Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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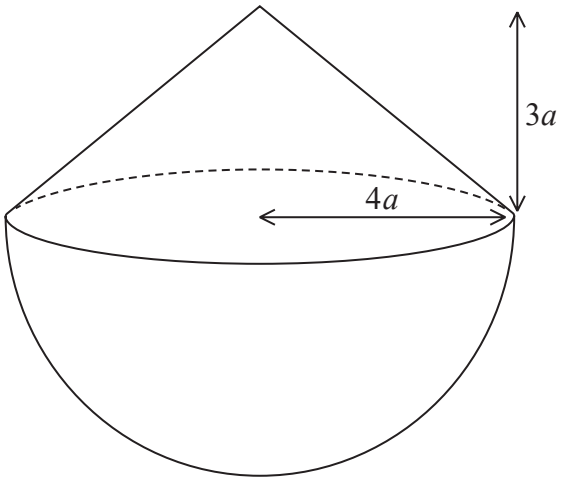


Figure 1

A hollow toy is formed by joining a uniform right circular conical shell C, with radius $4a$ and height $3a$, to a uniform hemispherical shell H, with radius $4a$. The circular edge of C coincides with the circular edge of H, as shown in Figure 1.

The mass per unit area of C is λ and the mass per unit area of H is $k\lambda$ where k is a constant.

Given that the centre of mass of the toy is a distance $4a$ from the vertex of the cone, find the value of k.

(6)

Slant height of cone $= \sqrt{((4a)^2 + (3a)^2)} = 5a$

Masses: H: $2\pi(4a)^2 \times k\lambda = 32\pi a^2 k\lambda$; C: $\pi(4a)(5a) \times \lambda = 20\pi a^2\lambda$

Mass ratio H : C : toy $= 8k : 5 : 8k + 5$

Distances of centres of mass from the vertex:

C: $\frac{2}{3} \times 3a = 2a$; H: $3a + \frac{4a}{2} = 5a$; toy: $4a$

Moments about the vertex:

$8k(5a) + 5(2a) = (8k + 5)(4a)$

$40ka + 10a = 32ka + 20a$

$8ka = 10a$

$k = \frac{10}{8} = \frac{5}{4}$ $(= 1.25)$

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Question 1 continued

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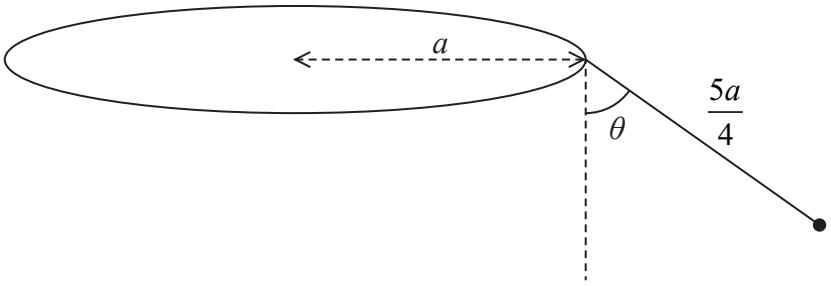


Figure 2

Figure 2 shows a fairground ride that consists of a chair of mass m attached to one end of a rigid arm of length $\frac{5a}{4}$. The other end of the arm is freely hinged to the rim of a thin horizontal circular disc of radius a. The disc rotates with constant angular speed ω about a vertical axis through the centre of the disc. As the ride rotates the arm remains in a vertical plane through the centre of the disc. The arm makes a constant angle θ with the vertical, where $\tan\theta = \frac{3}{4}$

The chair is modelled as a particle and the arm is modelled as a light rod.

(a) Find the tension in the arm in terms of m and g

(3)

(b) Find ω in terms of a and g

(6)

(a) $\tan\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

$R(\uparrow)$: $T\cos\theta = mg$

$\frac{4}{5}T = mg \Rightarrow T = \frac{5mg}{4}$

(b) Radius of circle: $r = a + \frac{5a}{4}\sin\theta = a + \frac{5a}{4} \times \frac{3}{5} = \frac{7a}{4}$

$R(\rightarrow)$: $T\sin\theta = mr\omega^2 = m\omega^2\frac{7a}{4}$

$\frac{3}{5} \times \frac{5mg}{4} = \frac{7}{4}ma\omega^2$

$\frac{3mg}{4} = \frac{7}{4}ma\omega^2 \Rightarrow \omega^2 = \frac{3g}{7a}$

$\omega = \sqrt{3g/(7a)}$ i.e. $\omega = \sqrt{\frac{3g}{7a}}$

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

3. The finite region enclosed by the curve with equation $y = 3 - \sqrt{x}$ and the lines $x = 0$ and $y = 0$ is rotated through 2π radians about the x-axis, to form a uniform solid S.

Use algebraic integration to

(a) show that the volume of S is $\frac{27}{2}\pi$

(4)

(b) find the x coordinate of the centre of mass of S.

(5)

(a) Curve meets the x-axis when $3 - \sqrt{x} = 0$, i.e. $x = 9$

$$V = \pi\int_0^9 y^2\,dx = \pi\int_0^9 (3 - \sqrt{x})^2\,dx$$

$$= \pi\int_0^9 (9 - 6x^{1/2} + x)\,dx$$

$$= \pi\left[9x - 4x^{3/2} + \frac{x^2}{2}\right]_0^9$$

$$= \pi\left(81 - 108 + \frac{81}{2}\right) - \pi(0)$$

$$\therefore\ V = \frac{27}{2}\pi \quad \text{as required}$$

(b) $\pi\int_0^9 xy^2\,dx = \pi\int_0^9 (9x - 6x^{3/2} + x^2)\,dx$

$$= \pi\left[\frac{9}{2}x^2 - \frac{12}{5}x^{5/2} + \frac{x^3}{3}\right]_0^9$$

$$= \pi\left(\frac{729}{2} - \frac{2916}{5} + \frac{729}{3}\right) - \pi(0) = \frac{243}{10}\pi$$

$$\bar{x} = \frac{\pi\int xy^2\,dx}{\pi\int y^2\,dx} = \frac{243\pi/10}{27\pi/2} = \frac{243}{10} \times \frac{2}{27}$$

$$\bar{x} = 1.8$$

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Question 3 continued

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Question 3 continued

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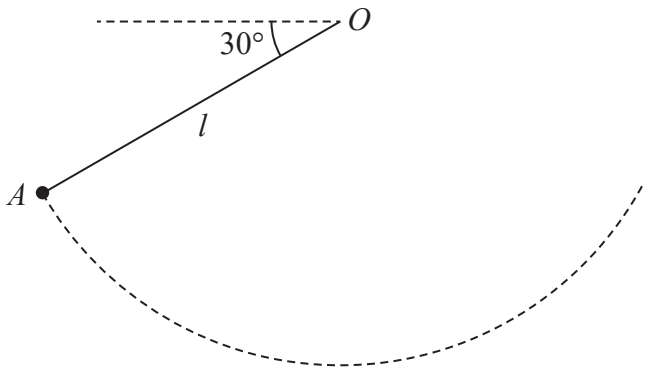


Figure 3

A circus performer has mass m. She is attached to one end of a cable of length l. The other end of the cable is attached to a fixed point O

Initially she is held at rest at point A with the cable taut and at an angle of 30° below the horizontal, as shown in Figure 3.

The circus performer is released from A and she moves on a vertical circular path with centre O

The circus performer is modelled as a particle and the cable is modelled as light and inextensible.

(a) Find, in terms of m and g, the tension in the cable at the instant immediately after the circus performer is released.

(2)

(b) Show that, during the motion following her release, the greatest tension in the cable is 4 times the least tension in the cable.

(7)

(a) At A, $v = 0$, so there is no acceleration towards O.

Resolving along AO: $T - mg\sin 30° = m(0)^2/l = 0$

$T = mg\cos 60° = \frac{mg}{2}$

(b) The performer swings down from A through the lowest point and up to the point at 30° below the horizontal on the other side, where she is again at rest.

Least tension: at the ends of the swing (speed 0, highest points), $T_{min} = \frac{mg}{2}$

Greatest tension: at the lowest point (greatest speed, cable along the weight).

Energy from A to the lowest point (drop $= l - l\sin 30°$):

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Question 4 continued

$\frac{1}{2}mv^2 - mgl = -mgl\sin 30° \Rightarrow v^2 = 2gl(1 - \frac{1}{2}) = gl$

At the lowest point: $T_{max} - mg = \frac{mv^2}{l} = \frac{m(gl)}{l}$

$T_{max} = mg + mg = 2mg$

$\frac{T_{max}}{T_{min}} = \frac{2mg}{mg/2} = 4$, so the greatest tension is 4 times the least tension.

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

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5. A particle P of mass 0.5 kg moves on the x-axis under the action of a single force.

At time t seconds, $t \geqslant 0$

- $OP = x$ metres, $0 \leqslant x < \frac{\pi}{2}$
- the force has magnitude $\sin 2x$ N and is directed towards the origin O
- P is moving in the positive x direction with speed $v\,\text{m s}^{-1}$

At time $t = 0$, P passes through the origin with speed $2\,\text{m s}^{-1}$

(a) Show that $v = 2\cos x$

(6)

(b) Show that $t = \frac{1}{2}\ln\left(\sqrt{2}+1\right)$ when $x = \frac{\pi}{4}$

(5)

(a) N2L (force towards O): $0.5v\frac{dv}{dx} = -\sin 2x$

$\int 0.5v\,dv = \int -\sin 2x\,dx$

$0.25v^2 = \frac{1}{2}\cos 2x + c$

$v^2 = 2\cos 2x + c'$

$x = 0, v = 2$: $4 = 2\cos 0 + c' \Rightarrow c' = 2$

$v^2 = 2\cos 2x + 2 = 2(2\cos^2 x - 1) + 2 = 4\cos^2 x$

$v = 2\cos x$ (positive root, since $v > 0$ and $\cos x > 0$ for $0 \leqslant x < \frac{\pi}{2}$)

(b) $\frac{dx}{dt} = 2\cos x$

$\int \sec x\,dx = \int 2\,dt$

$\ln|\sec x + \tan x| = 2t + k$

$t = 0, x = 0$: $\ln|1 + 0| = 0 + k \Rightarrow k = 0$

$t = \frac{1}{2}\ln|\sec x + \tan x|$

$x = \frac{\pi}{4}$: $t = \frac{1}{2}\ln\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) = \frac{1}{2}\ln(\sqrt{2}+1)$ as required

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Question 5 continued

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Question 5 continued

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6. A particle P of mass 0.4 kg is attached to one end of a light elastic string, of natural length 0.8 m and modulus of elasticity 0.6 N. The other end of the string is fixed to a point A on a rough horizontal table. The coefficient of friction between P and the table is $\frac{1}{7}$

The particle P is projected from A, with speed $1.8\,\text{m s}^{-1}$, along the surface of the table.

After travelling 0.8 m from A, the particle passes through the point B on the table.

(a) Find the speed of P at the instant it passes through B.

(5)

The particle P comes to rest at the point C on the table, where ABC is a straight line.

(b) Find the total distance travelled by P as it moves directly from A to C.

(6)

(c) Show that P remains at rest at C.

(3)

(a) $R = 0.4g$, $F_r = \frac{1}{7} \times 0.4 \times 9.8 = 0.56$ N

From A to B the string is slack (AB = 0.8 m = natural length).

Work-energy:

$\frac{1}{2} \times 0.4v^2 = \frac{1}{2} \times 0.4 \times 1.8^2 - 0.56 \times 0.8$

$0.2v^2 = 0.648 - 0.448 = 0.2 \quad \Rightarrow \quad v^2 = 1.00$

Speed at B: $v = 1.0\ \text{m s}^{-1}$

(b) Let BC = x m (extension of the string at C).

$\frac{1}{2} \times 0.4 \times 1.0^2 = 0.56x + \frac{0.6x^2}{2 \times 0.8}$

$0.2 = 0.56x + 0.375x^2$

$0.375x^2 + 0.56x - 0.2 = 0$

$x = \frac{-0.56 + \sqrt{(0.56^2 + 4 \times 0.375 \times 0.2)}}{2 \times 0.375} = 0.29776...$

Total distance AC = 0.8 + 0.29776... = 1.09776... = 1.10 m (3 s.f.)

(c) At C: $T = \frac{0.6 \times 0.29776...}{0.8} = 0.2233...$ N

Maximum friction = $\mu R = 0.56$ N

0.223 < 0.56, so the tension is less than the maximum friction

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Question 6 continued

and P remains at rest at C.

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Question 6 continued

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Question 6 continued

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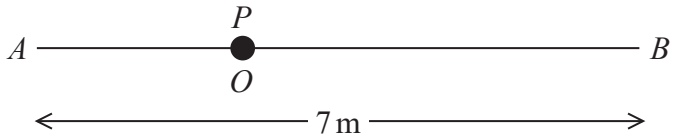


Figure 4

The fixed points A and B are 7 m apart on a smooth horizontal surface.

A light elastic string has natural length 2 m and modulus of elasticity 4 N. One end of the string is attached to a particle P of mass 2 kg and the other end is attached to A

Another light elastic string has natural length 3 m and modulus of elasticity 2 N. One end of this string is attached to P and the other end is attached to B

The particle P rests in equilibrium at the point O, where AOB is a straight line, as shown in Figure 4.

(a) Show that $OA = 2.5$ m.

(4)

The particle P now receives an impulse of magnitude 6 N s in the direction OB

(b) (i) Show that P initially moves with simple harmonic motion with centre O

(ii) Determine the amplitude of this simple harmonic motion.

(8)

The point C lies on OB. As P passes through C the string attached to B becomes slack.

(c) Find the speed of P as it passes through C

(2)

(d) Find the time taken for P to travel directly from O to C

(3)

(a) Let OA = x m. Extension of AP = x − 2, extension of BP = 7 − x − 3 = 4 − x

Equilibrium: $T_A = T_B \Rightarrow \frac{4(x-2)}{2} = \frac{2(7-x-3)}{3}$

$6(x-2) = 2(4-x)$

$6x - 12 = 8 - 2x \Rightarrow 8x = 20$

$x = 2.5$, so OA = 2.5 m

(b)(i) Extensions at O: AP: 0.5 m, BP: 1.5 m.

Let y m be the displacement of P from O, measured towards B.

$T_A = \frac{4(0.5+y)}{2}$, $T_B = \frac{2(1.5-y)}{3}$

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Question 7 continued

N2L (towards B): $2\ddot{y} = \frac{2(1.5 - y)}{3} - \frac{4(0.5 + y)}{2}$

$2\ddot{y} = 1 - \frac{2y}{3} - 1 - 2y = -\frac{8y}{3}$

$\ddot{y} = -\frac{4}{3}y$, which is of the form $\ddot{y} = -\omega^2 y$

$\therefore$ P moves with SHM with centre O (while both strings are taut).

(ii) $\omega^2 = \frac{4}{3}$, $\omega = \frac{2}{\sqrt{3}}$

Impulse: $2v_{max} = 6 \Rightarrow v_{max} = 3 \text{ m s}^{-1}$

$v_{max} = a\omega \Rightarrow 3 = \frac{2a}{\sqrt{3}}$

Amplitude $a = \frac{3\sqrt{3}}{2}$ m $(= 2.60 \text{ m})$

(c) String BP becomes slack when its extension is zero, i.e. $y = 1.5$

$v^2 = \omega^2(a^2 - y^2) = \frac{4}{3}\left(\frac{27}{4} - 1.5^2\right) = \frac{4}{3} \times \frac{9}{2} = 6$

Speed at C: $v = \sqrt{6} = 2.449\ldots = 2.45 \text{ m s}^{-1}$

(d) $y = a \sin \omega t$: $\frac{3}{2} = \frac{3\sqrt{3}}{2} \sin\left(\frac{2t}{\sqrt{3}}\right)$

$\sin\left(\frac{2t}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} \Rightarrow \frac{2t}{\sqrt{3}} = 0.61547\ldots$

$t = \frac{\sqrt{3}}{2} \times 0.61547\ldots = 0.53301\ldots$

Time from O to C = 0.533 s (3 s.f.)

Question 7 continued

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Question 7 continued

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TOTAL FOR PAPER IS 75 MARKS

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