Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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Pearson

1. *Spany* sells seeds and claims that 5% of its pansy seeds do not germinate. A packet of pansy seeds contains 20 seeds. Each seed germinates independently of the other seeds.

(a) Find the probability that in a packet of *Spany*'s pansy seeds

(i) more than 2 but fewer than 5 seeds do not germinate,

(ii) more than 18 seeds germinate.

(5)

Jem buys 5 packets of *Spany*'s pansy seeds.

(b) Calculate the probability that all of these packets contain more than 18 seeds that germinate.

(2)

Jem believes that *Spany*'s claim is incorrect. She believes that the percentage of pansy seeds that do not germinate is greater than 5%

(c) Write down the hypotheses for a suitable test to examine Jem's belief.

(1)

Jem planted all of the 100 seeds she bought from *Spany* and found that 8 did not germinate.

(d) Using a suitable approximation, carry out the test using a 5% level of significance.

(6)

(a) X = number of seeds that do not germinate, $X \sim B(20, 0.05)$

(i) $P(2 < X < 5) = P(X \leq 4) - P(X \leq 2)$

$= 0.9974 - 0.9245$

$= 0.0729$

(ii) more than 18 germinate $\Rightarrow X \leq 1$

$P(X \leq 1) = 0.7358$

(b) W = number of packets with more than 18 germinating

$P(W = 5) = (0.7358)^5$

$= 0.21573... = 0.216$

(c) $H_0 : p = 0.05$ $H_1 : p > 0.05$

(d) V = number of seeds out of 100 that do not germinate

$V \sim B(100, 0.05)$; n large, p small $\Rightarrow V \approx Po(5)$

$P(V \geq 8) = 1 - P(V \leq 7)$

$= 1 - 0.8666$

$= 0.1334$

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Question 1 continued

0.1334 > 0.05 so not significant: do not reject H_0

There is insufficient evidence to support Jem's belief that the percentage of seeds not germinating is more than 5%.

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 14 marks)

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2. Luis makes and sells rugs. He knows that faults occur randomly in his rugs at a rate of 3 every $4\,m^2$

(a) Find the probability of there being exactly 5 faults in one of his rugs that is $4\,m^2$ in size.

(2)

(b) Find the probability that there are more than 5 faults in one of his rugs that is $6\,m^2$ in size.

(2)

Luis makes a rug that is $4\,m^2$ in size and finds it has exactly 5 faults in it.

(c) Write down the probability that the next rug that Luis makes, which is $4\,m^2$ in size, will have exactly 5 faults. Give a reason for your answer.

(2)

A small rug has dimensions 80 cm by 150 cm. Faults still occur randomly at a rate of 3 every $4\,m^2$
Luis makes a profit of £80 on each small rug he sells that contains no faults but a profit of £60 on any small rug he sells that contains faults.

Luis sells n small rugs and expects to make a profit of at least £4000

(d) Calculate the minimum value of n

(4)

Luis wishes to increase the productivity of his business and employs Rhiannon. Faults also occur randomly in Rhiannon's rugs and independently to faults made by Luis.
Luis randomly selects 10 small rugs made by Rhiannon and finds 13 faults.

(e) Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that the rate at which faults occur is higher for Rhiannon than for Luis. State your hypotheses clearly.

(5)

(a) X = number of faults in 4 m^2, $X \sim Po(3)$

$P(X = 5) = P(X \leq 5) - P(X \leq 4) = 0.9161 - 0.8153$

$= 0.1008$

(b) Y = number of faults in 6 m^2, $Y \sim Po(4.5)$

$P(Y > 5) = 1 - P(Y \leq 5) = 1 - 0.7029$

$= 0.2971$

(c) 0.1008

Faults occur independently (randomly), so the number of faults in the previous rug does not affect the next rug.

(d) Area = $0.8 \times 1.5 = 1.2$ m^2, F = faults in a small rug, $F \sim Po(0.9)$

$P(F = 0) = e^{-0.9} = 0.40656...$, $P(F \geq 1) = 0.59343...$

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Question 2 continued

Expected profit: $n(80e^{-0.9} + 60(1 - e^{-0.9})) \geqslant 4000$

$68.131... n \geqslant 4000$

$n \geqslant \frac{4000}{20e^{-0.9} + 60} = 58.71...$

minimum $n = 59$

(e) $H_0 : \lambda = 9$ $H_1 : \lambda > 9$

R = faults in 10 small rugs, under H_0 $R \sim Po(9)$

$P(R \geqslant 13) = 1 - P(R \leqslant 12) = 1 - 0.8758$

$= 0.1242$

$0.1242 > 0.05$ so not significant: do not reject H_0

There is insufficient evidence that the rate at which faults occur is higher for Rhiannon than for Luis.

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 15 marks)

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3. The continuous random variable Y has the following probability density function

$$f(y) = \begin{cases} \frac{6}{25}(y-1) & 1 \leqslant y < 2 \\ \frac{3}{50}(4y^2 - y^3) & 2 \leqslant y < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch f(y)

(2)

(b) Find the mode of Y

(3)

(c) Use algebraic integration to calculate E(Y^2)

(4)

Given that E(Y) = 2.696

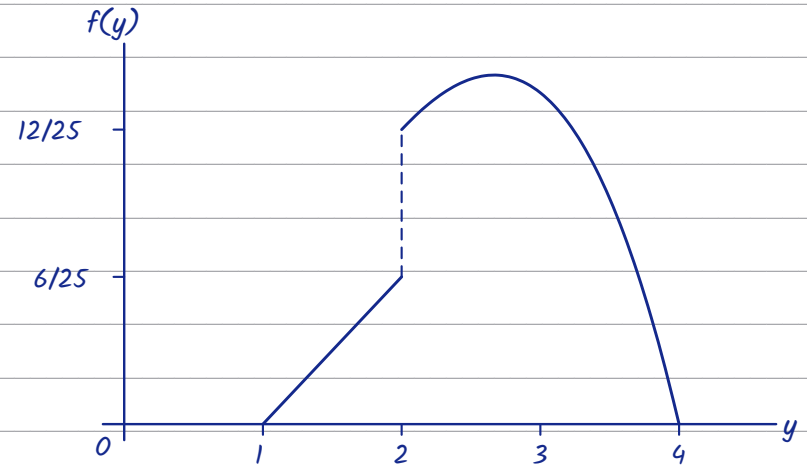
(d) find Var(Y)

(2)

(e) Find the value of y for which P($Y \geqslant y$) = 0.9
Give your answer to 3 significant figures.

(4)

(a)



(b) $\frac{d}{dy}\left(\frac{3}{50}(4y^2 - y^3)\right) = \frac{3}{50}(8y - 3y^2)$

$\frac{3}{50}(8y - 3y^2) = 0 \Rightarrow y(8 - 3y) = 0$

mode $y = \frac{8}{3}$ ($y = 0$ rejected, not in $2 \leqslant y < 4$)

(c) $E(Y^2) = \int_1^2 \left(\frac{6}{25}y^3 - \frac{6}{25}y^2\right) dy + \int_2^4 \left(\frac{12}{50}y^4 - \frac{3}{50}y^5\right) dy$

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$= \left[\frac{6}{100}y^4 - \frac{6}{75}y^3\right]_1^2 + \left[\frac{12}{250}y^5 - \frac{3}{300}y^6\right]_2^4$

$= \left[\frac{8}{25} - \left(-\frac{1}{50}\right)\right] + \left[\frac{1024}{125} - \frac{112}{125}\right]$

$= \frac{17}{50} + \frac{912}{125} = \frac{1909}{250} = 7.636$

(d) $\text{Var}(Y) = E(Y^2) - [E(Y)]^2 = \frac{1909}{250} - 2.696^2$

$= 7.636 - 7.268416 = 0.367584 = 0.368$

(e) $P(Y \geq y) = 0.9 \Rightarrow P(Y < y) = 0.1$; $P(Y < 2) = \frac{1}{2} \times 1 \times \frac{6}{25} = 0.24 > 0.1$

so $1 < y < 2$:

$\int_1^y \frac{6}{25}(t-1)\,dt = 0.1 \Rightarrow \frac{6}{25}\left[\frac{t^2}{2} - t\right]_1^y = 0.1$

$\frac{3}{25}(y-1)^2 = 0.1 \Rightarrow (y-1)^2 = \frac{5}{6}$

$y = 1 + \sqrt{\frac{5}{6}}$ $(1 - \sqrt{(5/6)} < 1$, rejected)

$y = 1.91287\ldots = 1.91$

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 15 marks)

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4. A bag contains a large number of balls, each with one of the numbers 1, 2 or 5 written on it in the ratio 2 : 3 : 4 respectively.

A random sample of 3 balls is taken from the bag.

The random variable B represents the range of the numbers written on the balls in the sample.

(i) Find $P(B = 4)$

(ii) Find the sampling distribution of B.

(10)

$P(1) = \frac{2}{9}$, $P(2) = \frac{3}{9} = \frac{1}{3}$, $P(5) = \frac{4}{9}$

(i) Samples with range 4: (1, 1, 5), (1, 5, 5), (1, 2, 5)

(1, 1, 5): $\frac{2}{9} \times \frac{2}{9} \times \frac{4}{9} \times 3 = \frac{16}{243}$

(1, 5, 5): $\frac{2}{9} \times \frac{4}{9} \times \frac{4}{9} \times 3 = \frac{32}{243}$

(1, 2, 5): $\frac{2}{9} \times \frac{1}{3} \times \frac{4}{9} \times 6 = \frac{16}{81}$

$P(B = 4) = \frac{16}{243} + \frac{32}{243} + \frac{16}{81} = \frac{96}{243} = \frac{32}{81}$

(ii) Possible ranges: 0, 1, 3, 4

$P(B = 0) = \left(\frac{2}{9}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(\frac{4}{9}\right)^3 = \frac{8 + 27 + 64}{729} = \frac{11}{81}$

$P(B = 1) = 3 \times \frac{2}{9} \times \left(\frac{1}{3}\right)^2 + 3 \times \frac{1}{3} \times \left(\frac{2}{9}\right)^2 = \frac{18 + 12}{243} = \frac{10}{81}$

$P(B = 3) = 1 - \frac{11}{81} - \frac{10}{81} - \frac{32}{81} = \frac{28}{81}$

check: $3 \times \frac{1}{3} \times \left(\frac{4}{9}\right)^2 + 3 \times \frac{4}{9} \times \left(\frac{1}{3}\right)^2 = \frac{48 + 36}{243} = \frac{28}{81}$

Sampling distribution of B:

b	0	1	3	4
$P(B = b)$	$\frac{11}{81}$	$\frac{10}{81}$	$\frac{28}{81}$	$\frac{32}{81}$

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 10 marks)

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5. A game uses two turntables, one red and one yellow. Each turntable has a point marked on the circumference that is lined up with an arrow at the start of the game. Jim spins both turntables and measures the distance, in metres, each point is from the arrow, around the circumference in an anticlockwise direction when the turntables stop spinning.

The continuous random variable Y represents the distance, in metres, the point is from the arrow for the yellow turntable. The cumulative distribution function of Y is given by $\mathrm{F}(y)$ where

$$\mathrm{F}(y) = \begin{cases} 0 & y < 0 \\ 1 - (\alpha + \beta y^2) & 0 \leqslant y \leqslant 5 \\ 1 & y > 5 \end{cases}$$

(a) Explain why (i) $\alpha = 1$

(ii) $\beta = -\dfrac{1}{25}$

(2)

(b) Find the probability density function of Y

(2)

The continuous random variable R represents the distance, in metres, the point is from the arrow for the red turntable. The distribution of R is modelled by a continuous uniform distribution over the interval $[d, 3d]$

Given that $\mathrm{P}\left(R > \dfrac{11}{5}\right) = \mathrm{P}\left(Y > \dfrac{5}{3}\right)$

(c) find the value of d

(3)

In the game each turntable is spun 3 times. The distance between the point and the arrow is determined for each spin. To win a prize, at least 5 of the distances the point is from the arrow when a turntable is spun must be less than $\dfrac{11}{5}$ m

Jo plays the game once.

(d) Calculate the probability of Jo winning a prize.

(4)

(a)(i) $F(0) = 0 \Rightarrow 1 - (\alpha + \beta \times 0) = 0 \Rightarrow \alpha = 1$

(ii) $F(5) = 1 \Rightarrow 1 - (1 + 25\beta) = 1$

$\Rightarrow 1 + 25\beta = 0 \Rightarrow \beta = -\dfrac{1}{25}$

(b) $F(y) = \dfrac{y^2}{25}$ for $0 \leqslant y \leqslant 5$, $f(y) = \dfrac{dF}{dy} = \dfrac{2y}{25}$

$f(y) = \dfrac{2y}{25}$ for $0 \leqslant y \leqslant 5$

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$f(y) = 0$ otherwise

(c) $P\left(Y > \frac{5}{3}\right) = 1 - \frac{1}{25}\left(\frac{5}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$

$R \sim U[d, 3d]$: $P\left(R > \frac{11}{5}\right) = \frac{3d - 11/5}{3d - d} = \frac{8}{9}$

$9(3d - 2.2) = 16d \Rightarrow 27d - 19.8 = 16d \Rightarrow 11d = 19.8$

$d = \frac{9}{5}$ $(= 1.8)$

(d) Red: $P\left(R < \frac{11}{5}\right) = \frac{2.2 - 1.8}{5.4 - 1.8} = \frac{1}{9} = p$

Yellow: $P\left(Y < \frac{11}{5}\right) = \frac{2.2^2}{25} = \frac{121}{625} = q$ $(= 0.1936)$

At least 5 of the 6 distances less than 2.2 m:

$P = p^3q^3 + 3p^2(1 - p)q^3 + 3p^3q^2(1 - q)$

$= \left(\frac{1}{9}\right)^3\left(\frac{121}{625}\right)^3 + 3\left(\frac{1}{9}\right)^2\left(\frac{8}{9}\right)\left(\frac{121}{625}\right)^3$

$+ 3\left(\frac{1}{9}\right)^3\left(\frac{121}{625}\right)^2\left(\frac{504}{625}\right)$

$= 0.000009954... + 0.000238887... + 0.000124385...$

$= 0.000373226... = 0.000373$

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Question 5 continued

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Question 5 continued

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6. The random variable $Y \sim B(225, p)$

Using a normal approximation, the probability that Y is at least 188 is 0.1056 to 4 decimal places.

(i) Show that p satisfies $145p^2 - 241p + 100 = 0$ when the normal probability tables are used.

(ii) Hence find the value of p, justifying your answer.

(10)

(i) $Y \approx X \sim N(225p, 225p(1-p))$, $\mu = 225p$, $\sigma = \sqrt{(225p(1-p))}$

$P(Y \geqslant 188) \approx P(X > 187.5) = 0.1056 \Rightarrow P(Z < z) = 0.8944, \quad z = 1.25$

$\frac{187.5 - \mu}{\sigma} = 1.25 \Rightarrow 187.5 - 225p = 1.25\sqrt{(225p(1-p))}$

Squaring: $(187.5 - 225p)^2 = 1.25^2 \times 225p(1-p)$

$37.5^2(5 - 6p)^2 = 1.5625 \times 225(p - p^2)$

$1406.25(5 - 6p)^2 = 351.5625(p - p^2)$

$\div 351.5625: \quad 4(5 - 6p)^2 = p - p^2$

$4(25 - 60p + 36p^2) = p - p^2$

$100 - 240p + 144p^2 = p - p^2$

$145p^2 - 241p + 100 = 0$ as required

(ii) $(29p - 25)(5p - 4) = 0$

$p = 0.8 \quad \text{or} \quad p = \frac{25}{29} = 0.86206...$

If $p = 0.86206...$, $\mu = 225 \times 0.86206... = 193.96... > 188$, so $P(Y \geqslant 188)$ would be greater than 0.5 ($187.5 - \mu < 0$, z negative), not 0.1056.

If $p = 0.8$, $\mu = 180$, $\sigma = \sqrt{36} = 6$, $z = (187.5 - 180)/6 = 1.25$ ✓

$\therefore \quad p = 0.8$

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Question 6 continued

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Question 6 continued

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TOTAL FOR PAPER: 75 MARKS

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