Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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| StemUp Model Solution | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA13/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P3

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**1.** The curve $C$ has equation

$$y = (3x - 2)^6$$

(a) Find $\dfrac{dy}{dx}$

**(2)**

Given that the point $P\left(\dfrac{1}{3}, 1\right)$ lies on $C$,

(b) find the equation of the normal to $C$ at $P$. Write your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers to be found.

**(4)**

(a) $y = (3x - 2)^6 \Rightarrow \dfrac{dy}{dx} = 6 \times 3(3x - 2)^5$

$\dfrac{dy}{dx} = 18(3x - 2)^5$

(b) At P, $x = \dfrac{1}{3}$ : $\dfrac{dy}{dx} = 18(3 \times \dfrac{1}{3} - 2)^5 = 18(-1)^5 = -18$

Gradient of normal $= -\dfrac{1}{-18} = \dfrac{1}{18}$

Normal: $y - 1 = \dfrac{1}{18}\left(x - \dfrac{1}{3}\right)$

$18y - 18 = x - 1/3$

$54y - 54 = 3x - 1$

$3x - 54y + 53 = 0$

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**Question 1 continued**

**Q1**

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**2.** The functions f and g are defined by

$$f(x) = \frac{5 - x}{3x + 2} \qquad x \in \mathbb{R}, x \neq -\frac{2}{3}$$

$$g(x) = 2x - 7 \qquad x \in \mathbb{R}$$

(a) Find the value of fg(5)

**(2)**

(b) Find $f^{-1}$

**(3)**

(c) Solve the equation

$$f\left(\frac{1}{a}\right) = g(a + 3)$$

**(4)**

(a) $g(5) = 2 \times 5 - 7 = 3$

$fg(5) = f(3) = \dfrac{5 - 3}{3 \times 3 + 2} = \dfrac{2}{11}$

(b) $y = \dfrac{5 - x}{3x + 2} \Rightarrow 3xy + 2y = 5 - x$

$3xy + x = 5 - 2y \Rightarrow x(3y + 1) = 5 - 2y$

$x = \dfrac{5 - 2y}{3y + 1}$

$f^{-1}(x) = \dfrac{5 - 2x}{3x + 1}, \quad x \in \mathbb{R}, \ x \neq -\dfrac{1}{3}$

(c) $f\left(\dfrac{1}{a}\right) = \dfrac{5 - 1/a}{3/a + 2} = \dfrac{5a - 1}{3 + 2a}$ (× a top and bottom)

$g(a + 3) = 2(a + 3) - 7 = 2a - 1$

$\dfrac{5a - 1}{2a + 3} = 2a - 1$

$5a - 1 = (2a - 1)(2a + 3) = 4a^2 + 4a - 3$

$4a^2 - a - 2 = 0$

$a = \dfrac{1 \pm \sqrt{(-1)^2 - 4(4)(-2)}}{2(4)} = \dfrac{1 \pm \sqrt{33}}{8}$

$a = \dfrac{1 + \sqrt{33}}{8}$ or $a = \dfrac{1 - \sqrt{33}}{8}$

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**Question 2 continued**

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**Question 2 continued**

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**Question 2 continued**

**Q2**

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**3.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Given that $k$ is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k} \, dx$$

**(2)**

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k} \, dx = \ln 8$$

(b) find the value of $k$

**(4)**

(a) $\int \frac{9x}{3x^2 + k} \, dx = \frac{3}{2} \int \frac{6x}{3x^2 + k} \, dx$

$= \frac{3}{2} \ln(3x^2 + k) + c$

(b) $\int_2^5 \frac{9x}{3x^2 + k} \, dx = \left[ \frac{3}{2} \ln(3x^2 + k) \right]_2^5$

$= \frac{3}{2} \ln(75 + k) - \frac{3}{2} \ln(12 + k) = \ln 8$

$\frac{3}{2} \ln\left( \frac{75 + k}{12 + k} \right) = \ln 8$

$\ln\left( \frac{75 + k}{12 + k} \right) = \frac{2}{3} \ln 8 = \ln 8^{2/3} = \ln 4$

$\frac{75 + k}{12 + k} = 4 \Rightarrow 75 + k = 48 + 4k$

$3k = 27$

$k = 9$

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**Question 3 continued**

**Q3**

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**4.**

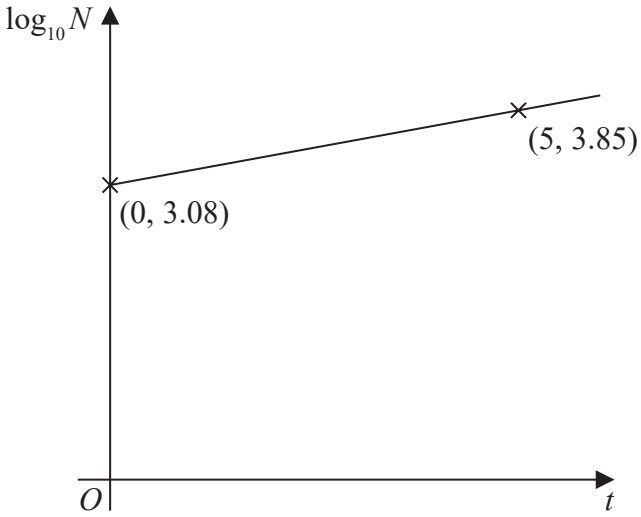



**Figure 1**

The number of subscribers to an online video streaming service, $N$, is modelled by the equation

$$N = ab^t$$

where $a$ and $b$ are constants and $t$ is the number of years since monitoring began.

The line in Figure 1 shows the linear relationship between $t$ and $\log_{10} N$

The line passes through the points (0, 3.08) and (5, 3.85)

Using this information,

(a) find an equation for this line.

**(2)**

(b) Find the value of $a$ and the value of $b$, giving your answers to 3 significant figures.

**(3)**

When $t = T$ the number of subscribers is 500 000

According to the model,

(c) find the value of $T$

**(2)**

(a) gradient $= \frac{3.85 - 3.08}{5 - 0} = \frac{0.77}{5} = 0.154$

$\log_{10} N = 3.08 + 0.154t$

(b) $N = ab^t \Rightarrow \log_{10} N = \log_{10} a + t \log_{10} b$

$\log_{10} a = 3.08 \Rightarrow a = 10^{3.08} = 1202.26... = 1200$ (3 s.f.)

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**Question 4 continued**

$\log_{10} b = 0.154 \Rightarrow b = 10^{0.154} = 1.42560... = 1.43$ (3 s.f.)

(c) $\log_{10} 500\,000 = 3.08 + 0.154T$

$$T = \frac{\log_{10} 500\,000 - 3.08}{0.154} = \frac{5.69897... - 3.08}{0.154}$$

$T = 17.006... \approx 17$

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**Question 4 continued**

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**Question 4 continued**

**Q4**

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**5.**

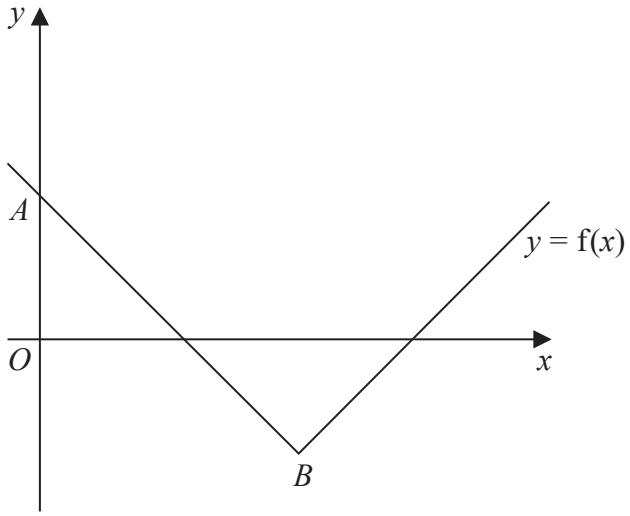



**Figure 2**

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 9| - 2 \qquad x \in \mathbb{R}$$

and $k$ is a positive constant.

The graph intersects the $y$-axis at the point $A$ and has a minimum point at $B$ as shown.

(a) (i) Find the $y$ coordinate of $A$

(ii) Find, in terms of $k$, the $x$ coordinate of $B$

**(2)**

(b) Find, in terms of $k$, the range of values of $x$ that satisfy the inequality

$$|kx - 9| - 2 < 0$$

**(3)**

Given that the line $y = 3 - 2x$ intersects the graph $y = f(x)$ at two distinct points,

(c) find the range of possible values of $k$

**(3)**

(a)(i) $x = 0$: $y = |-9| - 2 = 7$, $y$ coordinate of $A = 7$

(ii) minimum when $kx - 9 = 0$, $x$ coordinate of $B = \frac{9}{k}$

(b) $|kx - 9| - 2 < 0 \Rightarrow |kx - 9| < 2$

$-2 < kx - 9 < 2$

$7 < kx < 11$

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**Question 5 continued**

$\frac{7}{k} < x < \frac{11}{k} \quad (k > 0)$

(c) $B = \left(\frac{9}{k}, -2\right)$. Two intersections $\Rightarrow$ B lies below $y = 3 - 2x$

$-2 < 3 - 2 \times \frac{9}{k}$

Boundary: $-2 = 3 - \frac{18}{k} \Rightarrow \frac{18}{k} = 5 \Rightarrow k = 3.6$

$\frac{18}{k} < 5 \Rightarrow 18 < 5k \quad (k > 0)$

$k > 3.6$

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**Question 5 continued**

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**Question 5 continued**

**Q5**

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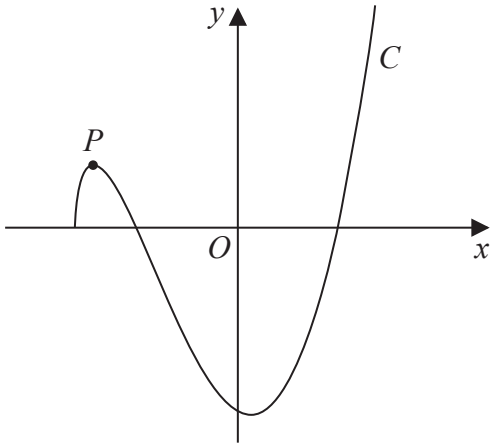



**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \qquad x \geqslant -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where $k$ is an integer to be found.

**(4)**

(b) Hence, find the values of $x$ for which $f'(x) = 0$

**(1)**

Figure 3 shows a sketch of the curve $C$ with equation $y = f(x)$.

The curve has a local maximum at the point $P$

(c) Find the exact coordinates of $P$

**(2)**

The function g is defined by

$$g(x) = 2f(x) + 4 \qquad -\frac{9}{4} \leqslant x \leqslant 0$$

(d) Find the range of g

**(3)**

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**Question 6 continued**

(a) $f'(x) = 5(x^2 - 2) \times \frac{1}{2} \times 4(4x + 9)^{-1/2} + 10x(4x + 9)^{1/2}$

$= 10(x^2 - 2)(4x + 9)^{-1/2} + 10x(4x + 9)^{1/2}$

$= \frac{10(x^2 - 2) + 10x(4x + 9)}{(4x + 9)^{1/2}}$

$= \frac{10x^2 - 20 + 40x^2 + 90x}{(4x + 9)^{1/2}} = \frac{50x^2 + 90x - 20}{(4x + 9)^{1/2}}$

$f'(x) = \frac{10(5x^2 + 9x - 2)}{(4x + 9)^{1/2}}$ $\quad k = 10$

(b) $f'(x) = 0 \Rightarrow 5x^2 + 9x - 2 = 0 \Rightarrow (5x - 1)(x + 2) = 0$

$x = -2$ or $x = \frac{1}{5}$

(c) $P$ is at $x = -2$: $y = 5((-2)^2 - 2)(4(-2) + 9)^{1/2}$

$y = 5 \times 2 \times 1^{1/2} = 10$

$P = (-2, 10)$

(d) On $-\frac{9}{4} \leqslant x \leqslant 0$: max $f = f(-2) = 10$

$f(0) = 5(0 - 2)(9)^{1/2} = -30$, $f\left(-\frac{9}{4}\right) = 0$

so $-30 \leqslant f(x) \leqslant 10$

$g$ max $= 2 \times 10 + 4 = 24$, $g$ min $= 2 \times (-30) + 4 = -56$

$-56 \leqslant g(x) \leqslant 24$

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**Question 6 continued**

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**Question 6 continued**

**Q6**

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**7.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that the equation

$$2\sin\theta(3\cot^2 2\theta - 7) = 13\sec\theta$$

can be written as

$$3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$$

**(4)**

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$, the equation

$$2\sin\theta(3\cot^2 2\theta - 7) = 13\sec\theta$$

giving your answers to 3 significant figures.

**(4)**

(a) $2\sin\theta(3\cot^2 2\theta - 7) = 13\sec\theta$

$\div 2\sin\theta$: $3\cot^2 2\theta - 7 = \dfrac{13}{2\sin\theta\cos\theta}$

using $\cot^2 2\theta = \operatorname{cosec}^2 2\theta - 1$: $3(\operatorname{cosec}^2 2\theta - 1) - 7 = \dfrac{13}{2\sin\theta\cos\theta}$

$2\sin\theta\cos\theta = \sin 2\theta$: $3\operatorname{cosec}^2 2\theta - 10 = \dfrac{13}{\sin 2\theta} = 13\operatorname{cosec} 2\theta$

$3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$ as required

(b) $(3\operatorname{cosec} 2\theta + 2)(\operatorname{cosec} 2\theta - 5) = 0$

$\operatorname{cosec} 2\theta = 5$ or $\operatorname{cosec} 2\theta = -\frac{2}{3}$ (no solution, $|\operatorname{cosec} 2\theta| \geq 1$)

$\sin 2\theta = \frac{1}{5}$, $0 < 2\theta < \pi$

$2\theta = \arcsin(0.2) = 0.20135\ldots$ or $2\theta = \pi - 0.20135\ldots = 2.94023\ldots$

$\theta = 0.10067\ldots$ or $\theta = 1.47011\ldots$

$\theta = 0.101$, $\theta = 1.47$

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**Question 7 continued**

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**Question 7 continued**

**Q7**

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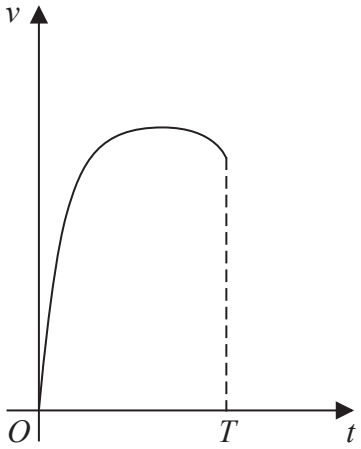



**Figure 4**

Figure 4 is a graph showing the velocity of a sprinter during a 100 m race.

The sprinter's velocity during the race, $v\,\text{m}\,\text{s}^{-1}$, is modelled by the equation

$$v = 12 - \text{e}^{t-10} - 12\text{e}^{-0.75t} \qquad t \geqslant 0$$

where $t$ seconds is the time after the sprinter begins to run.

According to the model,

(a) find, using calculus, the sprinter's maximum velocity during the race.

**(5)**

Given that the sprinter runs 100 m in $T$ seconds, such that

$$\int_0^T v\,\text{d}t = 100$$

(b) show that $T$ is a solution of the equation

$$T = \frac{1}{12}\left(116 - 16\text{e}^{-0.75T} + \text{e}^{T-10} - \text{e}^{-10}\right)$$

**(4)**

The iteration formula

$$T_{n+1} = \frac{1}{12}\left(116 - 16\text{e}^{-0.75T_n} + \text{e}^{T_n-10} - \text{e}^{-10}\right)$$

is used to find an approximate value for $T$

Using this iteration formula with $T_1 = 10$

(c) find, to 4 decimal places,

(i) the value of $T_2$

(ii) the time taken by the sprinter to run the race, according to the model.

**(3)**

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**Question 8 continued**

(a) $v = 12 - e^{t-10} - 12e^{-0.75t} \Rightarrow \frac{dv}{dt} = -e^{t-10} + 9e^{-0.75t}$

$\frac{dv}{dt} = 0 \Rightarrow 9e^{-0.75t} = e^{t-10} \Rightarrow 9 = e^{1.75t-10}$

$\ln 9 = 1.75t - 10 \Rightarrow t = \frac{10 + \ln 9}{1.75} = 6.9698...$

$v = 12 - e^{6.9698... - 10} - 12e^{-0.75 \times 6.9698...}$

$v = 12 - 0.04830... - 0.06441... = 11.887...$

Maximum velocity = 11.9 m $s^{-1}$

(b) $\int (12 - e^{t-10} - 12e^{-0.75t})\, dt = 12t - e^{t-10} + \frac{12}{0.75} e^{-0.75t}$

$= 12t - e^{t-10} + 16e^{-0.75t}$

$\int_0^T v\, dt = \left[12t - e^{t-10} + 16e^{-0.75t}\right]_0^T = 100$

$(12T - e^{T-10} + 16e^{-0.75T}) - (0 - e^{-10} + 16) = 100$

$12T = 116 - 16e^{-0.75T} + e^{T-10} - e^{-10}$

$T = \frac{1}{12}(116 - 16e^{-0.75T} + e^{T-10} - e^{-10})$ as required

(c)(i) $T_2 = \frac{1}{12}(116 - 16e^{-0.75 \times 10} + e^{10-10} - e^{-10})$

$T_2 = 9.749258... = 9.7493$ (4 d.p.)

(ii) $T_3 = 9.730624...$, $T_4 = 9.729415...$, $T_5 = 9.729337...$

$T_6 = 9.7293322...$, $T_7 = 9.7293319...$

Time taken = 9.7293 seconds (4 d.p.)

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**Question 8 continued**

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**Question 8 continued**

**Q8**

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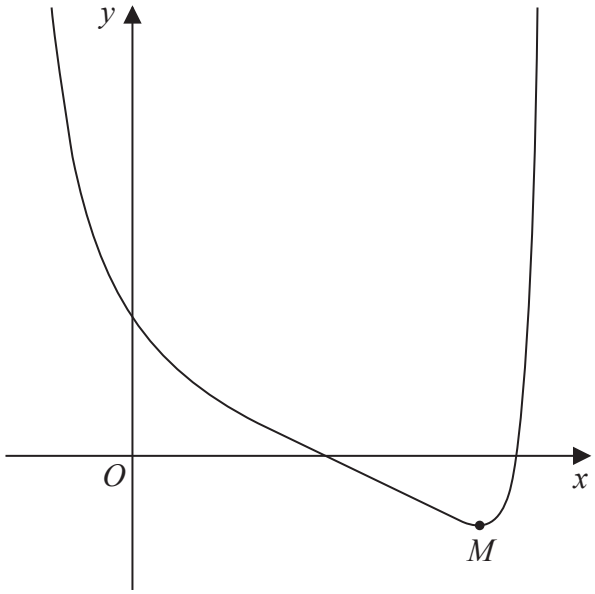



**Figure 5**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 5 shows the curve with equation

$$y = \frac{1 + 2\cos x}{1 + \sin x} \qquad -\frac{\pi}{2} < x < \frac{3\pi}{2}$$

The point $M$, shown in Figure 5, is the minimum point on the curve.

(a) Show that the $x$ coordinate of $M$ is a solution of the equation

$$2\sin x + \cos x = -2$$

**(4)**

(b) Hence find, to 3 significant figures, the $x$ coordinate of $M$.

**(5)**

(a) $\frac{dy}{dx} = \frac{(1 + \sin x)(-2\sin x) - (1 + 2\cos x)\cos x}{(1 + \sin x)^2}$

At M, $dy/dx = 0$: $(1 + \sin x)(-2\sin x) - (1 + 2\cos x)\cos x = 0$

$-2\sin x - 2\sin^2 x - \cos x - 2\cos^2 x = 0$

$-2\sin x - \cos x - 2(\sin^2 x + \cos^2 x) = 0$

$-2\sin x - \cos x - 2 = 0$

$2\sin x + \cos x = -2$ as required

(b) $2\sin x + \cos x \equiv R\sin(x + \alpha) = R\sin x \cos\alpha + R\cos x \sin\alpha$

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**Question 9 continued**

$R\cos\alpha = 2, \quad R\sin\alpha = 1 \Rightarrow R = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\tan\alpha = \frac{1}{2} \Rightarrow \alpha = 0.46364...$

$\sqrt{5}\sin(x + 0.46364...) = -2$

$\sin(x + 0.46364...) = -\frac{2}{\sqrt{5}} \quad (-1.107 < x + \alpha < 5.176)$

$x + 0.46364... = \pi + 1.10714... = 4.24873... \quad \text{or} \quad 2\pi - 1.10714... = 5.17604...$

$x = 3.78509... \quad \text{or} \quad x = 4.71238... = \frac{3\pi}{2}$ (not in range)

x coordinate of M = 3.79 (3 s.f.)

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**Question 9 continued**

**Q9**

**(Total 9 marks)**

**TOTAL FOR PAPER: 75 MARKS**

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