

$P(X \geqslant 11) = 1 - P(X \leqslant 10) = 1 - 0.9574 = 0.0426$

Two-tailed test at 5%: $0.0426 > 0.025$

Not significant, do not reject H_0 (11 is not in the critical region)

There is not significant evidence to suggest that the proportion of pears weighing more than 180 g has changed.

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

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5. The number of particles per millilitre in a solution is modelled by a Poisson distribution with mean 0.15

A randomly selected 50 millilitre sample of the solution is taken.

(a) Find the probability that

(i) exactly 10 particles are found,

(ii) between 6 and 11 particles (inclusive) are found.

(4)

Petra takes 12 independent samples of m millilitres of the solution.

The probability that at least 2 of these samples contain no particles is 0.1184

(b) Using the Statistical Tables provided, find the value of m

(6)

(a) X = number of particles in 50 ml, $X \sim Po(50 \times 0.15) = Po(7.5)$

(i) $P(X = 10) = \dfrac{e^{-7.5} \times 7.5^{10}}{10!} = 0.8622 - 0.7764 = 0.08583... = 0.0858$

(ii) $P(6 \leqslant X \leqslant 11) = P(X \leqslant 11) - P(X \leqslant 5)$

$= 0.9208 - 0.2414 = 0.6794 = 0.679$

(b) S = number of particles in m ml, $S \sim Po(0.15m)$

p = P(sample contains no particles) $= P(S = 0) = e^{-0.15m}$

Y = number of the 12 samples that contain no particles, $Y \sim B(12, p)$

$P(Y \geqslant 2) = 1 - P(Y \leqslant 1) = 0.1184 \Rightarrow P(Y \leqslant 1) = 0.8816$

From the tables, $B(12, 0.05)$ gives $P(Y \leqslant 1) = 0.8816$, so $p = 0.05$

$e^{-0.15m} = 0.05$

$-0.15m = \ln(0.05)$

$m = \dfrac{\ln(0.05)}{-0.15} = 19.9715... = 20.0$

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

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6. The continuous random variable X has probability density function

$$f(x) = \begin{cases} 0.1x & 0 \leqslant x < 2 \\ kx(8-x) & 2 \leqslant x < 4 \\ a & 4 \leqslant x < 6 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are constants.

It is known that $P(X < 4) = \frac{31}{45}$

(a) Find the exact value of k

(4)

(b) (i) Find the exact value of a

(ii) Find the exact value of $P(0 \leqslant X \leqslant 5.5)$

(3)

(c) Specify fully the cumulative distribution function of X

(6)

(a) $\int_0^2 0.1x\,dx + \int_2^4 kx(8-x)\,dx = \frac{31}{45}$

$\left[\frac{0.1x^2}{2}\right]_0^2 + k\left[4x^2 - \frac{x^3}{3}\right]_2^4 = \frac{31}{45}$

$0.2 + k\left(64 - \frac{64}{3} - \left(16 - \frac{8}{3}\right)\right) = \frac{31}{45} \Rightarrow \frac{88}{3}k = \frac{31}{45} - \frac{9}{45} = \frac{22}{45}$

$k = \frac{22}{45} \times \frac{3}{88} = \frac{1}{60}$

(b)(i) $2a = 1 - \frac{31}{45} = \frac{14}{45} \Rightarrow a = \frac{7}{45}$

(ii) $P(0 \leq X \leq 5.5) = \frac{31}{45} + 1.5 \times \frac{7}{45} = \frac{62}{90} + \frac{21}{90} = \frac{83}{90}$

(c) $0 \leq x < 2$: $F(x) = \int_0^x 0.1t\,dt = \left[\frac{0.1t^2}{2}\right]_0^x = 0.05x^2$

$2 \leq x < 4$: $F(x) = 0.2 + \int_2^x \frac{1}{60}t(8-t)\,dt = 0.2 + \frac{1}{60}\left[4t^2 - \frac{t^3}{3}\right]_2^x$

$= 0.2 + \frac{1}{60}\left(4x^2 - \frac{x^3}{3} - 16 + \frac{8}{3}\right) = \frac{1}{60}\left(4x^2 - \frac{x^3}{3} - \frac{4}{3}\right)$

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Question 6 continued

$4 \leq x < 6$: $F(x) = \frac{31}{45} + \int_4^x \frac{7}{45}\,dt = \frac{31}{45} + \frac{7(x-4)}{45} = \frac{7x}{45} + \frac{1}{15}$

Check: $F(6) = \frac{42}{45} + \frac{3}{45} = 1$

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.05x^2 & 0 \leq x < 2 \\ \frac{1}{60}\left(4x^2 - \frac{x^3}{3} - \frac{4}{3}\right) & 2 \leq x < 4 \\ \frac{7x}{45} + \frac{1}{15} & 4 \leq x < 6 \\ 1 & x \geq 6 \end{cases}$$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 13 marks)

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7. A bag contains 10 counters each with exactly one number written on it.

There are 6 counters with the number 7 on them
There are 3 counters with the number 8 on them
There is 1 counter with the number 9 on it

A random sample of 3 counters is taken from the bag (without replacement).
These counters are then put back in the bag.

This process is then repeated until 20 samples have been taken.

The random variable Y represents the number of these 20 samples that contain the counter with the number 9 on it.

(a) (i) Find the mean of Y

(ii) Find the variance of Y

(5)

A random sample of 3 counters is chosen from the bag (without replacement).

(b) List all possible samples where the median of the numbers on the 3 counters is 7

(2)

(c) Find the sampling distribution of the median of the numbers on the 3 counters.

(5)

(a) $p = P(\text{sample contains the counter with 9 on it})$

$$p = 1 - \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8} = 1 - \frac{7}{10} = \frac{3}{10}$$

$$Y \sim B\left(20, \frac{3}{10}\right)$$

(i) $E(Y) = np = 20 \times \frac{3}{10} = 6$

(ii) $\text{Var}(Y) = np(1-p) = 20 \times \frac{3}{10} \times \frac{7}{10} = 4.2$

(b) (7, 7, 7), (7, 7, 8), (7, 7, 9)

(c) There is only one 9, so the median can only be 7 or 8

$$P(M = 7) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} + 3 \times \frac{6}{10} \times \frac{5}{9} \times \frac{3}{8} + 3 \times \frac{6}{10} \times \frac{5}{9} \times \frac{1}{8}$$

$$= \frac{120}{720} + \frac{270}{720} + \frac{90}{720} = \frac{480}{720} = \frac{2}{3}$$

$$P(M = 8) = 1 - P(M = 7) = 1 - \frac{2}{3} = \frac{1}{3}$$

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Question 7 continued

m	7	8
$P(M = m)$	$\frac{2}{3}$	$\frac{1}{3}$

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Question 7 continued

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TOTAL FOR PAPER: 75 MARKS

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