Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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1. A motorcyclist and his motorcycle have a combined mass of 480 kg.

The motorcyclist drives down a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{12}$, with the engine of the motorcycle working at 3.5 kW. The motorcycle is moving at a constant speed of $V\,\text{m s}^{-1}$. The resistance to the motion of the motorcycle is modelled as a constant force with magnitude $20V$ newtons.

Find the value of V.

(5)

Driving force $F = \frac{P}{v} = \frac{3500}{V}$

Down the slope, constant speed so resultant force = 0:

$F + 480g \sin\theta - 20V = 0$

$\frac{3500}{V} + 480 \times 9.8 \times \frac{1}{12} - 20V = 0$

$\frac{3500}{V} + 392 - 20V = 0$

$20V^2 - 392V - 3500 = 0$

$5V^2 - 98V - 875 = 0$

$V = \frac{98 + \sqrt{98^2 + 4 \times 5 \times 875}}{10} = 26.322...$ (reject negative root)

$V = 26.3$

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Question 1 continued

Q1

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2. A particle P of mass 1.5 kg moves under the action of a single force **F** newtons.

At time t seconds, $t \geqslant 0$, P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = (5t^2 - t^3)\mathbf{i} + (2t^3 - 8t)\mathbf{j}$$

(a) Find **F** when $t = 2$

(4)

At time $t = 0$, P is at the origin O.

(b) Find the position vector of P relative to O at the instant when P is moving in the direction of the vector **j**

(4)

(a) $a = \frac{dv}{dt} = (10t - 3t^2)i + (6t^2 - 8)j$

$t = 2$: $a = (20 - 12)i + (24 - 8)j = 8i + 16j$

$F = ma = 1.5(8i + 16j)$

$F = (12i + 24j)$ N

(b) Moving in direction of j when i component of v = 0 ($t > 0$):

$5t^2 - t^3 = 0 \Rightarrow t^2(5 - t) = 0 \Rightarrow t = 5$

$\int v\,dt = \left\langle \frac{5}{3}t^3 - \frac{1}{4}t^4 \right\rangle i + \left\langle \frac{1}{2}t^4 - 4t^2 \right\rangle j + c$

$t = 0, r = 0 \Rightarrow c = 0$

$t = 5$: $r = \left\langle \frac{625}{3} - \frac{625}{4} \right\rangle i + \left\langle \frac{625}{2} - 100 \right\rangle j$

$r = \left\langle \frac{625}{12} i + \frac{425}{2} j \right\rangle$ m $(= 52.1i + 212.5j)$

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Question 2 continued

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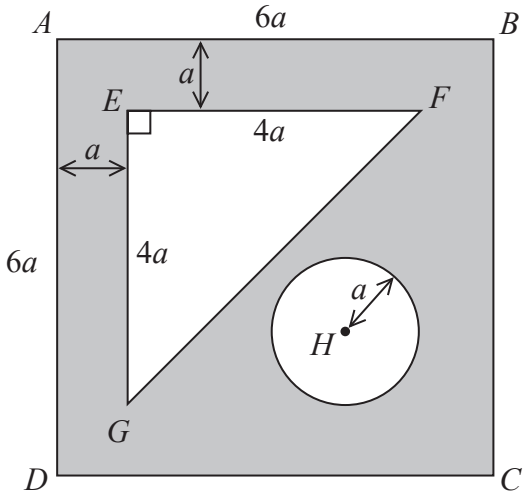


Figure 1

The uniform lamina $ABCD$ is a square of side $6a$. The template T, shown shaded in Figure 1, is formed by removing the right-angled triangle EFG and the circle, centre H and radius a, from the square lamina.

Triangle EFG has $EF = EG = 4a$, with EF parallel to AB and EG parallel to AD. The distance between AB and EF is a and the distance between AD and EG is a.

The point H lies on AC and the distance of H from BC is $2a$.

(a) Show that the centre of mass of T is a distance $\dfrac{4(67 - 3\pi)}{3(28 - \pi)}a$ from AD.

(5)

The template T is suspended from the ceiling by two light inextensible vertical strings. One string is attached to T at A and the other string is attached to T at B so that T hangs in equilibrium with AB horizontal.

The weight of T is W. The tension in the string attached to T at B is kW, where k is a constant.

(b) Find the value of k, giving your answer to 2 decimal places.

(3)

(a)

	square	triangle EFG	circle	T
mass ratio	$36a^2$	$8a^2$	πa^2	$(28 - \pi)a^2$
dist. from AD	$3a$	$\frac{7}{3}a\ (a + \frac{4a}{3})$	$4a$	d

M(AD):

$$36 \times 3a - 8 \times \frac{7}{3}a - \pi \times 4a = (28 - \pi)d$$

$$108a - \frac{56}{3}a - 4\pi a = (28 - \pi)d$$

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Question 3 continued

$d = \frac{324 - 56 - 12\pi}{3(28 - \pi)}a = \frac{268 - 12\pi}{3(28 - \pi)}a$

$d = \frac{4(67 - 3\pi)}{3(28 - \pi)}a$ as required

(b) M(A): $W \times d = kW \times 6a$

$\frac{4(67 - 3\pi)}{3(28 - \pi)}a = 6ak$

$k = \frac{4(67 - 3\pi)}{18(28 - \pi)} = \frac{3.0883...}{6} = 0.51473...$

$k = 0.51$

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Question 3 continued

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Question 3 continued

Q3

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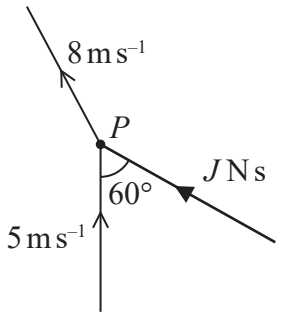


Figure 2

A particle P of mass 0.3 kg is moving with speed 5 m s^{-1} along a straight line on a smooth horizontal plane. The particle receives a horizontal impulse of magnitude J N s. The speed of P immediately after receiving the impulse is 8 m s^{-1}. The angle between the direction of motion of P before it receives the impulse and the direction of the impulse is 60°, as shown in Figure 2.

Find the value of J.

(6)

Initial momentum = $0.3 \times 5 = 1.5$ N s; final momentum = $0.3 \times 8 = 2.4$ N s

Let final velocity make angle α with the original direction. $I = mv - mu$:

Parallel to original motion: $2.4 \cos \alpha = 1.5 + J \cos 60°$

Perpendicular to original motion: $2.4 \sin \alpha = J \sin 60°$

Square and add:

$$2.4^2 = \left(1.5 + \frac{J}{2}\right)^2 + \left(\frac{\sqrt{3}J}{2}\right)^2$$

$$5.76 = 2.25 + 1.5J + \frac{J^2}{4} + \frac{3J^2}{4}$$

$$J^2 + 1.5J - 3.51 = 0$$

$$J = \frac{-1.5 + \sqrt{1.5^2 + 4 \times 3.51}}{2} = 1.2680... \quad (J > 0)$$

$$J = 1.27$$

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Question 4 continued

Q4

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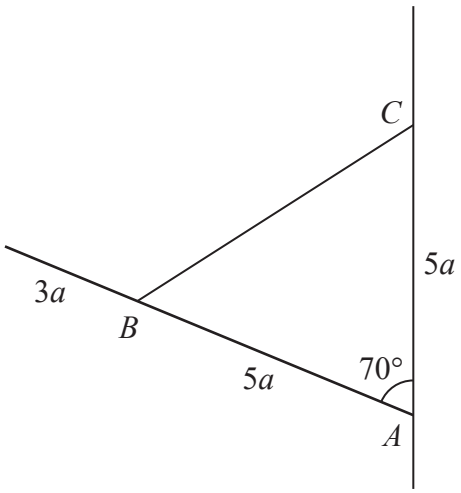


Figure 3

A uniform rod, of length $8a$ and mass M, has one end freely hinged to a fixed point A on a vertical wall. One end of a light inextensible string is attached to the rod at the point B, where $AB = 5a$. The other end of the string is attached to the wall at the point C, where $AC = 5a$ and C is vertically above A. The rod rests in equilibrium in a vertical plane perpendicular to the wall with angle $BAC = 70°$, as shown in Figure 3.

(a) Find, in terms of M and g, the tension in the string.

(3)

The magnitude of the force acting on the rod at A is λMg, where λ is a constant.

(b) Find, to 2 significant figures, the value of λ.

(6)

(a) Triangle ABC is isosceles (AB = AC = 5a), so ∠ABC = ∠ACB = 55°

M(A):

$T \times 5a \sin 55° = Mg \times 4a \sin 70°$ $(= Mg \times 4a \cos 20°)$

$T = \dfrac{4 \cos 20°}{5 \sin 55°} Mg = 0.91772...Mg$

$T = 0.918Mg$

(b) String makes 55° with the vertical (∠ACB = 55°).

Let the force at A have components V (up) and H (horizontal).

R(↑): $V + T \cos 55° = Mg$

$V = Mg - 0.91772...Mg \cos 55° = 0.47361...Mg$

R(→): $H = T \sin 55° = 0.91772...Mg \sin 55° = 0.75175...Mg$

Magnitude $= \sqrt{V^2 + H^2} = Mg\sqrt{0.47361...^2 + 0.75175...^2}$

$= 0.88849...Mg$

$\lambda = 0.89$

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Question 5 continued

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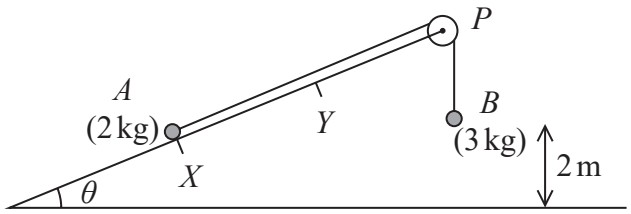


Figure 4

Two particles, A and B, of mass 2 kg and 3 kg respectively, are connected by a light inextensible string. Particle A is held at rest at the point X on a fixed rough ramp that is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The string passes over a small smooth pulley P that is fixed at the top of the ramp. Particle B hangs vertically below P, 2 m above the ground, as shown in Figure 4.

The particles are released from rest with the string taut so that A moves up the ramp and the section of the string from A to P is parallel to a line of greatest slope of the ramp. The coefficient of friction between A and the ramp is $\frac{3}{8}$

Air resistance is ignored.

(a) Find the potential energy lost by the system as A moves 2 m up the ramp.

(3)

(b) Find the work done against friction as A moves 2 m up the ramp.

(4)

When B hits the ground, B is brought to rest by the impact and does not rebound and A continues to move up the ramp.

(c) Use the work-energy principle to find the speed of B at the instant before it hits the ground.

(4)

Particle A comes to instantaneous rest at the point Y on the ramp, where $XY = (2 + d)$ m.

(d) Use the work-energy principle to find the value of d.

(4)

(a) $\tan\theta = \frac{5}{12} \Rightarrow \sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

PE lost by $B = 3g \times 2$; PE gained by $A = 2g \times 2\sin\theta$

PE lost $= 6g - 4g \times \frac{5}{13} = \frac{58g}{13}$

$= 43.7$ J

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Question 6 continued

(b) $R = 2g\cos\theta = \frac{24g}{13}$

$F = \mu R = \frac{3}{8} \times \frac{24g}{13} = \frac{9g}{13}$

Work done against friction $= F \times 2 = \frac{18g}{13}$

$= 13.6\ J$

(c) Work-energy: KE gained = PE lost − work done against friction

$\frac{1}{2}(2+3)v^2 = \frac{58g}{13} - \frac{18g}{13} = \frac{40g}{13}$

$v^2 = \frac{16g}{13} \Rightarrow v = \sqrt{(16 \times 9.8) \div 13} = 3.4729...$

speed of B $= 3.47\ m\ s^{-1}$

(d) After B lands, A moves a further d m up the ramp. Work-energy for A:

KE lost = PE gained + work done against friction

$\frac{1}{2} \times 2 \times \frac{16g}{13} = 2g \times d\sin\theta + \frac{9g}{13} \times d$

$\frac{16g}{13} = \frac{10g}{13}d + \frac{9g}{13}d$

$16 = 19d$

$d = \frac{16}{19} = 0.842$

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Question 6 continued

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Question 6 continued

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7. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

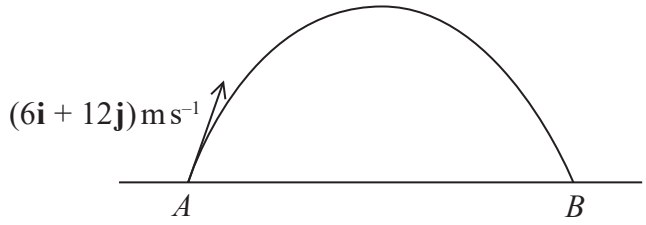


Figure 5

A small ball is projected with velocity (6**i** + 12**j**) m s^{-1} from a fixed point A on horizontal ground. The ball hits the ground at the point B, as shown in Figure 5. The motion of the ball is modelled as a particle moving freely under gravity.

(a) Find the distance AB.

(4)

When the height of the ball above the ground is more than h metres, the speed of the ball is less than 10 m s^{-1}

(b) Find the smallest possible value of h.

(4)

When the ball is at the point C on its path, the direction of motion of the ball is perpendicular to the direction of motion of the ball at the instant before it hits the ground at B.

(c) Find, in terms of **i** and **j**, the velocity of the ball when it is at C.

(3)

(a) (↑) $v = u + at$: $-12 = 12 - 9.8t$

$t = \frac{24}{9.8} = 2.4489...$ s

(→) $AB = 6t = 6 \times 2.4489...$

$AB = 14.693... = 14.7$ m

(b) Speed 10 m s^{-1}: $6^2 + v_y^2 = 10^2 \Rightarrow v_y = \pm 8$

Speed < 10 when $|v_y| < 8$. Height when $v_y = \pm 8$:

(↑) $v^2 = u^2 + 2as$: $8^2 = 12^2 - 2 \times 9.8 \times h$

$h = \frac{144 - 64}{19.6} = 4.0816...$

smallest $h = 4.08$

(c) Velocity just before B = $6i - 12j$ (by symmetry)

Let velocity at C = $6i + vj$. Perpendicular ⇒ scalar product = 0:

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Question 7 continued

$\begin{pmatrix} 6 \\ -12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ v \end{pmatrix} = 36 - 12v = 0 \Rightarrow v = 3$

velocity at C = $(6\mathbf{i} + 3\mathbf{j})$ m s^{-1}

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Question 7 continued

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Question 7 continued

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8. Particles A, B and C, of masses $2m$, m and $3m$ respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particle A is projected towards particle B with speed $2u$ and collides directly with B.

The coefficient of restitution between each pair of particles is e.

(a) (i) Show that the speed of B immediately after the collision with A is $\frac{4}{3}u(1+e)$

(ii) Find the speed of A immediately after the collision with B.

(7)

At the instant when A collides with B, particle C is projected with speed u towards B so that B and C collide directly.

(b) Show that there will be a second collision between A and B.

(6)

(a)(i) Let A have velocity v and B velocity w after the collision (→).

CLM: $2m(2u) = 2mv + mw \Rightarrow 4u = 2v + w$

NEL: $w - v = e(2u) \Rightarrow v = w - 2ue$

$4u = 2(w - 2ue) + w \Rightarrow 3w = 4u + 4ue$

$w = \frac{4}{3}u(1+e)$ as required

(a)(ii) $v = w - 2ue = \frac{4u + 4ue - 6ue}{3}$

speed of A $= \frac{2}{3}u(2-e)$

(b) $e \leqslant 1 < 2$, so $v = \frac{2}{3}u(2-e) > 0$: A keeps moving towards C.

B (speed w, →) collides with C (speed u, ←). After: B has x, C has y (→).

CLM: $mw - 3mu = mx + 3my \Rightarrow x + 3y = \frac{4u}{3} + \frac{4ue}{3} - 3u$

NEL: $y - x = e(w+u) \Rightarrow y - x = eu + \frac{4u}{3}e + \frac{4u}{3}e^2$

i.e. $x + 3y = \frac{4ue}{3} - \frac{5u}{3}$, $3y - 3x = 7ue + 4ue^2$

Subtract: $4x = \frac{4ue}{3} - \frac{5u}{3} - 7ue - 4ue^2$

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Question 8 continued

$x = -\frac{5}{12}u - \frac{17}{12}ue - ue^2$

$u > 0, e > 0 \Rightarrow x < 0$, so B moves back towards A after hitting C.

A is moving towards C and B is moving towards A, so they move towards each other: there will be a second collision between A and B.

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Question 8 continued

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Question 8 continued

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TOTAL FOR PAPER: 75 MARKS

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