Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

1. $$f(x) = \frac{5x + 10}{(1 - x)(2 + 3x)}$$

(a) Write f(x) in partial fraction form.

(3)

(b) (i) Hence find, in ascending powers of x up to and including the terms in x^2, the binomial series expansion of f(x). Give each coefficient as a simplified fraction.

(5)

(ii) Find the range of values of x for which this expansion is valid.

(1)

(a) $\frac{5x + 10}{(1 - x)(2 + 3x)} \equiv \frac{A}{1 - x} + \frac{B}{2 + 3x} \Rightarrow 5x + 10 \equiv A(2 + 3x) + B(1 - x)$

$x = 1 : 15 = 5A \Rightarrow A = 3$

$x = -\frac{2}{3} : \frac{20}{3} = \frac{5}{3}B \Rightarrow B = 4$

$f(x) = \frac{3}{1 - x} + \frac{4}{2 + 3x}$

(b)(i) $\frac{3}{1 - x} = 3(1 - x)^{-1} = 3(1 + x + x^2 + \ldots)$

$\frac{4}{2 + 3x} = 4(2 + 3x)^{-1} = \frac{4}{2}\left(1 + \frac{3x}{2}\right)^{-1}$

$= 2\left(1 + (-1)\left(\frac{3x}{2}\right) + \frac{(-1)(-2)}{2!}\left(\frac{3x}{2}\right)^2 + \ldots\right)$

$= 2\left(1 - \frac{3x}{2} + \frac{9x^2}{4} + \ldots\right) = 2 - 3x + \frac{9}{2}x^2 + \ldots$

$f(x) = 3 + 3x + 3x^2 + 2 - 3x + \frac{9}{2}x^2 + \ldots$

$f(x) = 5 + \frac{15}{2}x^2 + \ldots$

(ii) Valid for $|x| < 1$ and $\left|\frac{3x}{2}\right| < 1$

$|x| < \frac{2}{3}$

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 9 marks)

2. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

(a) $y = \frac{6}{2t+1} \Rightarrow 2t + 1 = \frac{6}{y} \Rightarrow t = \frac{6-y}{2y}$

$x = \frac{t-1}{2t+1} = (t-1) \times \frac{y}{6}$ and $t - 1 = \frac{6-y-2y}{2y} = \frac{6-3y}{2y}$

$x = \frac{6-3y}{2y} \times \frac{y}{6} = \frac{6-3y}{12} = \frac{1}{2} - \frac{y}{4}$

$4x = 2 - y \Rightarrow y = 2 - 4x = 2(1 - 2x)$

This is of the form $y = mx + c$, so all points P lie on a straight line.

(b) $2(1 - 2x) = x + 12 \Rightarrow 2 - 4x = x + 12$

$-5x = 10 \Rightarrow x = -2$

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Question 2 continued

(Total for Question 2 is 6 marks)

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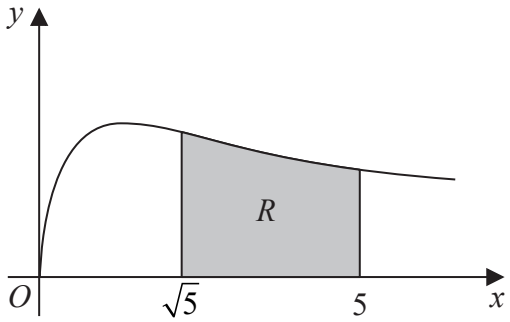


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region R is rotated through 360° about the x-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(5)

$V = \pi \int_{\sqrt{5}}^{5} \left(\frac{\sqrt{(3x)}}{\sqrt{(3x^2 + 5)}} \right)^2 dx = \pi \int_{\sqrt{5}}^{5} \frac{3x}{3x^2 + 5} dx$

$= \pi \left[\frac{1}{2} \ln(3x^2 + 5) \right]_{\sqrt{5}}^{5}$

$= \pi \left(\frac{1}{2} \ln(3 \times 25 + 5) - \frac{1}{2} \ln(3 \times 5 + 5) \right)$

$= \frac{\pi}{2} (\ln 80 - \ln 20) = \frac{\pi}{2} \ln 4$

$= \pi \ln 2$

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Question 3 continued

(Total for Question 3 is 5 marks)

4. (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}}\, dx$$

may be expressed in the form

$$\int_a^b ku^2 e^u\, du$$

where a, b and k are constants to be found.

(4)

(b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}}\, dx$$

giving your answer in simplest form.

(5)

(a) $u = \sqrt{(2x+1)} \Rightarrow u^2 = 2x+1 \Rightarrow 2u\frac{du}{dx} = 2 \Rightarrow dx = u\,du$

$\sqrt{(8x+4)} = \sqrt{(4(2x+1))} = 2\sqrt{(2x+1)} = 2u$

$x = 4 : u = \sqrt{9} = 3$; $x = 12 : u = \sqrt{25} = 5$

$\int_4^{12} \sqrt{(8x+4)}\, e^{\sqrt{(2x+1)}}\, dx = \int_3^5 2u\, e^u \times u\, du = \int_3^5 2u^2 e^u\, du$

$a = 3$, $b = 5$, $k = 2$

(b) $\int 2u^2 e^u\, du = 2u^2 e^u - \int 4u e^u\, du$

$= 2u^2 e^u - \left(4u e^u - \int 4e^u\, du\right)$

$= 2u^2 e^u - 4u e^u + 4e^u$

$\int_3^5 2u^2 e^u\, du = \left[2u^2 e^u - 4u e^u + 4e^u\right]_3^5$

$= (50e^5 - 20e^5 + 4e^5) - (18e^3 - 12e^3 + 4e^3)$

$= 34e^5 - 10e^3$

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

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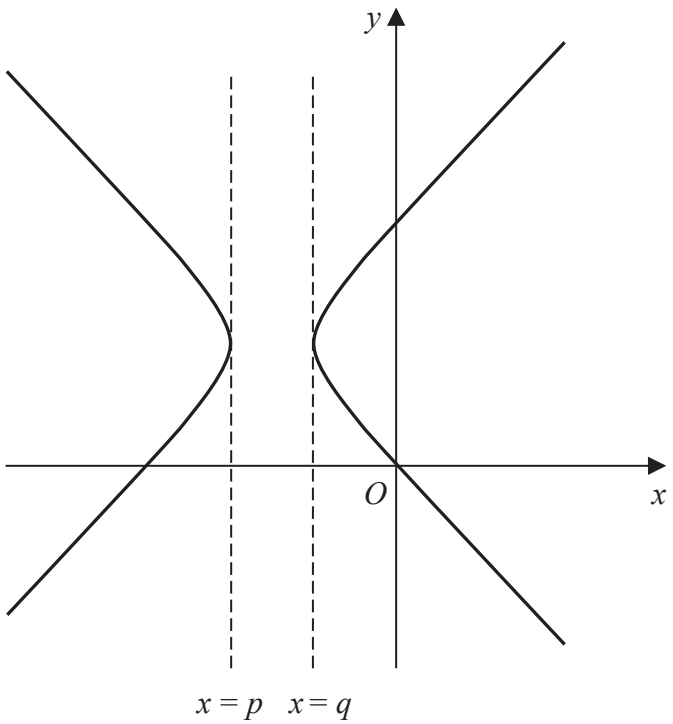


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

The curve is not defined for values of x in the interval (p, q), as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of p and the value of q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

(a) $2y\frac{dy}{dx} = 4x + 15 + 10\frac{dy}{dx}$

$(2y - 10)\frac{dy}{dx} = 4x + 15$

$\frac{dy}{dx} = \frac{4x + 15}{2y - 10}$

(b) Vertical tangents at $x = p$ and $x = q$, so $\frac{dy}{dx}$ is undefined:

$2y - 10 = 0 \Rightarrow y = 5$

$25 = 2x^2 + 15x + 50 \Rightarrow 2x^2 + 15x + 25 = 0$

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Question 5 continued

$(2x + 5)(x + 5) = 0 \Rightarrow x = -5$ or $x = -5/2$

$p = -5, \quad q = -\frac{5}{2}$

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 7 marks)

6. Relative to a fixed origin O.

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

(a)(i) $AB = OB - OA = (8i + 3j - 7k) - (2i - 3j + 5k)$

$AB = 6i + 6j - 12k$

(ii) $r = (2i - 3j + 5k) + \lambda(i + j - 2k)$

(b) $OP = (2 + \lambda)i + (-3 + \lambda)j + (5 - 2\lambda)k$

$CP = OP - OC = (\lambda - 1)i + (\lambda - 8)j + (3 - 2\lambda)k$

$CP \cdot (i + j - 2k) = 0$

$(\lambda - 1) + (\lambda - 8) - 2(3 - 2\lambda) = 0$

$6\lambda - 15 = 0 \quad \Rightarrow \quad \lambda = \frac{5}{2}$

$OP = (2 + \frac{5}{2})i + (-3 + \frac{5}{2})j + (5 - 5)k$

$OP = \frac{9}{2}i - \frac{1}{2}j$

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

7. The volume $V\,\text{cm}^3$ of a spherical balloon with radius $r\,\text{cm}$ is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\dfrac{dV}{dr}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

(5)

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

(a) $\dfrac{dV}{dr} = 4\pi r^2$

(b)(i) $V = \displaystyle\int \frac{900}{(2t+3)^2}\,dt = \int 900(2t+3)^{-2}\,dt$

$V = -\dfrac{450}{2t+3} + c$

$t = 0,\ V = 0: \quad 0 = -\dfrac{450}{3} + c \quad \Rightarrow \quad c = 150$

$V = 150 - \dfrac{450}{2t+3} = \dfrac{150(2t+3) - 450}{2t+3} = \dfrac{300t + 450 - 450}{2t+3}$

$V = \dfrac{300t}{2t+3}$ as required

(ii) As $t \to \infty$, $V = \dfrac{300}{2 + 3/t} \to \dfrac{300}{2} = 150\ \text{cm}^3$ (upper limit)

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Question 7 continued

(c) $t = 3$: $V = \frac{300 \times 3}{2 \times 3 + 3} = \frac{900}{9} = 100$

$100 = \frac{4}{3}\pi r^3 \Rightarrow r^3 = \frac{75}{\pi} = 23.873...$

$r = 2.8794... = 2.88 \text{ cm}$

(d) $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{900}{(2t+3)^2} = 4\pi r^2 \times \frac{dr}{dt}$

$t = 3$: $\frac{900}{81} = 4\pi(2.8794...)^2 \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{11.111...}{104.187...} = 0.10664...$

$\frac{dr}{dt} = 0.11 \text{ cm s}^{-1}$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

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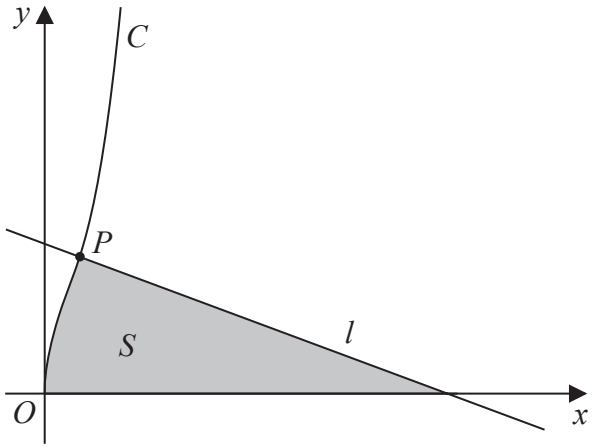


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The line l is the normal to C at P, as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17$$

(5)

The region S, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Find, using calculus, the exact area of S.

(6)

(a) $t = \frac{\pi}{4}$: $x = \sin^2 \frac{\pi}{4} = \frac{1}{2}$, $y = 2\tan\frac{\pi}{4} = 2$; $P\left(\frac{1}{2}, 2\right)$

$\frac{dx}{dt} = 2\sin t \cos t$, $\frac{dy}{dt} = 2\sec^2 t$

$\frac{dy}{dx} = \frac{2\sec^2 t}{2\sin t \cos t}$; at $t = \frac{\pi}{4}$: $\frac{dy}{dx} = \frac{2 \times 2}{2 \times (\sqrt{2}/2)(\sqrt{2}/2)} = \frac{4}{1} = 4$

Gradient of normal $= -\frac{1}{4}$

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Question 8 continued

$y - 2 = -\frac{1}{4}\left(x - \frac{1}{2}\right) \Rightarrow 8y - 16 = -2x + 1$

$8y + 2x = 17$ as required

(b) Area under C from $x = 0$ to $x = \frac{1}{2}$: $\int y \frac{dx}{dt}\, dt$

$= \int_0^{\pi/4} 2 \tan t \times 2 \sin t \cos t \, dt = \int_0^{\pi/4} 4 \sin^2 t \, dt$

$= \int_0^{\pi/4} (2 - 2 \cos 2t) \, dt = \left[2t - \sin 2t\right]_0^{\pi/4}$

$= \left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right) - (0 - 0) = \frac{\pi}{2} - 1$

l meets x-axis when $y = 0$: $2x = 17 \Rightarrow x = \frac{17}{2}$

Triangle area $= \frac{1}{2} \times \left(\frac{17}{2} - \frac{1}{2}\right) \times 2 = \frac{1}{2} \times 8 \times 2 = 8$

Area of S $= \frac{\pi}{2} - 1 + 8 = \frac{\pi}{2} + 7$

Question 8 continued

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Question 8 continued

(Total for Question 8 is 11 marks)

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9. A student was asked to prove, for $p \in \mathbb{N}$, that

"if p^3 is a multiple of 3, then p must be a multiple of 3"

The start of the student's proof by contradiction is shown in the box below.

> Assumption:
>
> There exists a number p, $p \in \mathbb{N}$, such that p^3 is a multiple of 3, and p is NOT a multiple of 3
>
> Let $p = 3k + 1$, $k \in \mathbb{N}$.
>
> Consider $p^3 = (3k + 1)^3 = 27k^3 + 27k^2 + 9k + 1$
>
> $= 3(9k^3 + 9k^2 + 3k) + 1$ which is not a multiple of 3

(a) Show the calculations and statements that are required to complete the proof.

(3)

(b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number.

(5)

(a) Let $p = 3k + 2$, $k \in \mathbb{N}$

$p^3 = (3k + 2)^3 = 27k^3 + 54k^2 + 36k + 8$

$= 3(9k^3 + 18k^2 + 12k + 3) - 1$ which is not a multiple of 3

So in both cases ($p = 3k + 1$ and $p = 3k + 2$) p^3 is not a multiple of 3. This contradicts the assumption that p^3 is a multiple of 3.

Hence if p^3 is a multiple of 3, then p must be a multiple of 3.

(b) Assume $\sqrt[3]{3}$ is rational, so $\sqrt[3]{3} = \frac{p}{q}$ where p, q are integers with no common factors (fraction in its simplest form).

$\sqrt[3]{3} = \frac{p}{q} \Rightarrow 3 = \frac{p^3}{q^3} \Rightarrow p^3 = 3q^3$

So p^3 is a multiple of 3 and so, by (a), p is a multiple of 3.

Let $p = 3m$: $(3m)^3 = 3q^3 \Rightarrow 27m^3 = 3q^3 \Rightarrow q^3 = 9m^3 = 3(3m^3)$

So q^3 is a multiple of 3 and so, by (a), q is a multiple of 3.

Then p and q have a common factor 3, which contradicts the assumption that p and q have no common factors.

Hence $\sqrt[3]{3}$ is an irrational number.

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Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

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