Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. A truck of mass 1500 kg is moving on a straight horizontal road.
The engine of the truck is working at a constant rate of 30 kW.
The resistance to the motion of the truck is modelled as a constant force of magnitude R newtons.
At the instant when the truck is moving at a speed of $20\,\text{m s}^{-1}$, the acceleration of the truck is $0.6\,\text{m s}^{-2}$

(a) Find the value of R.

(4)

Later on, the truck is moving up a straight road that is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{8}$

The resistance to the motion of the truck from non-gravitational forces is modelled as a constant force of magnitude 500 N.
The engine of the truck is again working at a constant rate of 30 kW.
At the instant when the speed of the truck is $V\,\text{m s}^{-1}$, the deceleration of the truck is $0.2\,\text{m s}^{-2}$

(b) Find the value of V

(4)

(a) Driving force $D = \frac{P}{v} = \frac{30\,000}{20} = 1500$ N

N2L: $D - R = ma$

$\frac{30\,000}{20} - R = 1500 \times 0.6$

$1500 - R = 900$

$R = 600$

(b) Driving force $= \frac{30\,000}{V}$ $\sin\alpha = \frac{1}{8}$

N2L up the slope (deceleration 0.2 m s^{-2}):

$\frac{30\,000}{V} - 1500g \sin\alpha - 500 = 1500 \times (-0.2)$

$\frac{30\,000}{V} - 1500 \times 9.8 \times \frac{1}{8} - 500 = -300$

$\frac{30\,000}{V} = 1837.5 + 500 - 300 = 2037.5$

$V = \frac{30\,000}{2037.5} = 14.723...$

$V = 14.7$

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Question 1 continued

(Total for Question 1 is 8 marks)

2. A particle P of mass 0.5 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$
The particle receives an impulse $(-2\mathbf{i} + \lambda\mathbf{j})\,\text{N s}$, where λ is a constant.
Immediately after receiving the impulse, the velocity of P is $(x\mathbf{i} + y\mathbf{j})\,\text{m s}^{-1}$
The kinetic energy gained by P as a result of receiving the impulse is 22 J.

Find the possible values of λ.

(7)

Impulse = change in momentum:

$0.5(xi + yj) - 0.5(5i + 3j) = -2i + \lambda j$

$0.5(x - 5) = -2 \Rightarrow x = 1$

$0.5(y - 3) = \lambda \Rightarrow y = 3 + 2\lambda$

Change in KE:

$\frac{1}{2} \times 0.5(x^2 + y^2) - \frac{1}{2} \times 0.5(5^2 + 3^2) = 22$

$0.25(x^2 + y^2) - 8.5 = 22$

$x^2 + y^2 = 122$

$1^2 + (3 + 2\lambda)^2 = 122$

$(3 + 2\lambda)^2 = 121$

$3 + 2\lambda = \pm 11$

$2\lambda = 8 \quad \text{or} \quad 2\lambda = -14$

$\lambda = 4 \quad \text{or} \quad \lambda = -7$

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Question 2 continued

(Total for Question 2 is 7 marks)

3.

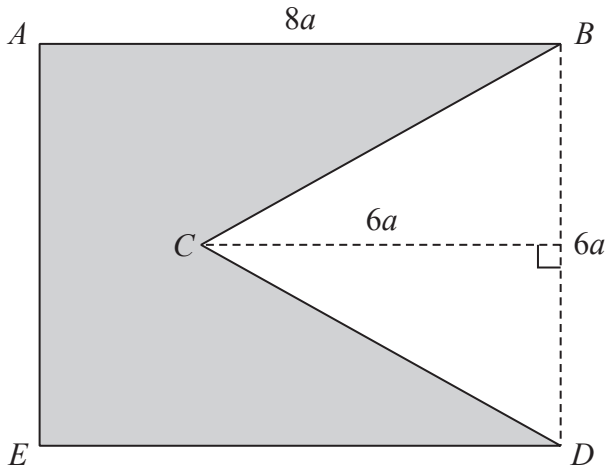


Figure 1

The uniform lamina $ABDE$ is in the shape of a rectangle with $AB = 8a$ and $BD = 6a$.
The triangle BCD is isosceles and has base $6a$ and perpendicular height $6a$.
The template $ABCDE$, shown shaded in Figure 1, is formed by removing the triangular lamina BCD from the lamina $ABDE$.

(a) Show that the centre of mass of the template is $\frac{14}{5}a$ from AE.

(5)

The template is freely suspended from A and hangs in equilibrium with AB at an angle of $\theta°$ to the downward vertical.

(b) Find the value of θ, giving your answer to the nearest whole number.

(3)

(a) Take mass per unit area as constant, so mass ∝ area.

	area	distance of centre of mass from AE
Rectangle	$8a \times 6a = 48a^2$	$4a$
Triangle	$\frac{1}{2} \times 6a \times 6a = 18a^2$	$8a - \frac{1}{3}(6a) = 6a$
Template	$48a^2 - 18a^2 = 30a^2$	$\bar{x}$

$M(AE)$: $48a^2 \times 4a - 18a^2 \times 6a = 30a^2 \bar{x}$

$192a^3 - 108a^3 = 30a^2 \bar{x}$

$84a^3 = 30a^2 \bar{x}$

$\bar{x} = \frac{84}{30}a = \frac{14}{5}a$ from AE, as required

(b) By symmetry the centre of mass G is 3a from AB.

When suspended from A, AG is vertical.

Angle between AB and the vertical = angle GAB:

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Question 3 continued

$\tan \theta^\circ = \frac{3a}{2.8a} = 1.0714...$

$\theta = 46.97...$

$\theta = 47$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

4. [*In this question, the perpendicular unit vectors* **i** *and* **j** *are in a horizontal plane.*]

A particle Q of mass 1.5 kg is moving on a smooth horizontal plane under the action of a single force **F** newtons. At time t seconds $(t \geqslant 0)$, the position vector of Q, relative to a fixed point O, is **r** metres and the velocity of Q is **v** m s^{-1}
It is given that

$$\mathbf{v} = (3t^2 + 2t)\mathbf{i} + (t^3 + kt)\mathbf{j}$$

where k is a constant.

Given that when $t = 2$ particle Q is moving in the direction of the vector $\mathbf{i} + \mathbf{j}$

(a) show that $k = 4$

(2)

(b) find the magnitude of **F** when $t = 2$

(4)

Given that $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ when $t = 0$

(c) find **r** when $t = 2$

(4)

(a) At $t = 2$ moving parallel to $i + j$, so i and j components of v are equal:

$3(2)^2 + 2(2) = (2)^3 + 2k$

$12 + 4 = 8 + 2k$

$2k = 8 \Rightarrow k = 4$ as required

(b) $v = (3t^2 + 2t)i + (t^3 + 4t)j$

$a = \frac{dv}{dt} = (6t + 2)i + (3t^2 + 4)j$

$t = 2$: $a = 14i + 16j$

$F = ma = 1.5(14i + 16j)$

$|F| = 1.5\sqrt{(14^2 + 16^2)} = 1.5\sqrt{452} = 3\sqrt{113}$

$|F| = 31.9$ N (31.890...)

(c) $r = \int v\,dt = (t^3 + t^2)i + \left(\frac{1}{4}t^4 + 2t^2\right)j + C$

$t = 0, r = 3i + 4j \Rightarrow C = 3i + 4j$

$r = (t^3 + t^2 + 3)i + \left(\frac{1}{4}t^4 + 2t^2 + 4\right)j$

$t = 2$: $r = (8 + 4 + 3)i + (4 + 8 + 4)j$

$r = 15i + 16j$

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Question 4 continued

(Total for Question 4 is 10 marks)

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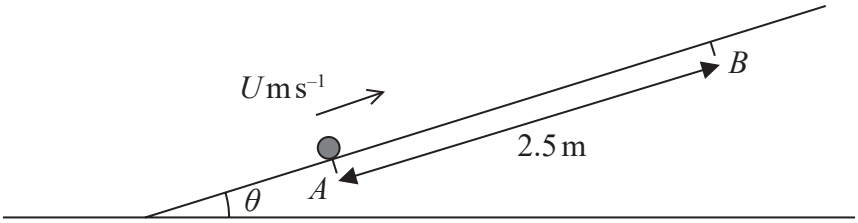


Figure 2

A rough straight ramp is fixed to horizontal ground. The ramp is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

The points A and B are on a line of greatest slope of the ramp, with $AB = 2.5\,\text{m}$ and B above A, as shown in Figure 2.

A package of mass 1.5 kg is projected up the ramp from A with speed $U\,\text{m s}^{-1}$ and first comes to instantaneous rest at B.

The coefficient of friction between the package and the ramp is $\frac{2}{7}$

The package is modelled as a particle.

(a) Find the work done against friction as the package moves from A to B.

(3)

(b) Use the work–energy principle to find the value of U.

(4)

After coming to instantaneous rest at B, the package slides back down the slope.

(c) Use the work–energy principle to find the speed of the package at the instant it returns to A.

(3)

(a) $\tan\theta = \frac{5}{12} \Rightarrow \sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

R(perpendicular to ramp): $R = 1.5g\cos\theta = 1.5 \times 9.8 \times \frac{12}{13} = 13.569\ldots$ N

$F = \mu R = \frac{2}{7} \times 13.569\ldots = 3.8769\ldots$ N

Work done against friction = $F \times 2.5 = 3.8769\ldots \times 2.5 = 9.6923\ldots$

Work done against friction = 9.69 J

(b) Work–energy, A to B:

Initial KE = gain in PE + work done against friction

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Question 5 continued

$\frac{1}{2} \times 1.5U^2 = 1.5 \times 9.8 \times 2.5 \sin\theta + 9.6923...$

$0.75U^2 = 1.5 \times 9.8 \times 2.5 \times \frac{5}{13} + 9.6923...$

$0.75U^2 = 14.134... + 9.6923... = 23.826...$

$U^2 = 31.769... \Rightarrow U = 5.636...$

$U = 5.64$

(c) Work–energy, B to A:

Loss in PE = gain in KE + work done against friction

$1.5 \times 9.8 \times 2.5 \times \frac{5}{13} = \frac{1}{2} \times 1.5v^2 + 9.6923...$

$14.134... = 0.75v^2 + 9.6923...$

$0.75v^2 = 4.4423... \Rightarrow v^2 = 5.923...$

$v = 2.433...$

Speed at A = 2.43 m s^{-1}

Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

6.

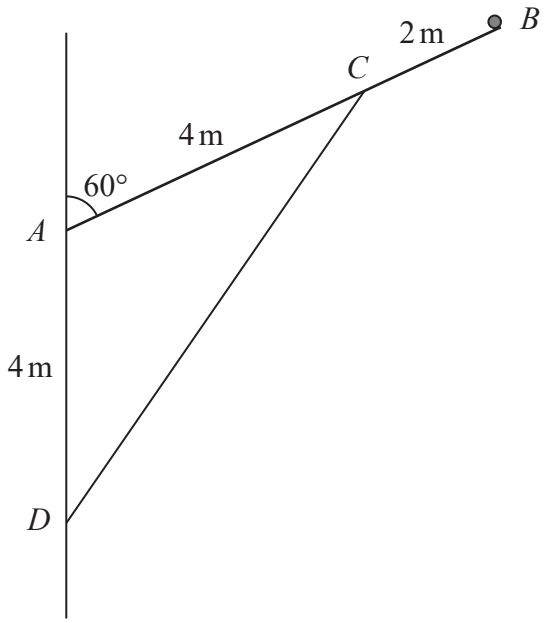


Figure 3

A uniform pole AB, of weight 50 N and length 6 m, has a particle of weight W newtons attached at its end B. The pole has its end A freely hinged to a vertical wall.
A light rod holds the particle and pole in equilibrium with the pole at 60° to the wall.
One end of the light rod is attached to the pole at C, where $AC = 4$ m.
The other end of the light rod is attached to the wall at the point D.
The point D is vertically below A with $AD = 4$ m, as shown in Figure 3.

The pole and the light rod lie in a vertical plane which is perpendicular to the wall.

The pole is modelled as a rod.

Given that the thrust in the light rod is $60\sqrt{3}$ N,

(a) show that $W = 15$

(4)

(b) find the magnitude of the resultant force acting on the pole at A.

(6)

(a) Angle DAC = 180° − 60° = 120° and AC = AD = 4 m,
so triangle ACD is isosceles with angle ACD = angle ADC = 30°.
Rod makes 30° with the vertical; perpendicular distance from A to DC = 4 sin 30° = 2 m.
Pole makes 30° with the horizontal.

M(A): 50 × 3 cos 30° + W × 6 cos 30° = 60√3 × 4 sin 30°
75√3 + 3√3 W = 120√3
3W = 45
W = 15 as required

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Question 6 continued

(b) Let the force at A have components V (up) and H (horizontal, away from wall).

Thrust $T = 60\sqrt{3}$ N acts along DC at C, at 30° to the vertical.

R(↑): $V + T\cos 30° = 50 + 15$

$V + 60\sqrt{3} \times \frac{\sqrt{3}}{2} = 65 \Rightarrow V + 90 = 65 \Rightarrow V = -25$

so vertical component has magnitude 25 N (acting downwards)

R(→): $H + T\sin 30° = 0$

$H = -60\sqrt{3} \times \frac{1}{2} = -30\sqrt{3}$ (magnitude $30\sqrt{3} = 51.96...$ N, towards the wall)

Magnitude of resultant force at A:

$|R| = \sqrt{(25^2 + (30\sqrt{3})^2)} = \sqrt{(625 + 2700)} = \sqrt{3325}$

$|R| = 5\sqrt{133} = 57.662...$

$|R| = 57.7$ N

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

7. Particle P has mass $3m$ and particle Q has mass km. The particles are moving towards each other on the same straight line on a smooth horizontal surface.
The particles collide directly.
Immediately **before** the collision, the speed of P is $2u$ and the speed of Q is $3u$.
Immediately **after** the collision, the speed of P is u and the speed of Q is v.

The direction of motion of P is unchanged by the collision.

(a) Show that $v = \dfrac{(3 - 3k)}{k} u$

(3)

(b) Find, in terms of m and u, the magnitude of the impulse received by Q in the collision.

(2)

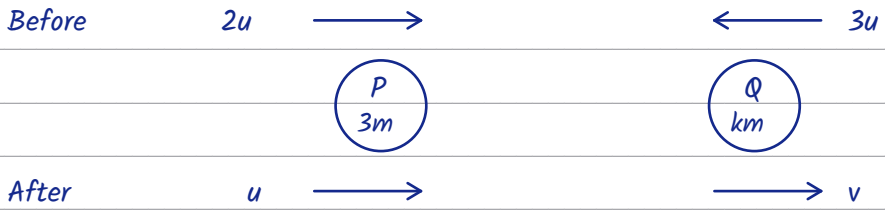
The coefficient of restitution between P and Q is e.

Given that $v \neq u$

(c) find the range of possible values of k.

(5)

(a)



CLM (direction of motion of P positive):

$3m(2u) - km(3u) = 3m(u) + km(v)$

$6u - 3ku = 3u + kv$

$kv = 3u - 3ku = (3 - 3k)u$

$v = \dfrac{(3 - 3k)}{k} u$ as required

(b) Impulse on Q = − impulse on P

Impulse on P = $3m(u - 2u) = -3mu$

Magnitude of impulse received by Q = $3mu$

(c) NEL: speed of separation = e × speed of approach

$v - u = e(2u + 3u) = 5ue$

$\dfrac{(3 - 3k)}{k} u - u = 5ue \Rightarrow e = \dfrac{3 - 4k}{5k}$

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Question 7 continued

$v \neq u$ and Q must move away faster than P, so $v > u$ $(e > 0)$:

$$\frac{3-3k}{k} > 1 \Rightarrow 3-3k > k \ (k > 0) \Rightarrow k < \frac{3}{4}$$

$$e \leq 1: \quad v - u \leq 5u \Rightarrow \frac{3-3k}{k} \leq 6 \Rightarrow 3-3k \leq 6k \Rightarrow k \geq \frac{1}{3}$$

Range: $\frac{1}{3} \leq k < \frac{3}{4}$

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Question 7 continued

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Question 7 continued

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8. A particle P is projected from a fixed point O. The particle is projected with speed $u\,\text{m s}^{-1}$ at angle α above the horizontal. The particle moves freely under gravity. At the instant when the horizontal distance of P from O is x metres, P is y metres vertically above the level of O.

(a) Show that $y = x\tan\alpha - \dfrac{gx^2}{2u^2}(1 + \tan^2\alpha)$

(6)

A small ball is projected from a fixed point A with speed $U\,\text{m s}^{-1}$ at $\theta°$ above the horizontal.
The point B is on horizontal ground and is vertically below the point A, with $AB = 20\,\text{m}$.
The ball hits the ground at the point C, where $BC = 30\,\text{m}$, as shown in Figure 4.

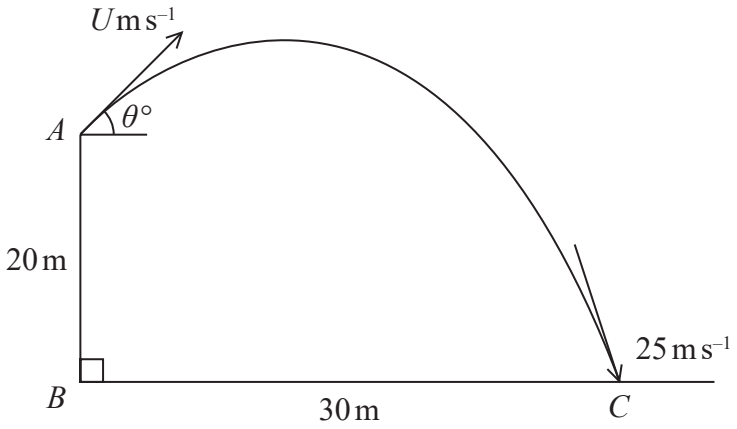


Figure 4

The speed of the ball immediately before it hits the ground is $25\,\text{m s}^{-1}$
The motion of the ball is modelled as that of a particle moving freely under gravity.

(b) Use the principle of conservation of mechanical energy to find the value of U.

(3)

(c) Find the value of θ

(3)

(a) Horizontal: $x = (u\cos\alpha)t \Rightarrow t = \dfrac{x}{u\cos\alpha}$

Vertical: $y = (u\sin\alpha)t - \frac{1}{2}gt^2$

Substitute for t:

$y = u\sin\alpha \times \dfrac{x}{u\cos\alpha} - \frac{1}{2}g\left(\dfrac{x}{u\cos\alpha}\right)^2$

$y = x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha}$

$y = x\tan\alpha - \dfrac{gx^2}{2u^2}\sec^2\alpha$

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Question 8 continued

Since $\sec^2 \alpha = 1 + \tan^2 \alpha$:

$y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2 \alpha)$ as required

(b) Conservation of mechanical energy, A to C (mass m):

$\frac{1}{2}mU^2 + mg(20) = \frac{1}{2}m(25)^2$

$\frac{1}{2}U^2 + 9.8 \times 20 = \frac{1}{2} \times 625$

$U^2 = 625 - 392 = 233$

$U = \sqrt{233} = 15.264...$

$U = 15.3$

(c) Using (a) with $x = 30$, $y = -20$, $u^2 = 233$, $\alpha = \theta°$:

$-20 = 30 \tan \theta - \frac{9.8 \times 30^2}{2 \times 233}(1 + \tan^2 \theta)$

$-20 = 30 \tan \theta - 18.927...(1 + \tan^2 \theta)$

$18.927... \tan^2 \theta - 30 \tan \theta - 1.0729... = 0$

$\tan \theta = \frac{30 \pm \sqrt{(30^2 + 4 \times 18.927... \times 1.0729...)}}{2 \times 18.927...}$

$\tan \theta = 1.6200...$ or $\tan \theta = -0.0349...$ (reject, $\theta°$ is above horizontal)

$\theta = 58.314...$

$\theta = 58.3$

Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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