Please check the examination details below before entering your candidate information

Candidate surname	Other names
StemUp Model Solution	

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Q:1/1/1/

P 7 1 3 8 1 A 0 1 3 2

Pearson

1. The binomial expansion of

$$(3 + kx)^{-2} \qquad |kx| < 3$$

where k is a non-zero constant, may be written in the form

$$A + Bx + Cx^2 + Dx^3 + \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that $C = 3B$

(b) show that

$$k^2 + 6k = 0$$

(3)

(c) Hence (i) find the value of k

(ii) find the value of D

(3)

(a) $(3 + kx)^{-2} = 3^{-2}\left(1 + \frac{kx}{3}\right)^{-2} = \frac{1}{9}\left(1 + \frac{kx}{3}\right)^{-2}$

$A = \frac{1}{9}$

(b) $= \frac{1}{9}\left[1 + (-2)\left(\frac{kx}{3}\right) + \frac{(-2)(-3)}{2!}\left(\frac{kx}{3}\right)^2 + \frac{(-2)(-3)(-4)}{3!}\left(\frac{kx}{3}\right)^3 + \ldots\right]$

$B = \frac{1}{9} \times (-2) \times \frac{k}{3} = -\frac{2k}{27}$ $\qquad C = \frac{1}{9} \times 3 \times \frac{k^2}{9} = \frac{k^2}{27}$

$C = 3B \Rightarrow \frac{k^2}{27} = 3 \times \left(-\frac{2k}{27}\right) = -\frac{6k}{27}$

$k^2 = -6k \Rightarrow k^2 + 6k = 0$ as required

(c) (i) $k(k + 6) = 0 \Rightarrow k = 0$ or $k = -6$

k is non-zero, so $k = -6$

(ii) $D = \frac{1}{9} \times \frac{(-2)(-3)(-4)}{3!} \times \left(\frac{k}{3}\right)^3 = \frac{1}{9} \times (-4) \times \left(\frac{-6}{3}\right)^3$

$= \frac{1}{9} \times (-4) \times (-8) = \frac{32}{9}$

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Question 1 continued

(Total for Question 1 is 7 marks)

2. (a) Express $\dfrac{1}{(1+3x)(1-x)}$ in partial fractions.

(3)

(b) Hence find the solution of the differential equation

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leqslant \frac{1}{2}$$

for which $x = \dfrac{1}{2}$ when $y = \dfrac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where n is an integer to be found.

(6)

(a) $\dfrac{1}{(1+3x)(1-x)} = \dfrac{A}{1+3x} + \dfrac{B}{1-x} \Rightarrow 1 = A(1-x) + B(1+3x)$

$x = 1: \quad 1 = 4B \Rightarrow B = \frac{1}{4}$

$x = -\frac{1}{3}: \quad 1 = \frac{4}{3}A \Rightarrow A = \frac{3}{4}$

$\dfrac{1}{(1+3x)(1-x)} = \dfrac{3}{4(1+3x)} + \dfrac{1}{4(1-x)}$

(b) $\dfrac{1}{\tan y}\dfrac{dy}{dx} = \dfrac{1}{(1+3x)(1-x)} \Rightarrow \int \cot y\, dy = \int \dfrac{1}{(1+3x)(1-x)}\, dx$

$\int \dfrac{\cos y}{\sin y}\, dy = \int \left(\dfrac{3}{4(1+3x)} + \dfrac{1}{4(1-x)} \right) dx$

$\ln \sin y = \frac{1}{4}\ln(1+3x) - \frac{1}{4}\ln(1-x) + c$

$x = \frac{1}{2}, y = \frac{\pi}{2}: \quad \ln \sin \frac{\pi}{2} = \frac{1}{4}\ln\frac{5}{2} - \frac{1}{4}\ln\frac{1}{2} + c$

$0 = \frac{1}{4}\ln 5 + c \Rightarrow c = -\frac{1}{4}\ln 5$

$4\ln \sin y = \ln(1+3x) - \ln(1-x) - \ln 5 = \ln\left(\dfrac{1+3x}{5(1-x)}\right)$

$\sin^4 y = \dfrac{1+3x}{5(1-x)} \qquad (n = 4)$

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

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3.

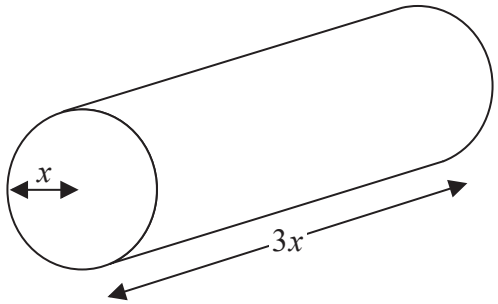


Figure 1

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At t seconds after the tablet is dropped into the water, the radius of the tablet is x mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of $0.5\,\text{mm}^2\,\text{s}^{-1}$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 7$

(4)

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$

(4)

(a) $A = \pi x^2 \Rightarrow \frac{dA}{dx} = 2\pi x$, $\frac{dA}{dt} = -0.5$

$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = \frac{-0.5}{2\pi x} = -\frac{1}{4\pi x}$

$x = 7$: $\frac{dx}{dt} = -\frac{1}{28\pi} = -0.011368... = -0.0114 \text{ mm s}^{-1}$

(b) $V = \pi x^2(3x) = 3\pi x^3$

$\frac{dV}{dx} = 9\pi x^2$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 9\pi x^2 \times \left(-\frac{1}{4\pi x}\right) = -\frac{9x}{4}$

$x = 4$: $\frac{dV}{dt} = -\frac{9 \times 4}{4} = -9$

Rate of decrease of volume = $9 \text{ mm}^3 \text{ s}^{-1}$

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Question 3 continued

(Total for Question 3 is 8 marks)

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4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where k is a constant.

(a) Show that

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

(4)

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of k

(4)

(a) $48x^2 - 9k\left(2xy + x^2\frac{dy}{dx}\right) + 24y^2\frac{dy}{dx} = 0$

$48x^2 - 18kxy - 9kx^2\frac{dy}{dx} + 24y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(24y^2 - 9kx^2) = 18kxy - 48x^2$

$\frac{dy}{dx} = \frac{18kxy - 48x^2}{24y^2 - 9kx^2} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$ as required

(b) Turning point: $\frac{dy}{dx} = 0 \Rightarrow 6kxy - 16x^2 = 0$

$x = \frac{5}{2}$: $6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2 = 0 \Rightarrow 15ky - 100 = 0 \Rightarrow y = \frac{20}{3k}$

$16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2\left(\frac{20}{3k}\right) + 8\left(\frac{20}{3k}\right)^3 = 875$

$250 - 375 + \frac{64000}{27k^3} = 875 \Rightarrow \frac{64000}{27k^3} = 1000$

$k^3 = \frac{64}{27} \Rightarrow k = \frac{4}{3}$

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $x = 2\sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x = \int_0^p \left(\frac{3}{2}\sec u\tan u + \frac{1}{2}\sec^2 u\right)\mathrm{d}u$$

where p is a constant to be found.

(4)

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

(4)

(a) $x = 2\sin u \Rightarrow \frac{dx}{du} = 2\cos u$

$x = 0 \Rightarrow u = 0$; $x = 1 \Rightarrow \sin u = \frac{1}{2} \Rightarrow u = \frac{\pi}{6}$, so $p = \frac{\pi}{6}$

$\int_0^1 \frac{3x+2}{(4-x^2)^{3/2}}\,dx = \int_0^{\pi/6} \frac{6\sin u + 2}{(4-4\sin^2 u)^{3/2}} \times 2\cos u\,du$

$= \int_0^{\pi/6} \frac{(6\sin u + 2)\,2\cos u}{(4\cos^2 u)^{3/2}}\,du = \int_0^{\pi/6} \frac{(6\sin u + 2)\,2\cos u}{8\cos^3 u}\,du$

$= \int_0^{\pi/6} \left(\frac{12\sin u}{8\cos^2 u} + \frac{4}{8\cos^2 u}\right)du$

$= \int_0^{\pi/6} \left(\frac{3}{2}\sec u\tan u + \frac{1}{2}\sec^2 u\right)du$ as required

(b) $= \left[\frac{3}{2}\sec u + \frac{1}{2}\tan u\right]_0^{\pi/6}$

$= \left(\frac{3}{2}\times\frac{2}{\sqrt{3}} + \frac{1}{2}\times\frac{1}{\sqrt{3}}\right) - \left(\frac{3}{2}\times 1 + \frac{1}{2}\times 0\right)$

$= \sqrt{3} + \frac{\sqrt{3}}{6} - \frac{3}{2} = \frac{7\sqrt{3}}{6} - \frac{3}{2} \quad \left(= \frac{7\sqrt{3}-9}{6}\right)$

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

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6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- the point C has position vector $3\mathbf{i} + p\mathbf{j} - \mathbf{k}$

where p is a constant.

The line l passes through A and B.

(a) Find a vector equation for the line l

(3)

Given that $\overrightarrow{AC}$ is perpendicular to l

(b) find the value of p

(3)

(c) Hence find the area of triangle ABC, giving your answer as a surd in simplest form.

(3)

(a) $AB = (5i + 3j - 2k) - (i - 4j + 3k) = 4i + 7j - 5k$

$l: \quad r = i - 4j + 3k + \lambda(4i + 7j - 5k)$

(b) $AC = (3i + pj - k) - (i - 4j + 3k) = 2i + (p + 4)j - 4k$

$AC \cdot AB = 0: \quad 2(4) + 7(p + 4) + (-4)(-5) = 0$

$8 + 7p + 28 + 20 = 0 \Rightarrow 7p = -56 \Rightarrow p = -8$

(c) $AC = 2i - 4j - 4k, \quad |AC| = \sqrt{(2^2 + (-4)^2 + (-4)^2)} = \sqrt{36} = 6$

$|AB| = \sqrt{(4^2 + 7^2 + (-5)^2)} = \sqrt{90} = 3\sqrt{10}$

Angle $BAC = 90°$, so Area $= \frac{1}{2} \times |AB| \times |AC| = \frac{1}{2} \times 3\sqrt{10} \times 6$

Area $= 9\sqrt{10}$

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

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7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \sin t - 3\cos^2 t \qquad y = 3\sin t + 2\cos t \qquad 0 \leqslant t \leqslant 5$$

(a) Show that $\dfrac{dy}{dx} = 3$ where $t = \pi$

(4)

The point P lies on C where $t = \pi$

(b) Find the equation of the tangent to the curve at P in the form $y = mx + c$ where m and c are constants to be found.

(3)

Given that the tangent to the curve at P cuts C at the point Q

(c) show that the value of t at point Q satisfies the equation

$$9\cos^2 t + 2\cos t - 7 = 0$$

(2)

(d) Hence find the exact value of the y coordinate of Q

(3)

(a) $\frac{dx}{dt} = \cos t + 6\cos t \sin t$, $\quad \frac{dy}{dt} = 3\cos t - 2\sin t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\cos t - 2\sin t}{\cos t + 6\cos t \sin t}$

$t = \pi$: $\frac{dy}{dx} = \frac{3\cos\pi - 2\sin\pi}{\cos\pi + 6\cos\pi\sin\pi} = \frac{3(-1) - 0}{-1 + 0} = \frac{-3}{-1} = 3$

(b) $t = \pi$: $x = \sin\pi - 3\cos^2\pi = -3$, $\quad y = 3\sin\pi + 2\cos\pi = -2$

$y - (-2) = 3(x - (-3)) \Rightarrow y + 2 = 3x + 9$

$y = 3x + 7$

(c) $3\sin t + 2\cos t = 3(\sin t - 3\cos^2 t) + 7$

$3\sin t + 2\cos t = 3\sin t - 9\cos^2 t + 7$

$9\cos^2 t + 2\cos t - 7 = 0$ as required

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Question 7 continued

(d) $(9\cos t - 7)(\cos t + 1) = 0 \Rightarrow \cos t = \frac{7}{9}$ ($\cos t = -1$ gives $t = \pi$, point P)

$\cos t = \frac{7}{9} \Rightarrow t = 0.680$ ($t = 2\pi - 0.680 = 5.60 > 5$ so rejected)

$\sin t = \sqrt{\left(1 - \frac{49}{81}\right)} = \frac{\sqrt{32}}{9} = \frac{4\sqrt{2}}{9}$

$y = 3 \times \frac{4\sqrt{2}}{9} + 2 \times \frac{7}{9} = \frac{4\sqrt{2}}{3} + \frac{14}{9}$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

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8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

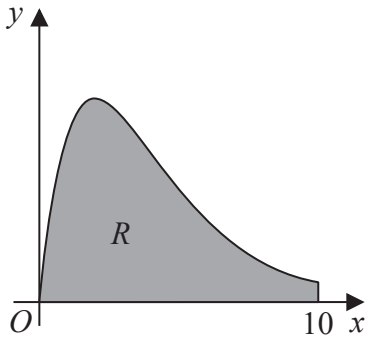


Figure 2

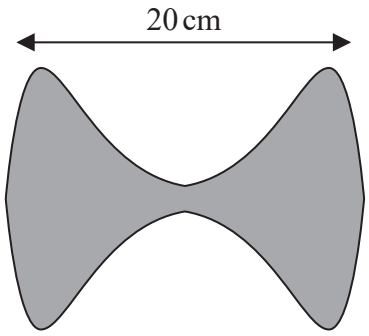


Figure 3

Figure 2 shows the curve with equation

$$y = 10xe^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant 10$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = 10$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of this solid is given by

$$V = k\int_0^{10} x^2e^{-x}\,dx$$

where k is a constant to be found.

(2)

(b) Find $\int x^2e^{-x}\,dx$

(3)

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per cm^3 to 3 significant figures.

(5)

(a) $V = \pi\int_0^{10}\left(10xe^{-\frac{1}{2}x}\right)^2 dx = \pi\int_0^{10} 100x^2e^{-x}\,dx$

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Question 8 continued

$V = 100\pi \int_0^{10} x^2 e^{-x}\, dx, \qquad k = 100\pi$

(b) $\int x^2 e^{-x}\, dx = -x^2 e^{-x} + \int 2xe^{-x}\, dx$

$= -x^2 e^{-x} + 2\left\{ -xe^{-x} + \int e^{-x}\, dx \right\}$

$= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + c$

(c) $\int_0^{10} x^2 e^{-x}\, dx = \left[-x^2 e^{-x} - 2xe^{-x} - 2e^{-x} \right]_0^{10}$

$= (-100e^{-10} - 20e^{-10} - 2e^{-10}) - (0 - 0 - 2)$

$= 2 - 122e^{-10} \quad = 1.99446\ldots$

Total volume $= 2 \times 100\pi \int_0^{10} x^2 e^{-x}\, dx = 200\pi(2 - 122e^{-10})$

$= 1253.156\ldots \text{ cm}^3$

Mass $= 5 \text{ kg} = 5000 \text{ g}$

$\text{Density} = \dfrac{5000}{200\pi(2 - 122e^{-10})} = \dfrac{5000}{1253.156\ldots} = 3.98992\ldots$

Density $= 3.99 \text{ g/cm}^3$ (3 s.f.)

Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

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9. Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(4)

Assume there is an integer n for which $n^2 - 2$ is divisible by 4.

Then $n^2 - 2 = 4k$ for some integer k.

So $n^2 = 4k + 2 = 2(2k + 1)$, which is even.

If n^2 is even then n is even, so $n = 2m$ for some integer m.

Then $n^2 = 4m^2$, so $n^2 - 2 = 4m^2 - 2 = 2(2m^2 - 1)$

$2m^2 - 1$ is odd, so $2(2m^2 - 1)$ is not a multiple of 4.

This contradicts $n^2 - 2 = 4k$, so the assumption is false.

Hence $n^2 - 2$ is never divisible by 4 for any integer n.

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Question 9 continued

Question 9 continued

(Total for Question 9 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS

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