Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

1. A shop sells shoes at a mean rate of 4 pairs of shoes per hour on a weekday.

(a) Suggest a suitable distribution for modelling the number of sales of pairs of shoes made per hour on a weekday.

(1)

(b) State one assumption necessary for this distribution to be a suitable model of this situation.

(1)

(c) Find the probability that on a weekday the shop sells

(i) more than 4 pairs of shoes in a one-hour period,

(ii) more than 4 pairs of shoes in each of 3 consecutive one-hour periods.

(4)

The area manager visits the shop on a weekday, the day after an advert for the shop appears in a local paper.

In a one-hour period during the manager's visit, the shop sells 7 pairs of shoes.
This leads the manager to believe that the advert has increased the shop's sales of pairs of shoes.

(d) Stating your hypotheses clearly, test at the 5% level of significance whether or not there is evidence of an increase in sales of pairs of shoes following the appearance of the advert.

(5)

(a) Poisson distribution, Po(4)

(b) Pairs of shoes are sold independently / at a constant (average) rate

(c)(i) X = number of sales per hour, $X \sim Po(4)$

$P(X > 4) = 1 - P(X \leq 4) = 1 - 0.6288$

$= 0.3712 \approx 0.371$

(ii) P(more than 4 in each of 3 hours) $= (0.3712)^3$

$= 0.051147... \approx 0.0511$

(d) $H_0 : \lambda = 4$ $H_1 : \lambda > 4$

Under H_0, $X \sim Po(4)$

$P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.8893$

$= 0.1107$

$0.1107 > 0.05$ so not significant: do not reject H_0

There is insufficient evidence of an increase in sales of pairs of shoes following the appearance of the advert, so the manager's belief is not supported.

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Question 1 continued

(Total for Question 1 is 11 marks)

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2. A bag contains a large number of coins. It only contains 20p and 50p coins. A random sample of 3 coins is taken from the bag.

(a) List all the possible combinations of 3 coins that might be taken.

(2)

Let $\bar{X}$ represent the mean value of the 3 coins taken.

Part of the sampling distribution of $\bar{X}$ is given below.

$\bar{x}$	20	a	b	50
$P(\bar{X} = \bar{x})$	$\frac{4913}{8000}$	c	d	$\frac{27}{8000}$

(b) Write down the value of a and the value of b

(1)

The probability of taking a 20p coin at random from the bag is p

The probability of taking a 50p coin at random from the bag is q

(c) Find the value of p and the value of q

(2)

(d) Hence, find the value of c and the value of d

(3)

Let M represent the mode of the 3 coins taken at random from the bag.

(e) Find the sampling distribution of M

(3)

(a) 20, 20, 20 20, 20, 50 20, 50, 50 50, 50, 50

(b) $a = 30$, $b = 40$

(c) $p^3 = \frac{4913}{8000} \Rightarrow p = \frac{17}{20} = 0.85$

$q = 1 - p = \frac{3}{20} = 0.15$ (check: $q^3 = \frac{27}{8000}$)

(d) c = P(two 20p, one 50p) $= 3 \times p^2 \times q$

$c = 3 \times \left(\frac{17}{20}\right)^2 \times \frac{3}{20} = \frac{2601}{8000}$ $(= 0.325125)$

d = P(one 20p, two 50p) $= 3 \times p \times q^2$

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Question 2 continued

$d = 3 \times \frac{17}{20} \times \left(\frac{3}{20}\right)^2 = \frac{459}{8000}$ $(= 0.057375)$

(e) $M = 20$ if at least two coins are 20p, $M = 50$ otherwise

$P(M = 20) = \frac{4913}{8000} + \frac{2601}{8000} = \frac{7514}{8000} = \frac{3757}{4000}$

$P(M = 50) = \frac{27}{8000} + \frac{459}{8000} = \frac{486}{8000} = \frac{243}{4000}$

m	20	50
$P(M = m)$	$\frac{3757}{4000}$	$\frac{243}{4000}$

Question 2 continued

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Question 2 continued

(Total for Question 2 is 11 marks)

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3. *Superbounce* is a manufacturer of tennis balls.

It knows from past records that 10% of its tennis balls fail a bounce test.

(a) Find the probability that from a random sample of 10 of these tennis balls

(i) at least 4 fail the bounce test

(ii) more than 1 but fewer than 5 fail the bounce test.

(4)

The managing director makes changes to the production process and claims that these changes will reduce the probability of its tennis balls failing the bounce test.

After the changes were made a random sample of 50 of the tennis balls were tested and it was found that 2 failed the bounce test.

(b) Test, at the 5% significance level, whether or not this result supports the managing director's claim.

(4)

In a second random sample of n tennis balls it was found that none failed the bounce test. As a result of this sample, the managing director's claim is supported at the 1% significance level.

(c) Find the smallest possible value of n

(3)

(a)(i) X = number failing, $X \sim B(10, 0.1)$

$P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - 0.9872$

$= 0.0128$

(ii) $P(1 < X < 5) = P(X \leqslant 4) - P(X \leqslant 1)$

$= 0.9984 - 0.7361$

$= 0.2623 \approx 0.262$

(b) $H_0 : p = 0.1$ $H_1 : p < 0.1$

Under H_0, $Y \sim B(50, 0.1)$

$P(Y \leqslant 2) = 0.1117$

$0.1117 > 0.05$ so not significant: do not reject H_0

There is insufficient evidence to support the managing director's claim of a reduction in the probability of a tennis ball failing the bounce test.

(c) $W \sim B(n, 0.1)$. Reject H_0 if $P(W = 0) < 0.01$

$P(W = 0) = {}^nC_0 \times 0.1^0 \times 0.9^n = 0.9^n < 0.01$

$0.9^{43} = 0.01077... > 0.01$

$0.9^{44} = 0.00969... < 0.01$

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Question 3 continued

(or $n > \frac{\ln 0.01}{\ln 0.9} = 43.7...$)

Smallest $n = 44$

Question 3 continued

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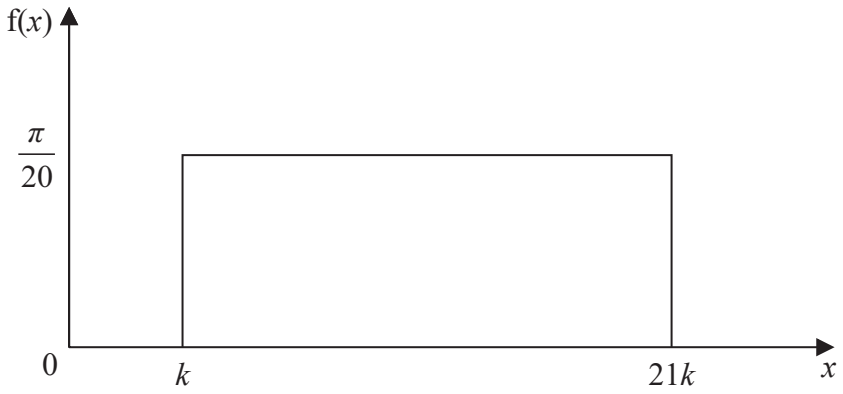
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Question 3 continued

(Total for Question 3 is 11 marks)

4. The continuous random variable X has probability density function f(x), shown in the diagram, where k is a constant.



(a) Find $P(X < 10k)$

(1)

(b) Show that $k = \frac{1}{\pi}$

(2)

(c) Find, in terms of π, the values of

(i) E(X)

(ii) Var(X)

(3)

Circles are drawn with area A, where

$$A = \pi\left(X + \frac{2}{\pi}\right)^2$$

(d) Find E(A)

(4)

(a) $P(X < 10k) = \frac{10k - k}{21k - k} = \frac{9k}{20k} = \frac{9}{20}$

(b) Total area = 1: $(21k - k) \times \frac{\pi}{20} = 1$

$20k \times \frac{\pi}{20} = 1 \Rightarrow k\pi = 1 \Rightarrow k = \frac{1}{\pi}$ as required

(c)(i) $E(X) = \frac{k + 21k}{2} = 11k = \frac{11}{\pi}$

(ii) $Var(X) = \frac{(21k - k)^2}{12} = \frac{400k^2}{12} = \frac{100}{3\pi^2}$

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Question 4 continued

(d) $A = \pi\left(X^2 + \frac{4X}{\pi} + \frac{4}{\pi^2}\right) = \pi X^2 + 4X + \frac{4}{\pi}$

$E(A) = \pi E(X^2) + 4E(X) + \frac{4}{\pi}$

$E(X^2) = Var(X) + [E(X)]^2 = \frac{100}{3\pi^2} + \frac{121}{\pi^2} = \frac{463}{3\pi^2}$

$E(A) = \pi \times \frac{463}{3\pi^2} + 4 \times \frac{11}{\pi} + \frac{4}{\pi} = \frac{463}{3\pi} + \frac{48}{\pi}$

$= \frac{607}{3\pi} = 64.4047\ldots \approx 64.4$

Question 4 continued

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Question 4 continued

(Total for Question 4 is 10 marks)

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5. A company produces steel cable.

Defects in the steel cable produced by this company occur at random, at a constant rate of 1 defect per 16 metres.

On one day the company produces a piece of steel cable 80 metres long.

(a) Find the probability that there are at most 5 defects in this piece of steel cable.

(2)

The company produces a piece of steel cable 80 metres long on each of the next 4 days.

(b) Find the probability that fewer than 2 of these 4 pieces of steel cable contain at most 5 defects.

(4)

The following week the company produces a piece of steel cable x metres long.

Using a normal approximation, the probability that this piece of steel cable has fewer than 26 defects is 0.5398

(c) Find the value of x

(8)

(a) X = number of defects in 80 m, $\lambda = 80 \div 16 = 5$, $X \sim Po(5)$

$P(X \leq 5) = 0.6160$

(b) Y = number of the 4 pieces with at most 5 defects

$Y \sim B(4, 0.616)$

$P(Y < 2) = P(Y \leq 1) = P(Y = 0) + P(Y = 1)$

$= 0.384^4 + 4 \times 0.616 \times 0.384^3$

$= 0.021743... + 0.139519...$

$= 0.161262... \approx 0.161$

(c) D = number of defects in x metres, $D \sim Po\left(\frac{x}{16}\right)$

Approximate by $D \approx N\left(\frac{x}{16}, \frac{x}{16}\right)$

$P(D < 26) \approx P\left(Z < \frac{25.5 - x/16}{\sqrt{x/16}}\right) = 0.5398$

From tables $\Phi(0.1) = 0.5398$, so $z = 0.1$

$\frac{25.5 - x/16}{\sqrt{x}/4} = 0.1 \Rightarrow 25.5 - \frac{x}{16} = \frac{\sqrt{x}}{40}$

$\frac{1}{16}x + \frac{1}{40}\sqrt{x} - 25.5 = 0 \quad (\times 80)$

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Question 5 continued

$5x + 2\sqrt{x} - 2040 = 0$ (quadratic in $\sqrt{x}$)

$\sqrt{x} = \dfrac{-2 \pm \sqrt{4 + 40800}}{10} = \dfrac{-2 \pm 202}{10}$

$\sqrt{x} = 20$ (or $\sqrt{x} = -20.4$, rejected since $\sqrt{x} > 0$)

$x = 20^2 = 400$

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 14 marks)

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6. The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ ax + bx^2 & 0 \leqslant x \leqslant k \\ 1 & x > k \end{cases}$$

where a, b and k are positive constants.

(a) Show that $ak = 1 - bk^2$

(1)

Using part (a) and given that $E(X) = \frac{6}{5}$

(b) show that $5bk^3 = 36 - 15k$

(6)

Using part (a) and given that $E(X) = \frac{6}{5}$ and $Var(X) = \frac{22}{75}$

(c) show that $5bk^4 = 52 - 10k^2$

(5)

Given that $k < 3$

(d) find the value of k

(4)

(e) Hence find the value of a and the value of b

(2)

(a) $F(k) = 1 \Rightarrow ak + bk^2 = 1 \Rightarrow ak = 1 - bk^2$ as required

(b) $f(x) = \frac{d}{dx} F(x) = a + 2bx$ for $0 \leqslant x \leqslant k$

$E(X) = \int_0^k x(a + 2bx)\,dx = \int_0^k (ax + 2bx^2)\,dx = \frac{6}{5}$

$\left[\frac{ax^2}{2} + \frac{2bx^3}{3}\right]_0^k = \frac{ak^2}{2} + \frac{2bk^3}{3} = \frac{6}{5}$

$(\times 30) \quad 15ak^2 + 20bk^3 = 36$

$ak^2 = k(ak) = k(1 - bk^2) = k - bk^3$

$15(k - bk^3) + 20bk^3 = 36$

$15k - 15bk^3 + 20bk^3 = 36$

$5bk^3 = 36 - 15k$ as required

(c) $E(X^2) = \int_0^k (ax^2 + 2bx^3)\,dx = \left[\frac{ax^3}{3} + \frac{bx^4}{2}\right]_0^k = \frac{ak^3}{3} + \frac{bk^4}{2}$

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Question 6 continued

$\text{Var}(X) = \frac{ak^3}{3} + \frac{bk^4}{2} - \left(\frac{6}{5}\right)^2 = \frac{22}{75}$

$\frac{ak^3}{3} + \frac{bk^4}{2} = \frac{22}{75} + \frac{36}{25} = \frac{130}{75} = \frac{26}{15}$

$(\times 30) \quad 10ak^3 + 15bk^4 = 52$

$ak^3 = k^2(ak) = k^2(1 - bk^2) = k^2 - bk^4$

$10k^2 - 10bk^4 + 15bk^4 = 52$

$5bk^4 = 52 - 10k^2$ as required

(d) Dividing: $\frac{5bk^4}{5bk^3} = k = \frac{52 - 10k^2}{36 - 15k}$

$36k - 15k^2 = 52 - 10k^2$

$5k^2 - 36k + 52 = 0$

$(k - 2)(5k - 26) = 0$

$k = 2$ or $k = 5.2$; since $k < 3$, $k = 2$

(e) $5b(2)^3 = 36 - 15(2) \Rightarrow 40b = 6 \Rightarrow b = \frac{3}{20}$

$2a = 1 - b(2)^2 = 1 - \frac{12}{20} = \frac{2}{5} \Rightarrow a = \frac{1}{5}$

$a = \frac{1}{5}, \quad b = \frac{3}{20}$

Question 6 continued

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Question 6 continued

Question 6 continued

(Total for Question 6 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS

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