Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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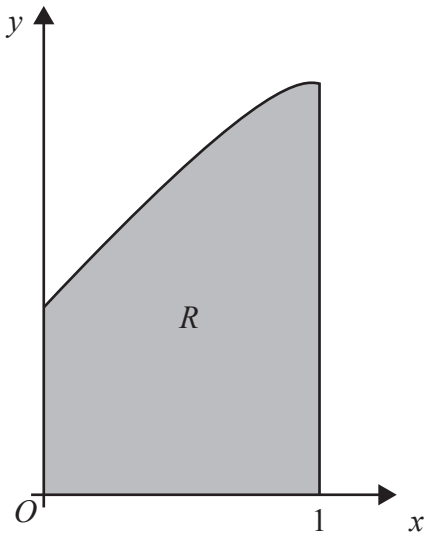


Figure 1

The shaded region R is bounded by the x-axis, the line with equation $x = 1$, the curve with equation $y = 1 + \sqrt{x}$ and the y-axis, as shown in Figure 1. The unit of length on both of the axes is 1 m.

The region R is rotated through 2π radians about the x-axis to form a solid of revolution which is used to model a uniform solid S.

Show, using the model and **algebraic integration**, that

(a) the volume of S is $\frac{17\pi}{6}$ m^3

(3)

(b) the centre of mass of S is $\frac{49}{85}$ m from O.

(5)

(a) $V = \pi\int_0^1 y^2\,dx = \pi\int_0^1 (1+\sqrt{x})^2\,dx = \pi\int_0^1 (1 + 2x^{1/2} + x)\,dx$

$= \pi\left[x + \frac{4}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^1$

$= \pi\left(1 + \frac{4}{3} + \frac{1}{2}\right) - 0$

$\therefore$ volume of $S = \frac{17\pi}{6}$ m^3

(b) $\pi\int_0^1 xy^2\,dx = \pi\int_0^1 x(1+\sqrt{x})^2\,dx = \pi\int_0^1 (x + 2x^{3/2} + x^2)\,dx$

$= \pi\left[\frac{1}{2}x^2 + \frac{4}{5}x^{5/2} + \frac{1}{3}x^3\right]_0^1$

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Question 1 continued

$= \pi\left(\frac{1}{2} + \frac{4}{5} + \frac{1}{3}\right) = \frac{49\pi}{30}$

$\bar{x} = \frac{\pi\int xy^2\,dx}{\pi\int y^2\,dx} = \frac{49\pi/30}{17\pi/6} = \frac{49 \times 6}{30 \times 17} = \frac{294}{510}$

$\therefore$ centre of mass of S is $\frac{49}{85}$ m from O (on the x-axis)

(Total for Question 1 is 8 marks)

2.



Figure 2

A light elastic string AB has natural length l and modulus of elasticity $2mg$.

The end A of the elastic string is attached to a fixed point. The other end B is attached to a particle of mass m. The particle is held in equilibrium, with the elastic string taut and horizontal, by a force of magnitude F. The line of action of the force and the elastic string lie in the same vertical plane. The direction of the force makes an angle α, where $\tan\alpha = \frac{3}{4}$, with the upward vertical, as shown in Figure 2.

Find, in terms of l, the length AB.

(6)

$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$ Let T = tension in AB.

$R(\uparrow)$: $F\cos\alpha = mg$

$R(\rightarrow)$: $F\sin\alpha = T$

Dividing: $T = mg\tan\alpha = \frac{3}{4}mg$

Hooke's law, extension x: $T = \frac{2mgx}{l}$

$\frac{3}{4}mg = \frac{2mgx}{l} \Rightarrow x = \frac{3l}{8}$

$AB = l + \frac{3l}{8} = \frac{11l}{8}$

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Question 2 continued

(Total for Question 2 is 6 marks)

3.

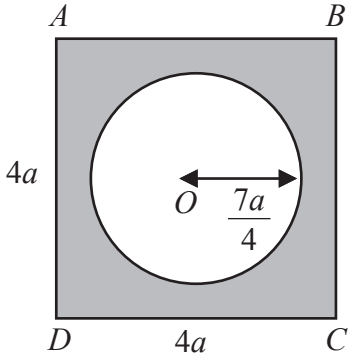


Figure 3

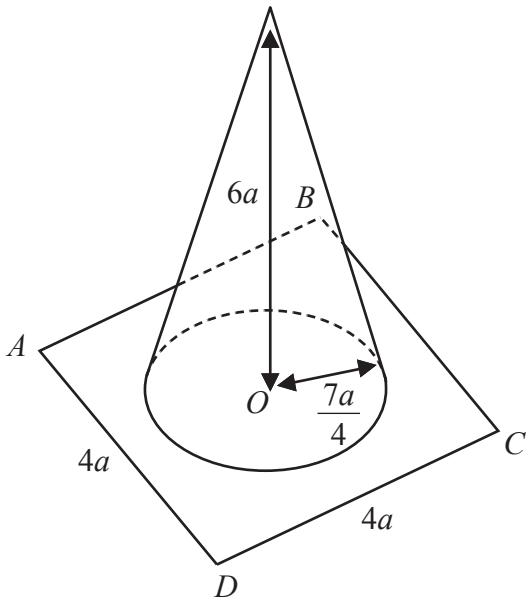


Figure 4

A square $ABCD$ of side $4a$ is made from thin uniform cardboard. The centre of the square is O. A circle with centre O and radius $\frac{7a}{4}$ is then removed from the square to form a template T, shown shaded in Figure 3.

A right conical shell, with no base, has radius $\frac{7a}{4}$ and perpendicular height $6a$.

The shell is made of the same thin uniform cardboard as T.

The shell is attached to T so that the circumference of the end of the shell coincides with the circumference of the circle centre O, to form the hat H, shown in Figure 4.

[*The surface area of a right conical shell of radius r and slant height l is* πrl.]

(a) Show that the exact distance of the centre of mass of H from O is

$$\frac{175\pi a}{(63\pi + 128)}$$

(8)

A fixed rough plane is inclined to the horizontal at an angle α. The hat H is placed on the plane, with $ABCD$ in contact with the plane, and AB parallel to a line of greatest slope of the plane. The plane is sufficiently rough to prevent the hat from sliding down the plane.

Given that the hat is on the point of toppling,

(b) find the exact value of $\tan \alpha$, giving your answer in simplest form.

(2)

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Question 3 continued

(a) Slant height $l = \sqrt{(7a/4)^2 + (6a)^2} = \sqrt{625a^2/16} = \frac{25a}{4}$

Mass proportional to area. Distances of centres of mass above O:

Square: $16a^2$ distance 0

Circle: $\pi(\frac{7a}{4})^2 = \frac{49\pi a^2}{16}$ distance 0

Conical shell: $\pi \times \frac{7a}{4} \times \frac{25a}{4} = \frac{175\pi a^2}{16}$ distance $\frac{1}{3} \times 6a = 2a$

Hat H: $16a^2 - \frac{49\pi a^2}{16} + \frac{175\pi a^2}{16} = 16a^2 + \frac{126\pi a^2}{16}$ distance $\bar{x}$

Moments about O:

$$\frac{175\pi a^2}{16} \times 2a = \left(16a^2 - \frac{49\pi a^2}{16} + \frac{175\pi a^2}{16}\right)\bar{x}$$

$\times 16$: $350\pi a^3 = (256a^2 + 126\pi a^2)\bar{x}$

$$\bar{x} = \frac{350\pi a}{256 + 126\pi} = \frac{175\pi a}{128 + 63\pi}$$

$\therefore$ distance of centre of mass of H from O $= \frac{175\pi a}{(63\pi + 128)}$

(b) On the point of toppling, the centre of mass G is vertically above the lowest edge of ABCD, which is 2a from O along the plane.

$$\tan\alpha = \frac{2a}{\bar{x}} = 2a \times \frac{63\pi + 128}{175\pi a}$$

$$\tan\alpha = \frac{126\pi + 256}{175\pi}$$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4. **In this question you must show all stages in your working.**

Solutions relying entirely on calculator technology are not acceptable.

A particle P is moving along the x-axis.

At time t seconds, where $0 \leqslant t \leqslant \frac{2}{3}$, P is x metres from the origin O and is moving with velocity v m s^{-1} in the positive x direction where

$$v = (2x + 1)^{\frac{3}{2}}$$

When $t = 0$, P passes through O.

(a) Find the value of x when the acceleration of P is 243 m s^{-2}

(4)

(b) Find v in terms of t.

(6)

(a) $a = v\frac{dv}{dx} = (2x + 1)^{3/2} \times \frac{3}{2}(2x + 1)^{1/2} \times 2$

$a = 3(2x + 1)^2$

$3(2x + 1)^2 = 243 \Rightarrow (2x + 1)^2 = 81$

$2x + 1 = 9$ ($2x + 1 = -9$ gives $x = -5$, rejected since $x \geqslant 0$)

$x = 4$

(b) $v = \frac{dx}{dt} = (2x + 1)^{3/2}$

$\int dt = \int (2x + 1)^{-3/2}\, dx$

$t = \frac{(2x + 1)^{-1/2}}{(-1/2) \times 2} + C = -(2x + 1)^{-1/2} + C$

$t = 0, x = 0$: $0 = -1 + C \Rightarrow C = 1$

$t = 1 - (2x + 1)^{-1/2} \Rightarrow (2x + 1)^{-1/2} = 1 - t$

$(2x + 1)^{1/2} = \frac{1}{1 - t}$

$v = (2x + 1)^{3/2} = \frac{1}{(1 - t)^3}$

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Question 4 continued

(Total for Question 4 is 10 marks)

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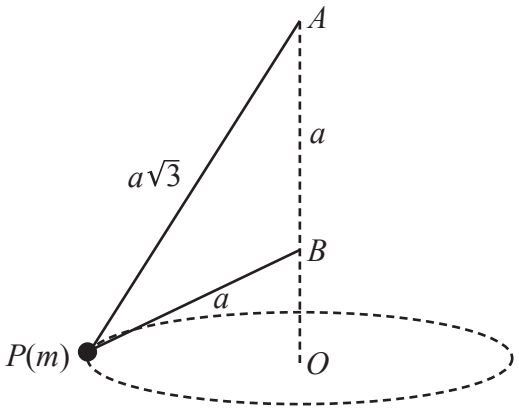


Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length $a\sqrt{3}$. The other end of the string is attached to a fixed point A. The particle P is also attached to one end of a second light inextensible string of length a. The other end of this string is attached to a fixed point B, where B is vertically below A, with $AB = a$.

The particle P moves in a horizontal circle with centre O, where O is vertically below B.

The particle P moves with constant angular speed ω, with both strings taut, as shown in Figure 5.

(a) Show that the upper string makes an angle of 30° with the downward vertical and the lower string makes an angle of 60° with the downward vertical.

(2)

(b) Show that the tension in the upper string is $\frac{1}{2}m\sqrt{3}(2g - a\omega^2)$.

(8)

(c) Show that $\frac{2g}{3a} < \omega^2 < \frac{2g}{a}$

(4)

(a) In triangle APB: AB = a, BP = a, AP = a√3. Cosine rule:

$\cos \angle PAB = \frac{(a\sqrt{3})^2 + a^2 - a^2}{2 \times a\sqrt{3} \times a} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \angle PAB = 30^\circ$

AB is vertical, so the upper string makes 30° with the downward vertical.

BA = BP so the triangle is isosceles: ∠APB = ∠PAB = 30°

∠ABP = 180° − 30° − 30° = 120°, so BP makes 180° − 120° = 60°

with the downward vertical.

(b) Let T_A = tension in AP, T_B = tension in BP.

Radius $r = a\sqrt{3} \sin 30^\circ = a \sin 60^\circ = (\sqrt{3}/2)a$

R(↑): $T_A \cos 30^\circ + T_B \cos 60^\circ = mg$

$\Rightarrow \sqrt{3}\, T_A + T_B = 2mg$ (1)

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Question 5 continued

$R(\leftarrow)$ towards O: $T_A \sin 30° + T_B \sin 60° = mr\omega^2$

$\Rightarrow \quad \frac{1}{2} T_A + \frac{\sqrt{3}}{2} T_B = m \times \frac{\sqrt{3}}{2} a \times \omega^2$

$\Rightarrow \quad T_A + \sqrt{3}\, T_B = \sqrt{3}\, ma\omega^2 \qquad (2)$

From (1): $T_B = 2mg - \sqrt{3}\, T_A$. Substitute in (2):

$T_A + \sqrt{3}(2mg - \sqrt{3}\, T_A) = \sqrt{3}\, ma\omega^2$

$T_A + 2\sqrt{3}\, mg - 3T_A = \sqrt{3}\, ma\omega^2$

$2T_A = 2\sqrt{3}\, mg - \sqrt{3}\, ma\omega^2 = \sqrt{3}\, m(2g - a\omega^2)$

$\therefore \quad T_A = \frac{1}{2} m\sqrt{3}(2g - a\omega^2)$

(c) Both strings taut: $T_A > 0$ and $T_B > 0$

$T_A > 0$: $\frac{1}{2} m\sqrt{3}(2g - a\omega^2) > 0 \quad \Rightarrow \quad \omega^2 < \frac{2g}{a}$

$T_B = 2mg - \sqrt{3} \times \frac{1}{2} m\sqrt{3}(2g - a\omega^2) = 2mg - 3mg + \frac{3}{2} ma\omega^2$

$T_B = \frac{3}{2} ma\omega^2 - mg$

$T_B > 0$: $\frac{3}{2} ma\omega^2 > mg \quad \Rightarrow \quad \omega^2 > \frac{2g}{3a}$

$\therefore \quad \frac{2g}{3a} < \omega^2 < \frac{2g}{a}$

Question 5 continued

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Question 5 continued

(Total for Question 5 is 14 marks)

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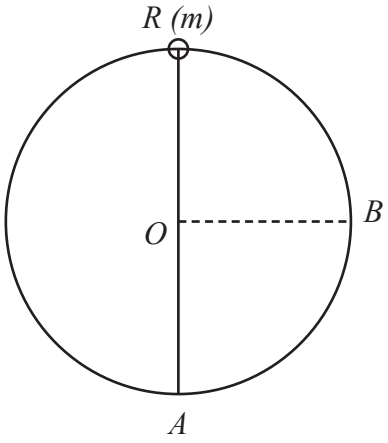


Figure 6

A small smooth ring R of mass m is threaded on to a smooth wire in the shape of a circle with centre O and radius l. The wire is fixed in a vertical plane. The ring R is attached to one end of a light elastic string of natural length l and modulus of elasticity mg. The other end of the elastic string is attached to A, the lowest point of the wire. The point B is on the wire and OB is horizontal.

The ring R is at rest at the highest point of the wire, as shown in Figure 6.

The ring R is slightly disturbed from rest and slides along the wire.

At the instant when R reaches the point B, the speed of R is v and the magnitude of the force exerted on R by the wire is N.

(a) Show that

$$v^2 = 2gl\sqrt{2}$$

(7)

(b) Show that

$$N = \frac{1}{2}mg\left(5\sqrt{2} - 2\right)$$

(7)

(a) At the top: AR = 2l, extension = l.

At B: AB = $\sqrt{(l^2 + l^2)} = l\sqrt{2}$, extension = $l\sqrt{2} - l$.

EPE at top = $\frac{mg\,l^2}{2l} = \frac{mgl}{2}$ EPE at B = $\frac{mg(l\sqrt{2} - l)^2}{2l}$

From top to B, R falls a height l. Conservation of energy:

$\frac{1}{2}mv^2 + \frac{mg(l\sqrt{2} - l)^2}{2l} = mgl + \frac{mg\,l^2}{2l}$

$(l\sqrt{2} - l)^2 = l^2(2 - 2\sqrt{2} + 1) = l^2(3 - 2\sqrt{2})$

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Question 6 continued

$\frac{1}{2}v^2 + \frac{gl(3 - 2\sqrt{2})}{2} = gl + \frac{gl}{2}$

$v^2 = 2gl + gl - 3gl + 2\sqrt{2}\,gl$

$\therefore\ v^2 = 2gl\sqrt{2}$

(b) Tension at B: $T = \frac{mg(l\sqrt{2} - l)}{l} = mg(\sqrt{2} - 1)$

BA makes 45° with BO. Towards O (horizontally), N towards O:

$N + T\cos 45° = \frac{mv^2}{l}$

$N + mg(\sqrt{2} - 1) \times \frac{\sqrt{2}}{2} = \frac{m}{l} \times 2gl\sqrt{2}$

$N = 2\sqrt{2}\,mg - mg\frac{2 - \sqrt{2}}{2} = 2\sqrt{2}\,mg - mg + \frac{\sqrt{2}}{2}mg$

$N = \frac{5\sqrt{2}}{2}mg - mg$

$\therefore\ N = \frac{1}{2}mg(5\sqrt{2} - 2)$

Question 6 continued

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Question 6 continued

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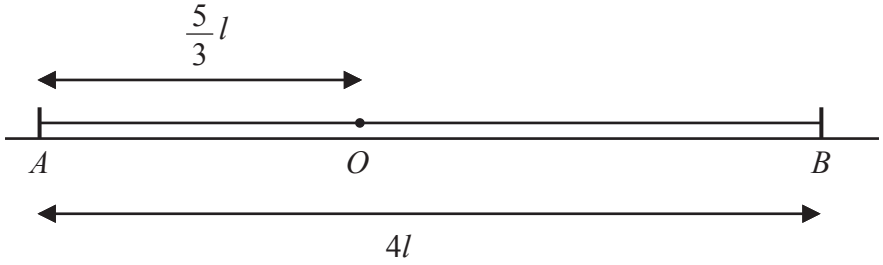


Figure 7

Two points A and B lie on a smooth horizontal table where $AB = 4l$.

A particle P of mass m is attached to one end of a light elastic spring of natural length l and modulus of elasticity $2mg$. The other end of the spring is attached to A. The particle P is also attached to one end of another light elastic spring of natural length l and modulus of elasticity mg. The other end of the spring is attached to B.

The particle P rests in equilibrium on the table at the point O, where $AO = \frac{5}{3}l$, as shown in Figure 7.

The particle P is moved a distance $\frac{1}{2}l$ along the table, from O towards A, and released from rest.

(a) Show that P moves with simple harmonic motion of period T, where

$$T = 2\pi\sqrt{\frac{l}{3g}}$$

(6)

(b) Find, in terms of l and g, the speed of P as it passes through O.

(1)

(c) Find, in terms of g, the maximum acceleration of P.

(1)

(d) Find the exact time, in terms of l and g, from the instant when P is released from rest to the instant when P is first moving with speed $\frac{3}{4}\sqrt{gl}$

(5)

(a) At O: extension of AP $= \frac{5l}{3} - l = \frac{2l}{3}$, BP $= 4l - \frac{5l}{3} = \frac{7l}{3}$, extension $\frac{4l}{3}$

(Check: $T_{AP} = \frac{2mg}{l} \times \frac{2l}{3} = \frac{4mg}{3} = \frac{mg}{l} \times \frac{4l}{3} = T_{BP}$)

Let P be displaced x from O towards A. N2L towards A:

$T_{AP} - T_{BP} = m\ddot{x}$

$\frac{2mg}{l}\left(\frac{2l}{3} - x\right) - \frac{mg}{l}\left(\frac{4l}{3} + x\right) = m\ddot{x}$

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Question 7 continued

$\frac{4mg}{3} - \frac{2mgx}{l} - \frac{4mg}{3} - \frac{mgx}{l} = m\ddot{x}$

$\ddot{x} = -\frac{3g}{l}x$

This is of the form $\ddot{x} = -\omega^2 x$, so P moves with SHM about O.

$\omega = \sqrt{(3g/l)}$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{(3g/l)}} = 2\pi\sqrt{(l/(3g))}$

(b) Amplitude $= \frac{1}{2}l$. Max speed (at O) $= a\omega = \frac{1}{2}l \times \sqrt{(3g/l)} = \frac{1}{2}\sqrt{3gl}$

(c) Max acceleration $= a\omega^2 = \frac{1}{2}l \times \frac{3g}{l} = \frac{3g}{2}$

(d) $x = \frac{l}{2}\cos \omega t \Rightarrow v = -\frac{l}{2}\omega \sin \omega t$

Speed $\frac{3}{4}\sqrt{gl}$: $\frac{l}{2}\sqrt{(3g/l)} \sin \omega t = \frac{3}{4}\sqrt{gl}$

$\frac{\sqrt{3}}{2}\sqrt{gl} \sin \omega t = \frac{3}{4}\sqrt{gl} \Rightarrow \sin \omega t = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

First time: $\omega t = \frac{\pi}{3} \Rightarrow t = \frac{\pi}{3} \times \sqrt{(l/(3g))}$

$\therefore$ time $= \frac{\pi}{3}\sqrt{(l/(3g))}$ s

Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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