Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Q:1/1/

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1. At time t seconds, $t \geqslant 0$, a particle P has position vector $\mathbf{r}$ metres with respect to a fixed origin O, where

$$\mathbf{r} = (t^3 - 8t)\mathbf{i} + \left(\frac{1}{3}t^3 - t^2 + 2t\right)\mathbf{j}$$

(a) Find the acceleration of P when $t = 4$

(5)

At time T seconds, $T \geqslant 0$, P is moving in the direction of $(2\mathbf{i} + \mathbf{j})$

(b) Find the value of T

(3)

(a) $v = \frac{dr}{dt} = (3t^2 - 8)i + (t^2 - 2t + 2)j$

$a = \frac{dv}{dt} = 6ti + (2t - 2)j$

$t = 4$: $a = 24i + (8 - 2)j$

$a = (24i + 6j)\ m\ s^{-2}$

(b) Moving in direction of $2i + j \Rightarrow i$ component $= 2 \times j$ component:

$3T^2 - 8 = 2(T^2 - 2T + 2)$

$3T^2 - 8 = 2T^2 - 4T + 4$

$T^2 + 4T - 12 = 0$

$(T + 6)(T - 2) = 0 \quad \Rightarrow \quad T = 2 \quad (T \geqslant 0)$

(check: $v = 4i + 2j = 2(2i + j)$ ✓)

$T = 2$

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Question 1 continued

Q1

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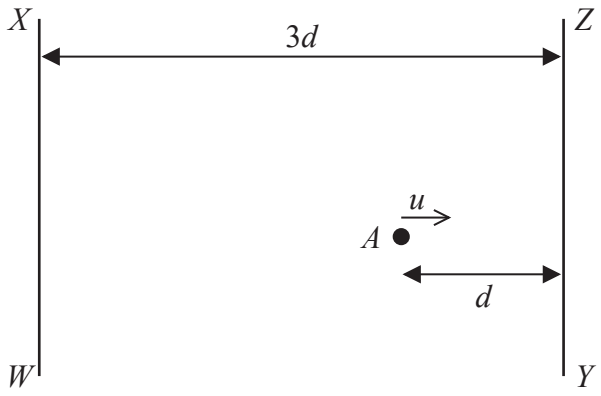


Figure 1

The point A lies on a smooth horizontal floor between two fixed smooth parallel vertical walls WX and YZ, as shown in the plan view in Figure 1.
The distance between WX and YZ is $3d$.
The distance of A from YZ is d.
A particle is projected from A along the floor with speed u towards YZ in a direction perpendicular to YZ.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$

The time taken for the particle to move from A, bounce off each wall once and return to A for the **first** time is T_1

(a) Find T_1 in terms of d and u.

(5)

The ball returns to A for the first time after bouncing off each wall once.
The further time taken for the particle to move from A, bounce off each wall once and return to A for the **second** time is T_2

(b) Find T_2 in terms of d and u.

(1)

(a) Speed after first impact (with YZ) $= \frac{2}{3}u$

Speed after second impact (with WX) $= \frac{2}{3} \times \frac{2}{3}u = \frac{4}{9}u$

$T_1 = \frac{d}{u} + \frac{3d}{(2/3)u} + \frac{2d}{(4/9)u}$

$T_1 = \frac{d}{u} + \frac{9d}{2u} + \frac{9d}{2u}$

$T_1 = \frac{10d}{u}$

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Question 2 continued

(b) Second circuit starts with speed $\frac{4}{9}u$ instead of u, so all times $\times \frac{9}{4}$:

$$T_2 = \frac{10d}{(4/9)u} = \frac{45d}{2u}$$

Q2

(Total 6 marks)

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3. A particle P of mass 0.5 kg is moving with velocity $\lambda(\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when P receives an impulse of magnitude $\sqrt{\frac{5}{2}}\,\text{N s}$

Immediately after P receives the impulse, the velocity of P is $4\mathbf{i}\,\text{m s}^{-1}$
Given that λ is a constant, find the two possible values of λ

(6)

$I = mv - mu$

$I = 0.5(4i) - 0.5\lambda(i + j)$

$I = 0.5[(4 - \lambda)i - \lambda j]$

$|I| = \sqrt{5/2}$:

$\frac{5}{2} = \frac{1}{4}\left[(4 - \lambda)^2 + (-\lambda)^2\right]$

$10 = 16 - 8\lambda + \lambda^2 + \lambda^2$

$0 = 2\lambda^2 - 8\lambda + 6$

$0 = 2(\lambda - 3)(\lambda - 1)$

$\lambda = 3$ or $\lambda = 1$

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Question 3 continued

Q3

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4. A truck of mass 900 kg is moving along a straight horizontal road with the engine of the truck working at a constant rate of P watts. The resistance to the motion of the truck is modelled as a constant force of magnitude R newtons.
At the instant when the speed of the truck is $15\,\text{m s}^{-1}$, the deceleration of the truck is $0.2\,\text{m s}^{-2}$

Later the same truck is moving down a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{30}$. The resistance to the motion of the truck is again modelled as a constant force of magnitude R newtons. The engine of the truck is again working at a constant rate of P watts.
At the instant when the speed of the truck is $12\,\text{m s}^{-1}$, the acceleration of the truck is $0.4\,\text{m s}^{-2}$

Find the value of R.

(8)

Driving force $F = \frac{P}{v}$

Horizontal road, speed 15 m s^{-1}, deceleration 0.2 m s^{-2}. N2L:

$$\frac{P}{15} - R = 900 \times (-0.2)$$

$$\frac{P}{15} - R = -180 \qquad ①$$

Down the hill, speed 12 m s^{-1}, acceleration 0.4 m s^{-2}, $\sin\theta = \frac{1}{30}$:

$$\frac{P}{12} + 900g\sin\theta - R = 900 \times 0.4$$

$$\frac{P}{12} + 900 \times 9.8 \times \frac{1}{30} - R = 360$$

$$\frac{P}{12} + 294 - R = 360 \Rightarrow \frac{P}{12} - R = 66 \qquad ②$$

② − ①:

$$\frac{P}{12} - \frac{P}{15} = 246 \Rightarrow \frac{P}{60} = 246 \Rightarrow P = 14760$$

$$R = \frac{14760}{15} + 180 = 984 + 180 = 1164$$

$R = 1160$ (3 s.f.)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

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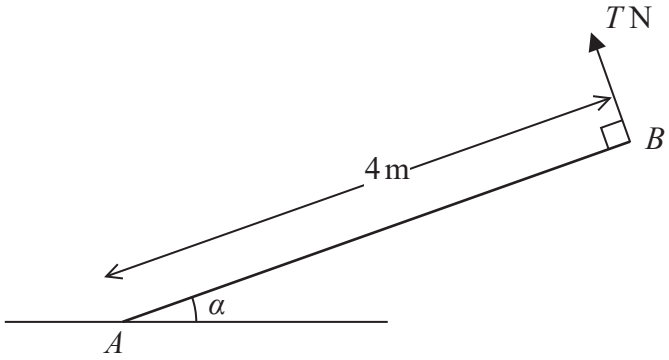


Figure 2

A uniform rod AB has length 4 m and weight 50 N.
The rod has its end A on rough horizontal ground. The rod is held in equilibrium at an angle α to the ground by a light inextensible cable attached to the rod at B, as shown in Figure 2. The cable and the rod lie in the same vertical plane and the cable is perpendicular to the rod. The tension in the cable is T newtons.

Given that $\sin\alpha = \frac{3}{5}$

(a) show that $T = 20$

(3)

Given also that the rod is in limiting equilibrium,

(b) find the value of the coefficient of friction between the rod and the ground.

(6)

(a) $\sin\alpha = \frac{3}{5} \Rightarrow \cos\alpha = \frac{4}{5}$

M(A): $T \times 4 = 50 \times 2\cos\alpha$

$4T = 2 \times \frac{4}{5} \times 50 = 80$

$T = 20$ as required

(b) Forces at A: normal reaction V (up), friction H (horizontal).

Cable is perpendicular to the rod, so it makes angle α with the vertical.

R(→): $H = T\sin\alpha$

$H = 20 \times \frac{3}{5} = 12$

R(↑): $V + T\cos\alpha = 50$

$V = 50 - 20 \times \frac{4}{5} = 34$

Limiting equilibrium: $H = \mu V$

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Question 5 continued

$12 = 34\mu$

$\mu = \frac{12}{34} = \frac{6}{17}$ $(= 0.353)$

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Question 5 continued

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Question 5 continued

Q5

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6. Two particles, P and Q, are moving in opposite directions along the same straight line on a smooth horizontal surface so that the particles collide directly.
The mass of P is km and the mass of Q is m.
Immediately before the collision, the speed of P is x and the speed of Q is y.
Immediately after the collision, P and Q are moving in the same direction, the speed of P is v and the speed of Q is $2v$.

The coefficient of restitution between P and Q is $\frac{1}{5}$

The magnitude of the impulse received by Q in the collision is $5mv$

(a) Find (i) y in terms of v
(ii) x in terms of v
(iii) the value of k

(9)

(b) Find, in terms of m and v, the total kinetic energy lost in the collision between P and Q.

(3)

(a) Take the direction of motion of P before the collision as positive (→).

Before: P (km) speed x →, Q (m) speed y ←

After: P speed v →, Q speed 2v →

(i) Impulse on Q: $I = mv - mu$

$5mv = m(2v - (-y))$

$5v = 2v + y$

$y = 3v$

(ii) Impact law: speed of separation = e × speed of approach

$2v - v = \frac{1}{5}(x + y)$

$x + y = 5v \quad \Rightarrow \quad x + 3v = 5v$

$x = 2v$

(iii) CLM: $kmx - my = kmv + m(2v)$

$2kv - 3v = kv + 2v$

$kv = 5v$

$k = 5$

(b) KE lost = KE before − KE after

$= \frac{1}{2}(5m)(2v)^2 + \frac{1}{2}m(3v)^2 - \left[\frac{1}{2}(5m)v^2 + \frac{1}{2}m(2v)^2\right]$

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Question 6 continued

$= 14.5mv^2 - 4.5mv^2$

$= 10mv^2$

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Question 6 continued

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Question 6 continued

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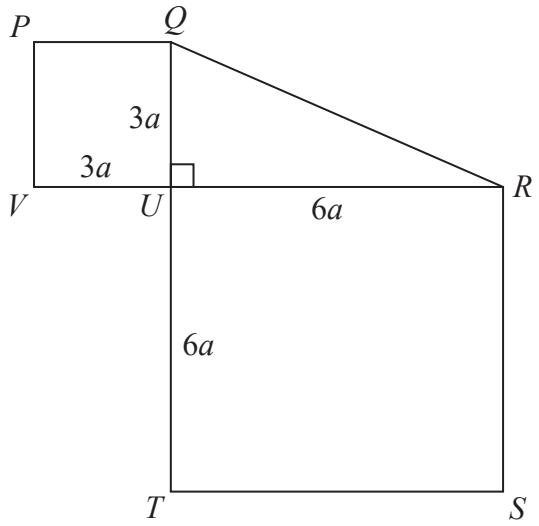


Figure 3

The template shown in Figure 3 is formed by joining together three separate laminas. All three laminas lie in the same plane.

- $PQUV$ is a uniform square lamina with sides of length $3a$
- $URST$ is a uniform square lamina with sides of length $6a$
- QRU is a uniform triangular lamina with $UQ = 3a$, $UR = 6a$ and angle $QUR = 90°$

The mass per unit area of $PQUV$ is k, where k is a constant.
The mass per unit area of $URST$ is k.
The mass per unit area of QRU is $2k$.

The distance of the centre of mass of the template from QT is d.

(a) Show that $d = \frac{29}{14}a$

(5)

The template is freely suspended from the point Q and hangs in equilibrium with QR at $\theta°$ to the downward vertical.

(b) Find the value of θ

(7)

(a)

	PQUV	URST	QRU	template
mass ratio	$9a^2k$	$36a^2k$	$18a^2k$	$63a^2k$
ratio	1	4	2	7
dist. from QT	$-\frac{3a}{2}$	$3a$	$2a$	d

(PQUV is on the other side of QT, so its distance is negative.)

M(QT):

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Question 7 continued

$\frac{3a}{2} \times (-1) + 4 \times 3a + 2 \times 2a = 7d$

$\frac{-3a + 24a + 8a}{2} = 7d \Rightarrow \frac{29a}{2} = 7d$

$d = \frac{29}{14} a$ as required

(b) Distances below Q (along QT):

PQUV: $\frac{3a}{2}$, URST: $3a + 3a = 6a$, QRU: $\frac{0 + 3a + 3a}{3} = 2a$

Let the centre of mass G be a distance v below PQ (through Q). M(PQ):

$\frac{3a}{2} \times 1 + 6a \times 4 + 2a \times 2 = 7v$

$\frac{3a}{2} + 24a + 4a = 7v \Rightarrow \frac{59a}{2} = 7v \Rightarrow v = \frac{59}{14} a$

Suspended from Q, so QG is vertical.

Angle between QG and QT: $\tan \beta = \frac{d}{v} = \frac{29/14}{59/14} = \frac{29}{59} \Rightarrow \beta = 26.175...°$

Angle between QR and QT: $\tan \gamma = \frac{UR}{QU} = \frac{6a}{3a} = 2 \Rightarrow \gamma = 63.434...°$

$\theta = \gamma - \beta = 63.434...° - 26.175...° = 37.259...°$

$\theta = 37.3$

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Question 7 continued

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Question 7 continued

Q7

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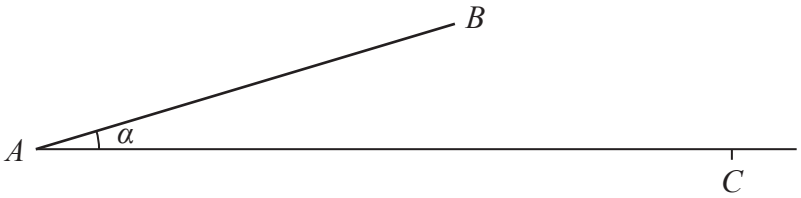


Figure 4

Figure 4 shows a rough ramp fixed to horizontal ground.

The ramp is inclined at angle α to the ground, where $\tan\alpha = \frac{1}{6}$

The point A is on the ground at the bottom of the ramp.
The point B is at the top of the ramp.
The line AB is a line of greatest slope of the ramp and $AB = 4\,\text{m}$.

A particle P of mass $3\,\text{kg}$ is projected with speed $U\,\text{m s}^{-1}$ from A directly towards B.

The coefficient of friction between the particle and the ramp is $\frac{3}{4}$

(a) Find the work done against friction as P moves from A to B.

(4)

Given that at the instant P reaches the point B, the speed of P is $5\,\text{m s}^{-1}$

(b) use the work-energy principle to find the value of U.

(4)

The particle leaves the ramp at B, and moves freely under gravity until it hits the ground at the point C.

(c) Find the horizontal distance from B to C.

(6)

(a) $\tan\alpha = \frac{1}{6} \Rightarrow \sin\alpha = \frac{1}{\sqrt{37}}$, $\cos\alpha = \frac{6}{\sqrt{37}}$

R(⊥ ramp): $R = 3g\cos\alpha = \frac{18g}{\sqrt{37}} = 29.0\ \text{N}$

$F = \mu R = \frac{3}{4} \times \frac{18g}{\sqrt{37}} = \frac{27g}{2\sqrt{37}} = 21.75\ldots\ \text{N}$

Work done $= F \times 4 = \frac{54g}{\sqrt{37}} = 86.99\ldots$

Work done against friction = 87.0 J

(b) Work-energy principle (A to B):

initial KE − work done against friction − PE gained = final KE

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Question 8 continued

$\frac{1}{2} \times 3U^2 - 86.99... - 3g \times 4 \sin\alpha = \frac{1}{2} \times 3 \times 5^2$

$1.5U^2 - 86.99... - 19.333... = 37.5$

$1.5U^2 = 143.83... \Rightarrow U^2 = 95.889...$

$U = 9.7922... = 9.79$

(c) Height of B above ground $= 4 \sin\alpha = \frac{4}{\sqrt{37}}$ m

At B: vertical velocity $= 5 \sin\alpha = \frac{5}{\sqrt{37}}$ ↑, horizontal $= 5 \cos\alpha = \frac{30}{\sqrt{37}}$

(↑) $s = ut + \frac{1}{2}at^2$: $-\frac{4}{\sqrt{37}} = \frac{5}{\sqrt{37}}t - 4.9t^2$

$4.9t^2 - \frac{5}{\sqrt{37}}t - \frac{4}{\sqrt{37}} = 0$

$t = \frac{0.82199... + \sqrt{0.82199...^2 + 4 \times 4.9 \times 0.65760...}}{9.8} = 0.45969...$ s

(→) BC (horizontal) $= \frac{30}{\sqrt{37}} \times 0.45969... = 2.2671...$

horizontal distance BC = 2.27 m

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Question 8 continued

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Question 8 continued

Q8

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