Please check the examination details below before entering your candidate information

Candidate surname: StemUp Model Solution | Other names:

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Monday 24 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1. A company produces NuttyChok bars that are made of chocolate and whole hazelnuts.

A 40 kg batch of the chocolate and hazelnut mixture used to make NuttyChok bars will contain 4750 hazelnuts.

(a) Show that the mean number of hazelnuts in an 80 g bar of NuttyChok is 9.5

(1)

A Poisson model is suggested for the number of hazelnuts in each 80 g bar of NuttyChok.

(b) State two assumptions that need to be made for a Poisson model to be suitable in this case.

(2)

(c) Find the probability that an 80 g bar of NuttyChok

(i) contains exactly 12 hazelnuts,

(ii) contains fewer than 8 hazelnuts.

(4)

(d) Find the probability that a pack of three 80 g bars of NuttyChok includes exactly one bar with fewer than 8 hazelnuts.

(3)

The company also produces a 200 g bar of NuttyChok using the same chocolate and hazelnut mixture.

(e) Use a suitable approximation to find the probability that a 200 g bar of NuttyChok contains at least 30 hazelnuts.

(5)

(a) $\text{Mean} = 4750 \times \frac{80}{40\,000} = 4750 \times 0.002 = 9.5$

(b) Hazelnuts occur singly and randomly (independently) in the mixture, and the mean number of hazelnuts per bar is constant (hazelnuts occur at a constant rate as the mixture is well mixed).

(c)(i) X = number of hazelnuts in an 80 g bar, $X \sim Po(9.5)$

$P(X = 12) = \frac{e^{-9.5} \times 9.5^{12}}{12!} = 0.084440... = 0.0844$

(ii) $P(X < 8) = P(X \leqslant 7)$

$= 0.26866... = 0.269$

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Question 1 continued

(d) R = number of bars with fewer than 8 hazelnuts, $R \sim B(3, 0.2687)$

$P(R = 1) = 3 \times 0.2687 \times (1 - 0.2687)^2$

$= 3 \times 0.2687 \times 0.7313^2$

$= 0.43110... = 0.431$

(e) Y = number in a 200 g bar: $\lambda = 9.5 \times \frac{200}{80} = 23.75$, $Y \sim Po(23.75)$

λ is large, so approximately $Y \sim N(23.75, 23.75)$

$P(Y \geqslant 30) \approx P\left(Z > \frac{29.5 - 23.75}{\sqrt{23.75}}\right) = P\left(Z > \frac{5.75}{4.8734...}\right)$

$= P(Z > 1.1798...) = P(Z > 1.18)$

$= 1 - 0.8810$

$= 0.1190 = 0.119$



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Question 1 continued

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Question 1 continued

(Total 15 marks)

Q1

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2. The random variable X has probability distribution

x	-1	0	1
$P(X = x)$	0.2	0.4	0.4

Samples of size 2 are selected at random from the distribution and the random variable T is the sum of the 2 values of X.

(a) Show that $P(T = 0) = 0.32$

(2)

(b) Find the sampling distribution of T.

(4)

(a) $T = 0$ from $(0, 0)$, $(-1, 1)$ or $(1, -1)$

$P(T = 0) = 0.4^2 + 2(0.2 \times 0.4)$

$= 0.16 + 0.16 = 0.32$

(b) $T = -2$: $(-1, -1)$ $0.2^2 = 0.04$

$T = -1$: $(-1, 0)$, $(0, -1)$ $2 \times 0.2 \times 0.4 = 0.16$

$T = 0$: 0.32

$T = 1$: $(0, 1)$, $(1, 0)$ $2 \times 0.4 \times 0.4 = 0.32$

$T = 2$: $(1, 1)$ $0.4^2 = 0.16$

t	-2	-1	0	1	2
$P(T = t)$	0.04	0.16	0.32	0.32	0.16

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Question 2 continued

Q2

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3. The continuous random variable Y is such that $Y \sim U[a, b]$, where a and b are constants.

The mean of Y is 6 and the variance of Y is 3

(a) Find the value of a and the value of b.

(3)

(b) Find the probability that Y is more than 1 standard deviation above the mean.

(2)

(c) Define fully the cumulative distribution function of Y.

(2)

(a) $\frac{a+b}{2} = 6 \Rightarrow a + b = 12$

$\frac{1}{12}(b-a)^2 = 3 \Rightarrow (b-a)^2 = 36 \Rightarrow b - a = 6$

Adding: $2b = 18 \Rightarrow a = 3, \quad b = 9$

(b) $\sigma = \sqrt{3}$, so find $P(Y > 6 + \sqrt{3})$

$P(Y > 6 + \sqrt{3}) = \frac{9 - (6 + \sqrt{3})}{9 - 3} = \frac{3 - \sqrt{3}}{6}$

$= 0.21132\ldots = 0.211$

(c)
$$F(y) = \begin{cases} 0 & y < 3 \\ \frac{y-3}{6} & 3 \leq y \leq 9 \\ 1 & y > 9 \end{cases}$$

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Question 3 continued

Q3

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4. A new supermarket opens with its entrance on a busy road.

In the 4 years before the supermarket opened there were 10 traffic accidents on the road. In the 12 months after opening there were 6 traffic accidents on the road.

The supermarket manager claims that there has been no change in the rate of traffic accidents on the road since the supermarket opened.

(a) Test the manager's claim at the 5% level of significance. State your hypotheses clearly.

(5)

A local resident claims that the rate of traffic accidents on the road has increased.

(b) Explain how a hypothesis test, with a 5% level of significance, to assess the resident's claim would be different from your hypothesis test in part (a). State the conclusion you would reach.

(3)

(a) X = number of accidents in 12 months. Before: 10 in 4 years

$\Rightarrow$ rate of 2.5 per year

$H_0 : \lambda = 2.5$ $\quad$ $H_1 : \lambda \neq 2.5$

Under H_0, $X \sim Po(2.5)$

$P(X \geqslant 6) = 1 - P(X \leqslant 5) = 1 - 0.9580$

$= 0.0420$

Two-tailed test: $0.0420 > 0.025$, so not significant;

do not reject H_0.

There is no evidence of a change in the rate of accidents:

the manager's claim is supported (not rejected).

(b) The test would be one-tailed: $H_0 : \lambda = 2.5$, $H_1 : \lambda > 2.5$

so compare $P(X \geqslant 6) = 0.0420$ with 5% (not 2.5%).

$0.0420 < 0.05$, so the result is significant; reject H_0.

There is evidence that the rate of accidents has increased:

the resident's claim is supported.

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Question 4 continued

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Question 4 continued

Q4

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5. A continuous random variable X has probability density function

$$f(x) = \begin{cases} k\{(x-2)^2 + 1\} & 0 \leqslant x \leqslant a \\ 0 & \text{otherwise} \end{cases}$$

where a and b are constants.
Given that the mode of X is 0

(a) find the range of possible values for a.

(2)

Given that $a = 3$

(b) (i) sketch the probability density function for all values of x.

(ii) Give the coordinates of the turning point.

(2)

(c) Use algebraic integration to show that $k = \frac{1}{6}$

(4)

Given that $E(X^2) = 2.1$

(d) use algebraic integration to find $\text{Var}(X)$.

(6)

(e) Show that $P(2 < X < 3) = \frac{2}{9}$

(2)

(f) State, giving a reason, whether the median of X is less than or greater than 1

(3)

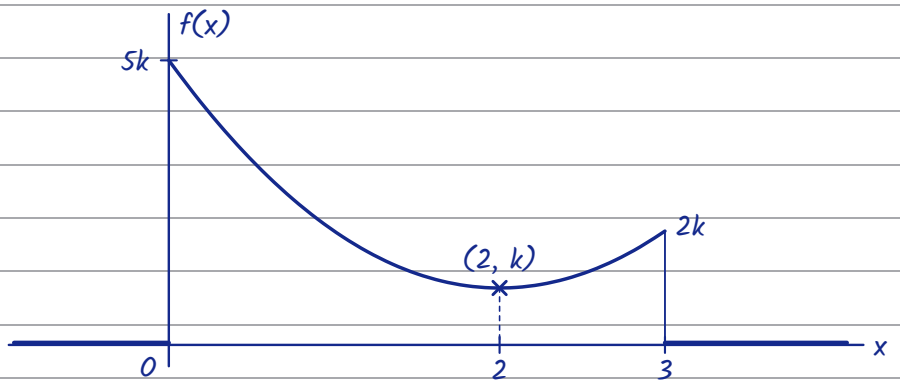
(a) Mode is 0, so $f(0) > f(a)$: $f(0) = k(4 + 1) = 5k$

$k\{(a-2)^2 + 1\} < 5k$

$(a-2)^2 < 4 \Rightarrow -2 < a - 2 < 2$

$0 < a < 4$

(b) (i)



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Question 5 continued

(ii) Turning point: $(2, k) = (2, \frac{1}{6})$

(c) $\int_0^3 k(x^2 - 4x + 5)\,dx = 1$

$k\left[\frac{x^3}{3} - 2x^2 + 5x\right]_0^3 = 1$

$k\left(\frac{27}{3} - 18 + 15 - 0\right) = 1$

$6k = 1 \quad \Rightarrow \quad k = \frac{1}{6}$

(d) $E(X) = \int_0^3 x \times \frac{1}{6}(x^2 - 4x + 5)\,dx = \frac{1}{6}\int_0^3 (x^3 - 4x^2 + 5x)\,dx$

$= \frac{1}{6}\left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{5x^2}{2}\right]_0^3$

$= \frac{1}{6}\left(\frac{81}{4} - 36 + \frac{45}{2} - 0\right) = \frac{1}{6} \times \frac{27}{4} = \frac{9}{8}$

$\text{Var}(X) = E(X^2) - [E(X)]^2 = 2.1 - \left(\frac{9}{8}\right)^2$

$= \frac{267}{320} \quad (= 0.834375)$

(e) $P(2 < X < 3) = \frac{1}{6}\left[\frac{x^3}{3} - 2x^2 + 5x\right]_2^3$

$= \frac{1}{6}\left[(9 - 18 + 15) - \left(\frac{8}{3} - 8 + 10\right)\right]$

$= \frac{1}{6}\left(6 - \frac{14}{3}\right) = \frac{1}{6} \times \frac{4}{3} = \frac{2}{9}$

(f) $P(X < 1) = \frac{1}{6}\left[\frac{x^3}{3} - 2x^2 + 5x\right]_0^1 = \frac{1}{6}\left(\frac{1}{3} - 2 + 5\right)$

$= \frac{1}{6} \times \frac{10}{3} = \frac{5}{9}$ and $\frac{5}{9} > 0.5$ $\left(P(X > 1) = \frac{4}{9} < 0.5\right)$

so $F(1) > 0.5$, therefore the median of X is less than 1.

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Question 5 continued

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Question 5 continued

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Q5

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6. Sarah makes clay pots in batches of size n. The clay pots need to be fired in a kiln. Sarah finds that 30% of the clay pots crack when they are fired. The remainder do not crack.

(a) Explain what assumptions need to be made to model with a binomial distribution the number of clay pots in a batch that crack when fired.

(2)

Sarah makes and fires a batch of 8 clay pots.

(b) Write down the expected number of clay pots that crack.

(1)

(c) Find the probability that exactly 2 clay pots crack.

(2)

The probability that at least k clay pots crack is 0.0113

(d) Find the value of k.

(2)

Sarah attends a course to improve her pottery skills. She claims that the course will reduce the proportion of clay pots that crack when fired.

After completing the course Sarah makes a batch of 20 clay pots to assess her claim.

(e) Stating your hypotheses clearly, determine the critical region for the hypothesis test she must use to assess her claim using a 5% significance level.

(3)

(f) Write down the actual significance level of this test.

(1)

(a) The probability that a pot cracks is constant (0.3) for each pot, and pots crack independently of each other. (The batch size n is fixed.)

(b) $8 \times 0.3 = 2.4$

(c) X = number of pots that crack, $X \sim B(8, 0.3)$

$P(X = 2) = {}^8C_2 \times 0.3^2 \times 0.7^6$

$= 0.296475... = 0.296$

(d) $P(X \geqslant k) = 0.0113 \Rightarrow P(X \leqslant k - 1) = 0.9887$

From tables $P(X \leqslant 5) = 0.9887$, so $k - 1 = 5$

$k = 6$

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Question 6 continued

(e) $H_0 : p = 0.3$ $H_1 : p < 0.3$

Under H_0, $Y \sim B(20, 0.3)$

$P(Y \leqslant 2) = 0.0355 < 0.05$, $P(Y \leqslant 3) = 0.1071 > 0.05$

Critical region: $Y \leqslant 2$

(f) Actual significance level = $P(Y \leqslant 2) = 0.0355$ (3.55%)

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Question 6 continued

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Question 6 continued

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7. A continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{3}{4}(x-1) & 1 \leqslant x < 2 \\ \frac{3}{32}x(x-4)^2 & 2 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the cumulative distribution function for all values of x.

(7)

(b) Verify that the median of X is 2.17 to 2 decimal places.

(2)

(a) For $1 \leq x < 2$:

$\int \frac{3}{4}(x-1)\,dx = \frac{3x^2}{8} - \frac{3x}{4} + c$

$F(1) = 0$: $\frac{3}{8} - \frac{3}{4} + c = 0 \Rightarrow c = \frac{3}{8}$

For $2 \leq x \leq 4$:

$\int \frac{3}{32}x(x-4)^2\,dx = \frac{3}{32}\int (x^3 - 8x^2 + 16x)\,dx$

$= \frac{3x^4}{128} - \frac{x^3}{4} + \frac{3x^2}{4} + d$

$F(4) = 1$: $\frac{3 \times 256}{128} - 16 + 12 + d = 1 \Rightarrow 2 + d = 1 \Rightarrow d = -1$

(Check: $F(2) = \frac{48}{128} - 2 + 3 - 1 = \frac{3}{8}$, which agrees with the first part.)

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{3x^2}{8} - \frac{3x}{4} + \frac{3}{8} & 1 \leq x < 2 \\ \frac{3x^4}{128} - \frac{x^3}{4} + \frac{3x^2}{4} - 1 & 2 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

(b) Median m satisfies $F(m) = 0.5$, using $2 \leq x \leq 4$:

$F(2.165) = 0.4933\ldots < 0.5$

$F(2.175) = 0.5001\ldots > 0.5$

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Question 7 continued

$F(2.165) < 0.5 < F(2.175)$, so $2.165 < m < 2.175$

$\therefore \ m = 2.17$ (to 2 decimal places)

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Question 7 continued

Q7

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TOTAL FOR PAPER: 75 MARKS

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