Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Thursday 20 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P55876A

©2019 Pearson Education Ltd.

1/1/1/1/1/

P 5 5 8 7 6 A 0 1 2 8

Pearson

stemup.co.uk

Leave blank

Answer ALL questions. Write your answers in the spaces provided.

1. A truck of mass 800 kg is moving on a straight road that is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{10}$. When the truck is moving up the road at a constant speed of 12 m s^{-1}, the engine of the truck is working at a constant rate of 15 kW. The resistance to the motion of the truck from non-gravitational forces is modelled as a constant force of magnitude R newtons.

(a) Find the value of R.

(4)

The truck now moves down the same road. The resistance to the motion of the truck is now modelled as a constant force of magnitude 500 N. The engine of the truck is again working at a constant rate of 15 kW.

(b) Find the acceleration of the truck at the instant when it is moving at 12 m s^{-1}.

(3)

(a) Driving force: $P = Fv \Rightarrow F = \frac{15000}{12} = 1250$ N

Constant speed up the road, so resultant force = 0:

$F - R - 800g \sin\alpha = 0$

$1250 - R - 800 \times 9.8 \times \frac{1}{10} = 0$

$R = 1250 - 80g = 1250 - 784 = 466$

$R = 466$ N

(b) Driving force $F = \frac{15000}{12} = 1250$ N (same power and speed)

Down the road, N2L: $F + 800g \sin\alpha - 500 = 800a$

$1250 + 784 - 500 = 800a$

$800a = 1534$

$a = 1.9175 = 1.92$ m s^{-2}

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 1 continued

Leave blank

Question 1 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 1 continued

Q1

(Total 7 marks)

stemup.co.uk

Leave blank

2. A particle P moves along the x-axis. At time t seconds, the acceleration of P is $a\,\text{m s}^{-2}$ in the positive x direction, where

$$a = 8 - 6t \quad t \geqslant 0$$

When $t = 0$, P is at the origin O and is moving with speed $3\,\text{m s}^{-1}$ in the positive x direction.

Find

(i) the distance of P from O at the instant when P is instantaneously at rest,

(ii) the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(10)

$v = \int (8 - 6t)\,dt = 8t - 3t^2 + C$

$t = 0,\ v = 3 \quad \Rightarrow \quad C = 3, \quad$ so $\quad v = 3 + 8t - 3t^2$

$s = \int (3 + 8t - 3t^2)\,dt = 3t + 4t^2 - t^3 \quad (s = 0 \text{ when } t = 0)$

(i) At rest when $v = 0$: $\quad 3t^2 - 8t - 3 = 0$

$(3t + 1)(t - 3) = 0 \quad \Rightarrow \quad t = 3 \quad (t \geqslant 0)$

$s(3) = 3(3) + 4(9) - 27 = 9 + 36 - 27 = 18$

Distance from $O = 18$ m

(ii) P moves in the positive direction for $0 \leqslant t < 3$ and in the negative direction for $3 < t \leqslant 4$.

$s(4) = 3(4) + 4(16) - 64 = 12 + 64 - 64 = 12$

Total distance $= s(3) + (s(3) - s(4))$

$= 18 + (18 - 12)$

$= 24$ m

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

P 5 5 8 7 6 A 0 6 2 8
stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

(Total 10 marks)

Q2

Leave blank

3. A particle P of mass 0.4 kg is moving with velocity $u\mathbf{i}$ m s^{-1}, where u is a positive constant. The particle receives an impulse $(3\mathbf{i} + 6\mathbf{j})$ N s.

Immediately after receiving the impulse, the speed of P is $2u$ m s^{-1}.

Find the value of u.

(5)

$I = mv - mu \Rightarrow (3i + 6j) = 0.4v - 0.4ui$

Final momentum $0.4v = (0.4u + 3)i + 6j$

Final speed $2u$, so $|0.4v| = 0.4 \times 2u = 0.8u$:

$(0.4u + 3)^2 + 6^2 = (0.8u)^2$

$0.16u^2 + 2.4u + 9 + 36 = 0.64u^2$

$0.48u^2 - 2.4u - 45 = 0$

$4u^2 - 20u - 375 = 0$

$(2u - 25)(2u + 15) = 0$

$u = \frac{25}{2} \quad (u > 0)$

$u = 12.5$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 5 5 8 7 6 A 0 8 2 8

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Q3

(Total 5 marks)

stemup.co.uk

Leave blank

4.

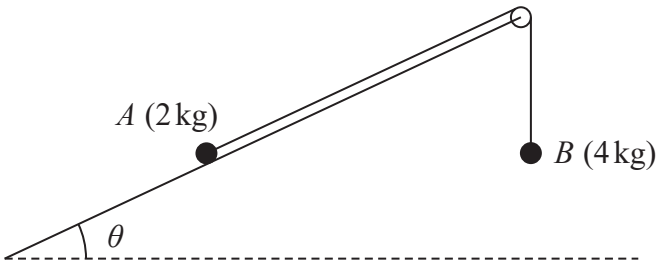


Figure 1

A particle A of mass 2 kg is attached to one end of a light inextensible string. A particle B of mass 4 kg is attached to the other end of the string. The string passes over a small smooth pulley. The pulley is fixed at the top of a fixed rough plane, which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{3}{4}$. Initially the particles are held at rest with A on the plane, B hanging freely below the pulley and the string taut, as shown in Figure 1. The part of the string from A to the pulley lies along a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{5}$.

At time $t = 0$, the particles are released from rest, with A more than 1.5 m from the pulley and B more than 1.5 m above the ground.

At time $t = T$ seconds, the speed of B is $v\,\text{m s}^{-1}$ and B is 1.5 m below its initial position.

(a) Find the total potential energy lost by the system in the interval $0 \leqslant t \leqslant T$.

(3)

(b) Find the work done against friction in the interval $0 \leqslant t \leqslant T$.

(3)

(c) Use the work-energy principle to find the value of v.

(3)

$\tan\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}$

(a) B falls 1.5 m; A moves 1.5 m up the plane, rising 1.5 sin θ

PE lost $= 4g \times 1.5 - 2g \times 1.5 \times \frac{3}{5}$

$= 6g - \frac{9g}{5} = \frac{21g}{5}$

$= 41.16 = 41.2$ J

(b) Perpendicular to plane: $R = 2g\cos\theta = 2g \times 0.8 = 1.6g$

Friction $F = \mu R = \frac{1}{5} \times 1.6g = 0.32g$

P 5 5 8 7 6 A 0 1 0 2 8

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Work done against friction = $0.32g \times 1.5 = \frac{12g}{25}$

$= 4.704 = 4.70$ J

(c) Work-energy: PE lost = KE gained + work done against friction

$\frac{21g}{5} - \frac{12g}{25} = \frac{1}{2} \times 4 \times v^2 + \frac{1}{2} \times 2 \times v^2$

$\frac{93g}{25} = 3v^2$

$v^2 = \frac{31g}{25} = 12.152$

$v = 3.4859... = 3.49$

stemup.co.uk

Leave blank

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Q4

(Total 9 marks)

stemup.co.uk

Leave blank

5.

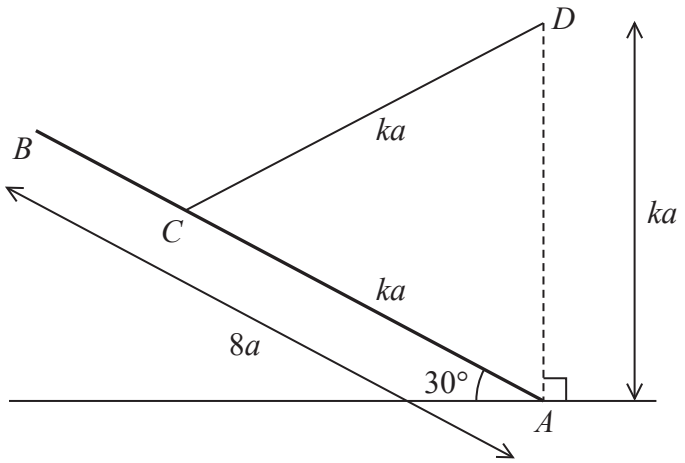


Figure 2

A uniform rod AB, of weight W and length $8a$, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at 30° to the horizontal by a light inextensible string of length ka, where k is a constant. One end of the string is attached to the rod at C, where $AC = ka$. The other end of the string is attached to the fixed point D which is vertically above A such that $AD = ka$, as shown in Figure 2. The string lies in the vertical plane which contains the rod.

The coefficient of friction between the rod and the ground is $\frac{\sqrt{3}}{2}$.

(a) Show that the tension in the string is $\frac{4W}{k}$.

(2)

(b) Find the value of k.

(6)

The magnitude of the force exerted on the rod by the ground at A is λW.

(c) Find the value of λ.

(3)

(a) String CD: triangle ACD is isosceles (AC = AD = ka), angle CAD = 60°, so ACD is equilateral and DC makes 60° with the rod: T at 60° to AB, perpendicular component of T on the rod = T sin 60° = T cos 30°.

Moments about A: $T \cos 30° \times ka = W \times 4a \cos 30°$

$T \times ka = 4aW \Rightarrow T = \frac{4W}{k}$ as required

(b) T acts along CD at 30° above the horizontal. At A: normal reaction R, friction F (limiting, $F = \mu R$) opposing the horizontal component of T.

Resolving vertically (↑): $R + T \cos 60° = W$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

$R = W - \frac{T}{2} = W - \frac{2W}{k} = W\left(1 - \frac{2}{k}\right)$

Resolving horizontally (↔): $F = T\cos 30°$

$F = \frac{4W}{k} \times \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}W}{k}$

Limiting: $F = \mu R \Rightarrow \frac{2\sqrt{3}W}{k} = \frac{\sqrt{3}}{2} W\left(1 - \frac{2}{k}\right)$

$\frac{4}{k} = 1 - \frac{2}{k} \Rightarrow \frac{6}{k} = 1$

$k = 6$

(c) $R = W\left(1 - \frac{2}{6}\right) = \frac{2W}{3}$, $F = \frac{2\sqrt{3}W}{6} = \frac{\sqrt{3}W}{3}$

Magnitude of force at A $= \sqrt{(R^2 + F^2)}$

$= W\sqrt{\left(\frac{4}{9} + \frac{3}{9}\right)} = \frac{\sqrt{7}}{3} W$

$\lambda = \frac{\sqrt{7}}{3}$ $(= 0.882)$

stemup.co.uk

Leave blank

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

(Total 11 marks)

Q5

P 5 5 8 7 6 A 0 1 7 2 8
stemup.co.uk

Leave blank

6. A particle P of mass m is moving in a straight line with speed v on a smooth horizontal surface. The particle P collides directly with a particle Q of mass km which is moving with speed w, $(w < v)$, along the same straight line and in the same direction as P. The direction of motion of P is unchanged by the collision and, immediately after the collision, the speed of P is w and the speed of Q is $2w$.

The coefficient of restitution between P and Q is $\frac{2}{3}$.

(a) Find the value of k.

(6)

When P and Q collide they are at the point A, which is a distance d from a smooth fixed vertical wall. The wall is perpendicular to the direction of motion of the particles. After the collision with P, particle Q hits the wall and rebounds towards P.

The coefficient of restitution between Q and the wall is $\frac{1}{3}$.

There is a second direct collision between P and Q at the point B.

(b) Find, in terms of d and w, the time taken for P to travel from A to B.

(5)

(a) Before: P (m) speed v →, Q (km) speed w →

After: P speed w →, Q speed 2w →

CLM: $mv + kmw = mw + 2kmw$

$v + kw = w + 2kw \Rightarrow v = w + kw$ (1)

NEL: $2w - w = \frac{2}{3}(v - w)$

$w = \frac{2}{3}v - \frac{2}{3}w \Rightarrow \frac{2}{3}v = \frac{5}{3}w \Rightarrow v = \frac{5}{2}w$

Sub in (1): $\frac{5}{2}w = w + kw \Rightarrow kw = \frac{3}{2}w$

$k = \frac{3}{2}$

(b) Speed of Q after hitting the wall $= \frac{1}{3} \times 2w = \frac{2w}{3}$

Time for Q to reach the wall $= \frac{d}{2w}$

Let t = time from A to B for P. P travels wt, and Q travels

$\frac{2w}{3}\left(t - \frac{d}{2w}\right)$ back from the wall, and together these make d:

P 5 5 8 7 6 A 0 1 8 2 8

stemup.co.uk

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

$$wt + \frac{2w}{3}\left(t - \frac{d}{2w}\right) = d$$

$$wt + \frac{2wt}{3} - \frac{d}{3} = d$$

$$\frac{5wt}{3} = \frac{4d}{3}$$

$$t = \frac{4d}{5w}$$

Leave blank

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

(Total 11 marks)

Q6

stemup.co.uk

Leave blank

7.

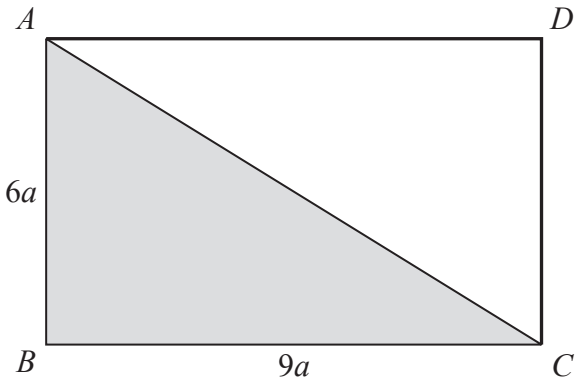


Figure 3

The model of the design for a pendant consists of a uniform triangular lamina, ABC, and two uniform rods AD and DC, as shown in Figure 3. The rods are in the same plane as the lamina. The lamina ABC has mass $8M$, sides $AB = 6a$ and $BC = 9a$, and angle $ABC = 90°$. The rod AD has mass $2M$ and length $9a$. The rod DC has mass kM and length $6a$.

(a) Show that the centre of mass of the model is $\left(\frac{33 + 9k}{10 + k}\right)a$ from AB.

(4)

The model is suspended from A and hangs freely in equilibrium with AC vertical.

(b) Find the value of k.

(7)

(a) Total mass = $8M + 2M + kM = (10 + k)M$

Distances from AB: lamina $3a$ (centroid at $\frac{1}{3} \times 9a$), AD $4.5a$, DC $9a$

Moments about AB: $(10 + k)M\,\bar{x} = 8M \times 3a + 2M \times \frac{9a}{2} + kM \times 9a$

$(10 + k)\,\bar{x} = 24a + 9a + 9ka = (33 + 9k)a$

$\bar{x} = \left(\frac{33 + 9k}{10 + k}\right)a$ as required

(b) Distances from AD: lamina $4a$ ($\frac{2}{3} \times 6a$), AD 0, DC $3a$

Moments about AD: $(10 + k)M\,\bar{y} = 8M \times 4a + 2M \times 0 + kM \times 3a$

$\bar{y} = \left(\frac{32 + 3k}{10 + k}\right)a$ from AD

Hanging from A with AC vertical, so G lies on AC.

G is $\bar{x}$ from AB and $\bar{y}$ from AD; on AC these satisfy

$\frac{\bar{x}}{\bar{y}} = \frac{BC}{AB} = \frac{9a}{6a} = \frac{3}{2}$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

$$\frac{33 + 9k}{32 + 3k} = \frac{3}{2}$$

$$2(33 + 9k) = 3(32 + 3k)$$

$$66 + 18k = 96 + 9k$$

$$9k = 30$$

$$k = \frac{10}{3}$$

Leave blank

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

(Total 11 marks)

Q7

stemup.co.uk

Leave blank

8.

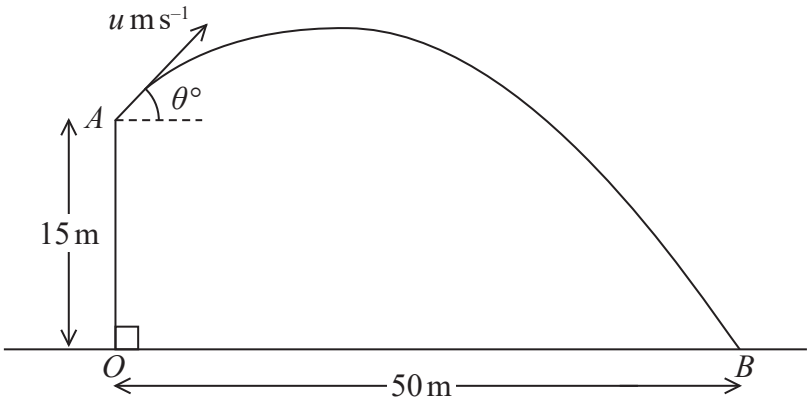


Figure 4

A small ball is thrown from the point A with speed $u\,\text{m s}^{-1}$ at an angle $\theta°$ above the horizontal. The point A is vertically above the point O, which is on horizontal ground, such that AO is 15 m.

The ball takes 3 seconds to travel from A to B, where B is on the ground and $OB = 50$ m, as shown in Figure 4. By modelling the motion of the ball as that of a particle moving freely under gravity,

find

(a) (i) the value of θ,

(ii) the value of u,

(6)

(b) the speed of the ball as it hits the ground at B,

(3)

(c) the direction of motion of the ball as it hits the ground at B.

(2)

(a) Horizontal (→): $3u\cos\theta = 50 \Rightarrow u\cos\theta = \frac{50}{3} = 16.666...$

Vertical (↑), $s = ut + \frac{1}{2}at^2$: $-15 = u\sin\theta \times 3 - \frac{1}{2} \times 9.8 \times 3^2$

$3u\sin\theta = 44.1 - 15 = 29.1 \Rightarrow u\sin\theta = 9.7$

$\tan\theta = \frac{u\sin\theta}{u\cos\theta} = \frac{29.1}{50} = 0.582$

(i) $\theta = 30.198... = 30.2$

$u = \sqrt{(16.666...^2 + 9.7^2)} = \sqrt{371.867...} = 19.283...$

(ii) $u = 19.3$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

P 5 5 8 7 6 A 0 2 6 2 8
stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

(b) Conservation of energy from A to B:

$\frac{1}{2}mv^2 = \frac{1}{2}m \times 19.283...^2 + m \times 9.8 \times 15$

$v^2 = 371.867... + 294 = 665.867...$

$v = 25.804... = 25.8 \text{ m s}^{-1}$

(c) At B: horizontal velocity $= \frac{50}{3} = 16.666...$ $(\rightarrow)$

vertical velocity $= 9.7 - 9.8 \times 3 = -19.7$, i.e. 19.7 downwards

Angle below horizontal $= \tan^{-1}\left(\frac{19.7}{16.666...}\right) = 49.766...°$

Direction: 49.8° below the horizontal (to the ground)

Leave blank

Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 5 5 8 7 6 A 0 2 8 2 8

stemup.co.uk