Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either two significant figures or three significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Q:1/1/

P 7 3 5 7 5 A 0 1 2 8

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1. A particle P moves in a straight line with simple harmonic motion between two fixed points A and B. The particle performs 2 complete oscillations per second. The midpoint of AB is O and the midpoint of OA is C

The length of AB is 0.6 m.

(a) Find the maximum speed of P

(4)

(b) Find the time taken by P to move directly from O to C

(2)

(a) 2 oscillations per second $\Rightarrow$ period $T = \frac{1}{2}$ s

$\frac{2\pi}{\omega} = \frac{1}{2} \Rightarrow \omega = 4\pi$

Amplitude $a = 0.6 \div 2 = 0.3$ m

$v_{max} = a\omega = 0.3 \times 4\pi = 1.2\pi$

Maximum speed $= 1.2\pi = 3.7699... = 3.77$ m s^{-1}

(b) $OC = 0.15$ m. Measuring x from O, with $x = 0$ at $t = 0$:

$x = a \sin \omega t \Rightarrow 0.15 = 0.3 \sin 4\pi t$

$\sin 4\pi t = \frac{1}{2} \Rightarrow 4\pi t = \frac{\pi}{6}$

$t = \frac{1}{4\pi} \times \frac{\pi}{6} = \frac{1}{24}$

Time from O to $C = 0.041666... = 0.0417$ s (3 s.f.)

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Question 1 continued

(Total for Question 1 is 6 marks)

2.

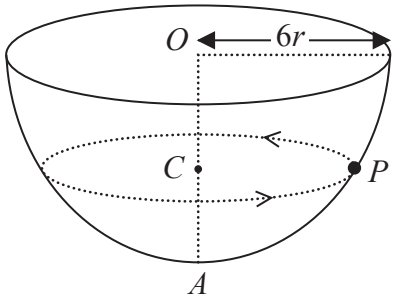


Figure 1

A hemispherical bowl of internal radius $6r$ is fixed with its circular rim horizontal. The centre of the circular rim is O and the point A on the surface of the bowl is vertically below O. A particle P moves in a horizontal circle, with centre C, on the smooth inner surface of the bowl. The particle moves with constant angular speed $\sqrt{\frac{g}{4r}}$. The point C lies on OA, as shown in Figure 1.

Find, in terms of r, the distance OC

(9)

Let θ = angle between OP and the vertical OA. Normal reaction R acts along PO.

Radius of circle: $CP = 6r \sin\theta$

N2L horizontally (towards C): $R\sin\theta = m(6r\sin\theta)\omega^2 = m \times 6r\sin\theta \times \frac{g}{4r}$

$R = 6m \times \frac{g}{4} = \frac{3}{2}mg$

Resolving vertically: $R\cos\theta = mg$

$\frac{3}{2}mg\cos\theta = mg$

$\cos\theta = \frac{2}{3}$

$OC = 6r\cos\theta = 6r \times \frac{2}{3}$

$OC = 4r$

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A particle P is moving along a straight line.

At time t seconds, P is a distance x metres from a fixed point O on the line and is moving away from O with speed $\frac{50}{2x+3}$ m s^{-1}

(a) Find the deceleration of P when $x = 12$

(5)

Given that $x = 4$ when $t = 1$

(b) find the value of t when $x = 12$

(5)

(a) $v = \frac{50}{2x+3} = 50(2x+3)^{-1}$

$\frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}$

$\frac{dv}{dx} = -100(2x+3)^{-2} = \frac{-100}{(2x+3)^2}$

Acceleration $= \frac{-100}{(2x+3)^2} \times \frac{50}{2x+3} = \frac{-5000}{(2x+3)^3}$

$x = 12$: acceleration $= \frac{-5000}{27^3} = \frac{-5000}{19683} = -0.254026...$

Deceleration $= 0.254$ m s^{-2} (3 s.f.)

(b) $v = \frac{dx}{dt} = \frac{50}{2x+3}$

$\int (2x+3)\,dx = \int 50\,dt$

$x^2 + 3x = 50t + c$

$t = 1, x = 4$: $16 + 12 = 50 + c \Rightarrow c = -22$

$x^2 + 3x = 50t - 22$

$x = 12$: $144 + 36 = 50t - 22 \Rightarrow 50t = 202$

$t = \frac{202}{50} = 4.04$

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

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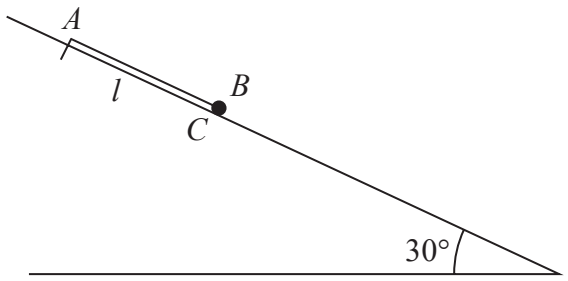


Figure 2

One end of a light elastic string, of natural length l and modulus of elasticity λ, is fixed to a point A on a smooth plane. The plane is inclined at 30° to the horizontal.

A small ball B of mass m is attached to the other end of the elastic string. Initially, B is held at rest at the point C on the plane with the elastic string lying along a line of greatest slope of the plane.

The point C is below A and $AC = l$, as shown in Figure 2.

The ball is released and comes to instantaneous rest at a point D on the plane.

The points A, C and D all lie along a line of greatest slope of the plane and $AD = \frac{5l}{4}$

The ball is modelled as a particle and air resistance is modelled as being negligible.

Using the model,

(a) show that $\lambda = 4mg$

(4)

(b) find, in terms of g and l, the greatest speed of B as it moves from C to D

(7)

(a) $CD = \frac{5l}{4} - l = \frac{l}{4}$, so the vertical drop from C to D is $\frac{l}{4}\sin 30° = \frac{l}{8}$

Energy from C to D (B at rest at C and at D): loss of GPE = gain of EPE

$mg \times \frac{l}{4}\sin 30° = \frac{\lambda}{2l}\left(\frac{l}{4}\right)^2$

$\frac{mgl}{8} = \frac{\lambda l}{32}$

$\lambda = 4mg$ as required

(b) Greatest speed when the acceleration of B is zero (equilibrium position).

Let the extension there be e. Resolving along the plane: $T = mg\sin 30°$

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Question 4 continued

$\frac{4mge}{l} = \frac{mg}{2} \Rightarrow e = \frac{1}{8}$

Vertical drop from C to this point $= \frac{l}{8} \sin 30° = \frac{l}{16}$

Energy from C to the equilibrium position:

$\frac{1}{2}mv^2 + \frac{4mg}{2l}\left(\frac{l}{8}\right)^2 = mg \times \frac{l}{8} \sin 30°$

$\frac{1}{2}mv^2 + \frac{mgl}{32} = \frac{mgl}{16}$

$\frac{1}{2}v^2 = \frac{gl}{32} \Rightarrow v^2 = \frac{gl}{16}$

Greatest speed $v = \sqrt{(gl/16)} = \frac{\sqrt{(gl)}}{4}$

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 11 marks)

5. (a) Use algebraic integration to show that the centre of mass of a uniform solid hemisphere of radius r is at a distance $\frac{3}{8}r$ from the centre of its plane face.

[*You may assume that the volume of a sphere of radius r is* $\frac{4}{3}\pi r^3$]

(5)

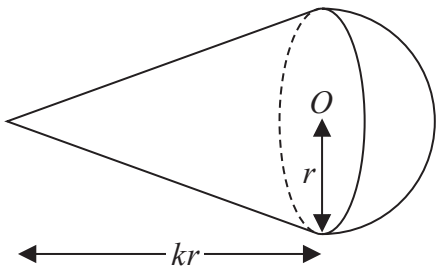


Figure 3

A uniform solid hemisphere of radius r is joined to a uniform solid right circular cone made of the **same material** to form a toy. The cone has base radius r and height kr. The centre of the base of the cone is O. The plane face of the cone coincides with the plane face of the hemisphere, as shown in Figure 3.

The toy can rest in equilibrium on a horizontal plane with any point of the curved surface of the hemisphere in contact with the plane.

(b) Find the exact value of k

(5)

(a) Take the plane face on the y-axis, centre at the origin: $y^2 = r^2 - x^2$

$\pi\rho\int_0^r xy^2\,dx = \pi\rho\int_0^r (r^2x - x^3)\,dx$

$= \pi\rho\left[\frac{1}{2}r^2x^2 - \frac{x^4}{4}\right]_0^r = \pi\rho\left(\frac{r^4}{2} - \frac{r^4}{4}\right) = \frac{\pi\rho r^4}{4}$

Mass of hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3\rho = \frac{2}{3}\pi r^3\rho$

$\frac{2}{3}\pi\rho r^3\,\bar{x} = \pi\rho\int xy^2\,dx = \frac{\pi\rho r^4}{4}$

$\bar{x} = \frac{\pi\rho r^4}{4} \div \frac{2\pi\rho r^3}{3} = \frac{3}{8}r$ from the centre of the plane face, as required

(b) It rests in any position, so the centre of mass of the toy is at O.

Masses: hemisphere $\frac{2}{3}\pi r^3\rho$, cone $\frac{1}{3}\pi r^2(kr)\rho = \frac{k}{3}\pi r^3\rho$

Distances of centres of mass from O (on opposite sides of O):

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Question 5 continued

hemisphere $\frac{3}{8}r$, cone $\frac{1}{4}kr$

Moments about O:

$$\frac{2}{3} \times \frac{3}{8}r = \frac{k}{3} \times \frac{1}{4}kr$$

$$\frac{1}{4} = \frac{k^2}{12} \Rightarrow k^2 = 3$$

$$k = \sqrt{3}$$

Question 5 continued

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Question 5 continued

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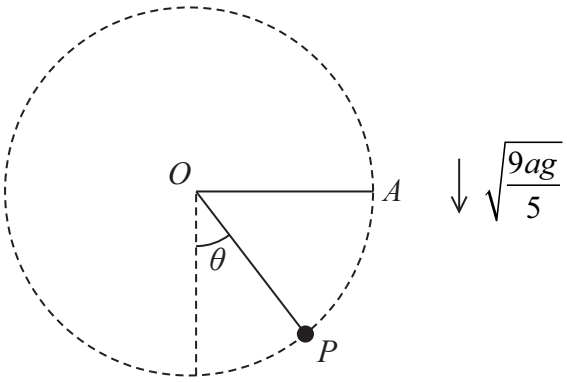


Figure 4

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where $OA = a$ and OA is horizontal, as shown in Figure 4.

The particle is projected vertically downwards with speed $\sqrt{\frac{9ag}{5}}$

When the string makes an angle θ with the downward vertical through O and the string is still taut, the tension in the string is S.

(a) Show that $S = \frac{3}{5}mg(5\cos\theta + 3)$

(6)

At the instant when the string becomes slack, the speed of P is v

(b) Show that $v = \sqrt{\frac{3ag}{5}}$

(3)

(c) Find the maximum height of P above the horizontal level of O

(4)

(a) Let the speed be u when the string makes angle θ with the downward vertical.

N2L along PO: $S - mg\cos\theta = \frac{mu^2}{a}$

Energy from A (P has fallen a cos θ):

$\frac{1}{2}mu^2 - \frac{1}{2}m \times \frac{9ag}{5} = mga\cos\theta$

$mu^2 = 2mga\cos\theta + \frac{9}{5}mga$

$S = mg\cos\theta + \frac{mu^2}{a} = mg\cos\theta + 2mg\cos\theta + \frac{9}{5}mg$

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Question 6 continued

$S = 3mg\cos\theta + \frac{9}{5}mg$

$S = \frac{3}{5}mg(5\cos\theta + 3)$ as required

(b) String becomes slack when $S = 0$: $5\cos\theta + 3 = 0 \Rightarrow \cos\theta = -\frac{3}{5}$

$v^2 = 2ga\cos\theta + \frac{9ag}{5} = 2ga\left(-\frac{3}{5}\right) + \frac{9ag}{5} = \frac{3ag}{5}$

$v = \sqrt{(3ag/5)}$ i.e. $v = \sqrt{\frac{3ag}{5}}$ as required

(c) At this instant P is a $\cos(180° - \theta) = \frac{3}{5}a$ above the level of O, and $\sin\theta = \frac{4}{5}$

Velocity is perpendicular to OP; vertical component $= v\sin\theta = \frac{4}{5}\sqrt{\frac{3ag}{5}}$

Vertically (as a projectile): $0 = \frac{16}{25} \times \frac{3ag}{5} - 2gs$

$2gs = \frac{48ag}{125} \Rightarrow s = \frac{24}{125}a$

Maximum height above O $= \frac{24}{125}a + \frac{3}{5}a = \frac{24a + 75a}{125} = \frac{99}{125}a$ $(= 0.792a)$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 13 marks)

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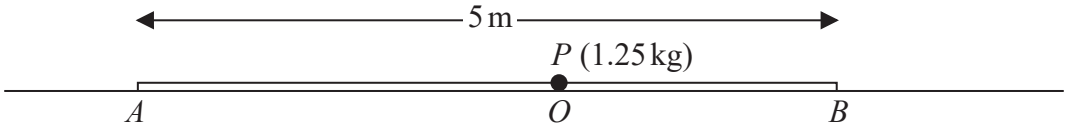


Figure 5

Figure 5 shows two fixed points, A and B, which are 5 m apart on a smooth horizontal floor.

A particle P of mass 1.25 kg is attached to one end of a light elastic string, of natural length 2 m and modulus of elasticity 20 N. The other end of the string is attached to A

A second light elastic string, of natural length 1.2 m and modulus of elasticity λ newtons, has one end attached to P and the other end attached to B

Initially P rests in equilibrium at the point O, where $AO = 3$ m

(a) Show that $\lambda = 15$

(3)

The particle is now projected along the floor towards B

At time t seconds, P is a displacement x metres from O in the direction OB

(b) Show that, while both strings are taut, P moves with simple harmonic motion where $\ddot{x} = -18x$

(4)

The initial speed of P is $10\,\text{m}\,\text{s}^{-1}$

(c) Find the speed of P at the instant when the string PB becomes slack.

(3)

Both strings are taut for T seconds during one complete oscillation.

(d) Find the value of T

(6)

(a) At O: extension of AP = 3 − 2 = 1 m , extension of BP = 2 − 1.2 = 0.8 m

Equilibrium: $T_{AP} = T_{BP} \Rightarrow \frac{20 \times 1}{2} = \frac{\lambda \times 0.8}{1.2}$

$10 = \frac{2\lambda}{3}$

$\lambda = 15$ as required

(b) Displacement x from O towards B: ext of AP = 1 + x , ext of BP = 0.8 − x

N2L (direction OB): $1.25\ddot{x} = \frac{15(0.8 - x)}{1.2} - \frac{20(1 + x)}{2}$

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Question 7 continued

$1.25\ddot{x} = 10 - 12.5x - 10 - 10x = -22.5x$

$\ddot{x} = -\frac{22.5}{1.25}x = -18x$

This is of the form $\ddot{x} = -\omega^2 x$, so P moves with SHM with centre O while both strings are taut.

(c) $\omega = \sqrt{18} = 3\sqrt{2}$; $v_{max} = a\omega \Rightarrow 10 = a\sqrt{18} \Rightarrow a = \frac{10}{\sqrt{18}} = \frac{5\sqrt{2}}{3}$ m

PB becomes slack when its extension is zero, i.e. $x = 0.8$

$v^2 = \omega^2(a^2 - x^2) = 18\left(\frac{100}{18} - 0.8^2\right) = 100 - 11.52 = 88.48$

$v = 9.40638... = 9.41 \text{ m s}^{-1}$ (3 s.f.)

(d) Use $x = a \sin \omega t$ with $t = 0$ at O. PB becomes slack at $x = 0.8$:

$0.8 = \frac{10}{\sqrt{18}} \sin(\sqrt{18}\, t_1)$

$t_1 = \frac{1}{\sqrt{18}} \sin^{-1}\left(\frac{0.8\sqrt{18}}{10}\right) = \frac{0.346290...}{\sqrt{18}} = 0.081621...$

PA becomes slack when its extension is zero, i.e. $x = -1$ (distance 1 m from O)

$1 = \frac{10}{\sqrt{18}} \sin(\sqrt{18}\, t_2)$

$t_2 = \frac{1}{\sqrt{18}} \sin^{-1}\left(\frac{\sqrt{18}}{10}\right) = \frac{0.438149...}{\sqrt{18}} = 0.103272...$

In one complete oscillation both strings are taut while P moves from $x = -1$ to $x = 0.8$ and back, each way taking $t_1 + t_2$:

$T = 2(t_1 + t_2) = 2(0.081621... + 0.103272...) = 0.369788...$

$T = 0.370$ (3 s.f.)

Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS

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